

## A Robust Pose Estimation Method for Nearly Coplanar Points

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**Abstract** The pose of a calibrated camera can be estimated from known correspondences with known scene structure. There is a number of pose estimation algorithms which solve this problem, however, all of these algorithms suffer from pose ambiguity. This paper analyses the reason of pose ambiguity from the aspect of the distribution of object points. Then, we develop a novel robust method to solve pose ambiguity for nearly coplanar points, through analyzing the influence of coplanar points and non-coplanar points comprehensively. Simulation experiments and the real image results have demonstrated the effectiveness of our new algorithm.

**Keywords** Pose Estimation, Pose Ambiguity, Coplanar Points

### 1. Introduction

The aim of pose estimation is to determine the position and orientation between the camera and the object. It has many applications in computer vision, such as hand-eye robot systems, augmented reality, photogrammetry and so on.

In the literatures of pose estimation, several approaches have also been proposed. Most of them work for 3D points configurations, and some have been extended to coplanar points configurations. It is a degenerate configuration when the object points are coplanar and pose ambiguity will arise. Oberkamp et al. have also discussed these pose ambiguities. They give a straightforward interpretation and develop their algorithm for planar points. Schweghofer and Pinz analyse the general case of perspective projection. They choose the correct pose with minimum re-projection error from two possible

solutions. However, as the noise increases, all the two possible solutions will be very close to each other, therefore, the rate of wrong pose will increase regardless of the selected strategy.

In this paper, we present a thorough analysis of the reason of pose ambiguity. We point out that pose ambiguity will also exist for nearly coplanar points. In this situation, the traditional pose estimation algorithms will result to large re-projection error and the coplanar algorithms cannot be adopted directly. Then we show that pose ambiguity has a compact relationship with the distribution of object points, not only for coplanar points but also for 3D object point configurations, in Section 2. Section 3 proposes a robust novel algorithm to calculate the pose. Section 4 presents experimental results thorough testing on both synthetic and real-data and comparison with state-of-the-art pose algorithms. Conclusions are drawn in Section 5.

### 2. Pose Ambiguity Description

In general, for algorithms which use objective function optimization, there are two kinds of objective function because of different projection methods. One is image space error function:

$$E_{is}(R, T) = \sum_{i=1}^N \left[ \left( x_i - \frac{R_1 \cdot p_i + T_x}{R_3 \cdot p_i + T_z} \right)^2 + \left( y_i - \frac{R_2 \cdot p_i + T_y}{R_3 \cdot p_i + T_z} \right)^2 \right] \quad (1)$$

And another is object space error function:

$$E_{os}(R, T) = \sum_{i=1}^N \left\| (I - V_i)(Rp_i + T) \right\|^2, \quad V_i = \frac{v_i v_i^T}{v_i^T v_i} \quad (2)$$

Pose ambiguity problem denotes situations where  $E_{is}$  or  $E_{os}$  have several local minima for a given configuration. We now illustrate pose ambiguity for a comprehensive example. For a

special object scene, which has five points, we change the distance  $\|t\|$  between camera and the object ceaselessly and keep all the other parameter fixed, and then we observe the variation trend of  $E_{OS}$  around some axis, as depicted in Fig.1.

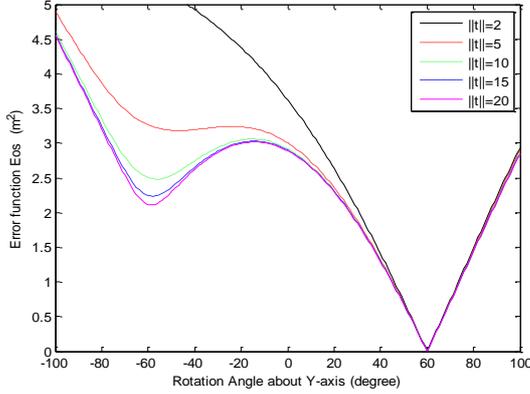


Fig.1. Error function for varying distances  $\|t\|$ .

As we know, when all the object points are coplanar, there will be two possible poses at most, and this situation is called coplanar pose ambiguity. In Fig.1 we can see that, with the increase of  $\|t\|$ , the second minimum becomes increasing significance. Hence, for the specific configuration of object points, pose ambiguity will come up, especially when the image or object noise level is heavy.

### 3. Interpretation and Robust Algorithm

In this section, we firstly present a thorough analysis of pose ambiguities. Then we show that pose ambiguities have a compact relationship with the distribution of object points, not only for coplanar points but also for three-dimensional points. Finally, we propose a robust novel algorithm to calculate the position and orientation between the object and the camera. Using the new algorithm, pose parameter can be uniquely determined and the pose ambiguity can be avoided.

We adopt perspective projection camera model to interpret the pose ambiguity, with the projection equation as follows:

$$l_i \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = RP_i + T = \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} P_i \\ 1 \end{bmatrix} \quad (3)$$

where  $P_i = [X_i \ Y_i \ Z_i]^T$  and  $v_i = l_i[x_i \ y_i \ 1]^T$  are the corresponding object point coordinates and normalized image coordinates. We can write projection function in a matrix form,

$$V = [R \ T] \tilde{P} = A \tilde{P} \quad (4)$$

where  $V = [v_1, v_2, \dots, v_N]$ ,  $\tilde{P} = [\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_N]$  and  $A = [R, T]$  is the pose to be determined.

From the equation (4), we can find that the stability of pose A has a compact relationship with the condition number of  $\tilde{P}$ . According to the theory of linear equations, the more condition number, the less stability of equations. In other words, the robustness of pose estimation has a direct relationship with the configuration of object points. When the object points have different distribution, the stability of solution will also have some difference, and this is the root cause of pose ambiguity.

According to the above interpretation, we proposed a novel robust method to solve pose ambiguities for nearly coplanar points, through analyzing the influence of coplanar points and non-coplanar points comprehensively. Firstly, the singular value decomposition in the object point set is done, and then a part of the object points, corresponding to the largest singular value and the second largest singular value, is selected as the main plane points. Secondly, using the scale orthogonal projection model, the pose can be estimated. As the presence of pose ambiguity, there are, in general, two local minima in most configurations of object points. Finally, a verification step will be adopted, using the remaining set of object points. The re-projection error is calculated according to these two local minima, in image space and object space respectively. Then the unique pose, according to the minimum re-projection error,

can be chosen.

## 4. Results

We compare the effect of our approach against that of state-of-the-art ones, both on simulated and real image data.

### 4.1 Synthetic Experiments

For all synthetic experiments, we use the following setup:

- 1) There are 9 object points, where 8 points are coplanar. The distance  $\|t\|=10$ .
- 2) For each test, we generate three random angle of rotation  $R$ , the interval is  $[-180,180]$ .
- 3) The corresponding image points are calculated through rotation  $R$  and translation  $T$ , and Gaussian noise is added.

For each noise level, we run 1000 independent simulations for each algorithm, such as Lu's algorithm, Fiore's algorithm and our novel algorithm. Fig.2 shows the rate of correct pose.

