

# Combinatorial Optimization by Cooperative Mechanism of Ant Colony and Aphid

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### Abstract

In this study, we propose an optimization method by the cooperative mechanism of ant and aphid as a new Ant Colony Optimization (ACO). This algorithm is named Ant Colony Optimization with Cooperative Aphid (ACOCA). In ACOCA algorithm, the aphid searches neighborhood solutions. This solution information is treated as a honey obtained from the aphid and the honey affects the search of ACO. Moreover, the aphid shares a solution information with the ant. We apply ACOCA to three Traveling Salesman Problems (TSPs) and confirm its effectiveness.

### 1. Introduction

The ant takes communication with other ants at feeding action by using a pheromone and find the shortest path from feeds to a nest. Ant Colony Optimization (ACO) was developed by this mechanism [1]. ACO is applied to various combination optimization problems, for example, Traveling Salesman Problem (TSP) [2], vehicle routing problem, graph coloring problem, and so on. In these problems, TSP is solved by various optimization algorithms. We provide a simple explanation about TSP. City coordinates are given and the salesman leaves first city, visits all cities, and returns to the first city. TSP is the problem which finds the shortest tour in the combination of the tours.

In ACO algorithm for solving TSP, a number of ants exist and they find a number of solutions. Ants trail a pheromone on the path between cities. The pheromone is updated depending on behavior of ants, and the ants find tours according to the strength of the pheromone trail on the path. In other words, when the ants take communication with other ants through the pheromone trail on the path, ACO can find the optimal solution. However, ACO has a problem of being easy to fall into a local minimum solution. Therefore, it is important to adjust the diversity of the algorithm and to improve performance.

By the way, there are ants which live in cooperation with aphids [3]. The aphid has a number of natural enemies and is

easily attacked because it has little defensive power. Therefore, in order to accommodate with natural enemies, the aphid has an interesting mechanism. The mechanism is a cooperative relation with the ant. The aphid sucks at nutrients from a plant and discharges them from the anus as honey. The aphid gives honey to ants. Since honey is a favorite food of ants, the ants obtain much honey in return for defend aphid from natural enemies [4]. In this way, there is cooperative relation in an ant and an aphid. Moreover, although the aphid is eaten by a lot of natural enemies, it has tremendous procreative power. If the number of the aphids increases too much, it sucks all nutrients of the plant. Because the plant which sucked all nutrients withers, the aphid cannot survive. Therefore, if a number of the aphid increases too much, it is carried to another plant by the ant and begins to propagate on new plant. The aphid repeats this step and propagates.

In this study, we propose Ant Colony Optimization with Cooperative Aphid (ACOCA) and we apply the feature of the aphid and this cooperative relation to optimization algorithm. In ACOCA algorithm, ant and aphid operate in parallel and give solution. The solution information obtained from aphid is treated as a honey and the honey affects the search solutions of ant. We apply ACOCA which have the cooperative relation of ant and aphid to three TSPs and confirm its effectiveness.

# 2. ACO [5]

ACO finds the solutions by repeating the following processes.

1. Ants are set on the cities at random.

2. Each ant choose a path by the probability based on pheromone and heuristic information.

3. Pheromone is updated depending on obtained solutions.

Figure 1 shows the choosing method of the path by the ant. In this figure, the ant tends to choose the path which is near and has many quantity of pheromone.

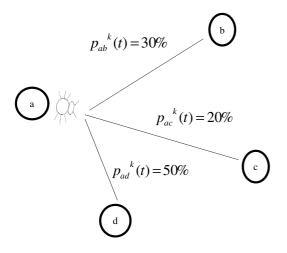


Figure 1: The ant tends to choose the path which is near and has many quantity of pheromone.

# 3. ACOCA

#### 3.1. Feature of ACOCA

In this study, we adopt the following three cooperative relations.

- 1. Ant likes honey of aphid.
- 2. Aphid propagates and increases gradually.

3. Instead of honey, ant carries aphid to feeding area (plant). In order to take in these three elements to the algorithm, we consider the following three features on a program.

1. The aphid performs local search, and searches the neighborhood solution of the present solution. There are various methods such as simulated annealing [6], hill-climbing [7], and tabu search [8] in a local search. Search of aphid are based on simulated annealing. The transition probability of the solution changes by the number of aphids. When the number of aphids is small, it is equal to random search. When the number of them increases, it changes from random search to local search. Moreover, the evaluation value of the solution obtained from the aphid called the honey. This honey affects the search of ant. When the ant chooses a path, "honey" is added as new information and affects choosing path of ant. Although honey is similar to the pheromone, search of aphid is unaffected by honey. Moreover, the aphid is unaffected by a pheromone.

2. Express the number of aphids as the search times. Moreover, the number of aphids increases while searching for solution. The ant and aphid operate in parallel and give solution. Although an ant gives only one solution, an aphid gives plural solutions.

3. The number of aphids increases while searching for the solution, but if it increases to a constant number, the aphid

will be moved by an ant. Movement of the aphid means receiving the solution of ant. At this time, the number of aphids decreases.

### 3.2. Algorithm of ACOCA

### [Step 1 (Initialization)]

Let iteration number t = 0.  $\tau_{ij}(t)$  is the quantity of pheromone trail on the path (ij) between cities *i* and *j* at time *t*, and  $\tau_{ij}(t)$  is initially set to  $\tau_0$ .  $\epsilon i j(t)$  is the quantity of honey trail on the path (ij) between cities *i* and *j* at time *t*, and  $\epsilon_{ij}(t)$  is initially set to  $\epsilon_0$ .

[Step 2 (Find tour)]

· Find tour of ACO:

For the k-th ant  $(k = 1, 2, \dots, M)$ , the visiting city is chosen by probability  $p_{ij}^k(t)$ . ACOCA uses two kinds of  $p_{ij}^k(t)$ according to conditions. The k-th ant finds a tour according to the following

$$p_{ij}^{k}(t) = \begin{cases} \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}[\epsilon_{ij}]}{\sum_{h=\Omega}[\tau_{ih}]^{\alpha}[\eta_{ih}]^{\beta}[\epsilon_{ih}]}, & \text{if } L_{a} \ge L_{A} \\ \frac{[\eta_{ij}]^{\beta}[\epsilon_{ij}]^{\gamma}}{\sum_{h=\Omega}[\eta_{ih}]^{\beta}[\epsilon_{ih}]^{\gamma}}, & \text{otherwise}, \end{cases}$$
(1)

where  $L_A$  is the best tour length of ants at t,  $L_a$  is the best tour length of CA at t.  $\Omega$  is a set of cities which ants are not visiting yet.  $\tau$  is quantity of pheromone,  $\eta$  is heuristic value (inverse the distance between cities) and  $\epsilon$  is quantity of honey.  $\alpha$ ,  $\beta$ , and  $\gamma$  are the control parameters of  $\tau$ ,  $\eta$ , and  $\epsilon$ , respectively.

· Find tour of CA:

1: Cities are set at random and tours are created. This tour length is the present state *s*.

2: Exchange two of the cities of city permutation at random and tours are created. This behavior is conducted for the number of cities, and minimum length of tours is the neighborhood state e.

3: Compare e with s and the present state is updated according to the result. CA finds a tour according to the following equations;

$$s_b = s(t), \quad if \ s_b \ge s(t) \tag{2}$$

$$s(t+1) = \begin{cases} e, & s(t) \ge e, \\ e, & s(t) < e, : follow the prob. \underline{p_a(t)} \\ s(t), & s(t) < e, : follow the prob. \underline{p_a(t)} \end{cases}$$
(3)

where  $s_b$  is the best state,  $p_a$  is transition probability and is shown by Eq. (4).

$$p_a(t) = \exp \frac{-l_{max}}{l_{start} - l_{max}},\tag{4}$$

where  $l_{max}$  is the maximum number of aphid,  $l_{start}$  is the default number of aphid. l is updated by Eq. (5).

$$\begin{cases} l(t+1) = l(t) + 1, \\ l(t+1) = l_{start}, & if \ l(t) = l_{max}, \end{cases}$$
(5)

where l is the number of aphids at t. When under equation is applied and t is bigger than  $t_e$ , the best tour of ant at t is given to the aphid. It performs local search for the given tour.

# [Step 3 (Pheromone update)]

Compute the tour length  $L_A(t)$  and update the quantity of the pheromone trail between i and j by the following equation;

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^{M} \Delta \tau_{ij}^{k}(t),$$
(6)

$$\Delta \tau_{ij}^k(t) = \begin{cases} 1/L_k, & if \ tour \ made \ by \ ant \\ 0, & otherwise, \end{cases}$$
(7)

where  $\rho \in [0, 1]$  is the evaporation of pheromone trail coefficient. The pheromone trail of all path evaporates and the pheromone is added to the path.

### [Step 4 (Honey update)]

Compute the tour length  $L_a(t)$  and update the quantity of the honey by the following equation;

$$\epsilon_{ij}(t+1) = 1/L_a,\tag{8}$$

where  $\epsilon$  is the quantity of honey.

### [Step 5]

Let t = t + 1. Go to [ACOCA 2] and repeat until the maximum time limit  $t = t_{max}$ .

## 4. Numerical Simulations

In order to confirm the effectiveness of ACOCA, we apply ACOCA to three TSPs, att48, st70 and kroA100. The number of simulations of att48, st70 and kroA100 is 100 times.

### [Parameter setup]

M (number of ants): 48 (att48), 70 (st70), 100 (kroA100),

- t (iteration count) = 2200,
- $\alpha$  (control parameter of pheromone) = 1,
- $\beta$  (control parameter of heuristic value) = 5,
- $\gamma$  (control parameter of honey)= 1~9,

 $\rho$  (evaporation coefficient) = 0.3,

$$l_{start}$$
 (the default number of aphid) = 400,

 $l_{max}$  (the maximum number of aphid) = 600,

$$t_e = 1500.$$

In order to investigate the effect of honey,  $\gamma = 1 \sim 9$  are used, and we confirm the optimal value of  $\gamma$ . Moreover, ACOCA uses the upper equation in Eq. (1) until t=500. Because the honey has a bad influence on ACO in the early step.

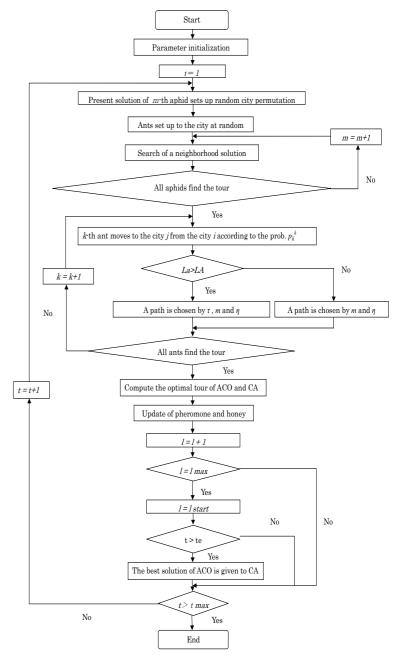


Figure 2: Flowchart of ACOCA.

Table 1: Results of ACOCA:  $\gamma = 1 \sim 9$ , for att48, st70 and kroA100.

			att48	st70	kroA100
	1	AVG	1.60 %	4.29 %	11.14 %
		MIN	0 %	0.40 %	7.76 %
	2	AVG	1.39 %	4.98 %	11.07 %
		MIN	0 %	1.96 %	8.39 %
$\gamma$	3	AVG	1.40 %	3.90 %	11.02 %
		MIN	0 %	0 %	3.16 %
	4	AVG	1.37 %	3.78 %	10.62 %
		MIN	0 %	0 %	2.93 %
	5	AVG	1.53 %	4.02 %	11.14 %
		MIN	0 %	1.00 %	6.45 %
	6	AVG	1.57 %	4.06 %	10.94 %
		MIN	0 %	0.10 %	6.63 %
	7	AVG	1.39 %	3.95 %	11.22 %
		MIN	0 %	0.48 %	7.50 %
	8	AVG	1.48 %	3.95 %	11.28 %
		MIN	0.10 %	1.25 %	7.02 %
	9	AVG	1.43 %	4.11 %	11.05 %
		MIN	0 %	0.74 %	5.41 %

Table 2: Results of the standard ACO and ACOCA for att48, st70 and kroA100.

		att48	st70	kroA100
ACO	AVG	2.59 %	5.41 %	12.49 %
	MIN	1.65 %	4.25 %	10.36 %
ACOCA	AVG	1.37 %	3.78 %	10.62 %
	MIN	0 %	0 %	2.93 %

### [Simulation results]

The simulation result of  $\gamma = 1 \sim 9$  is shown in Table 1. The numerical value in the Table 1 is the average error and minimum error of solution obtained from simulations. The minimum error is given by Eq. (9).

$$MIN = \frac{(Best \ tour \ in \ all \ sims) - \ (Opt. \ tour)}{Opt. \ tour} \times 100$$
(9)

AVG is an average error of all simulations. From Table 1, it turns out that  $\gamma = 4$  is the best parameter in att48, st70

and kroA100. Moreover, the simulation result of ACO and ACOCA is shown in Table 2. In Table 2, we can confirm that ACOCA obtains better results than the standard ACO in three TSPs. From this result, we consider that honey has a good effect on search solution of ACO. Moreover, although the standard ACO can not find the optimal tour, ACOCA can find the optimal tour by exchange of solution information.

# 5. Conclusion

In this study, we proposed an optimization method by the cooperative mechanism of ant and aphid as new ACO. This algorithm is named Ant Colony Optimization with Cooperative Aphid (ACOCA) and ACOCA obtained the better results than the standard ACO. From this result, we can say that ACOCA is more effective algorithm than the standard ACO for solving TSPs.

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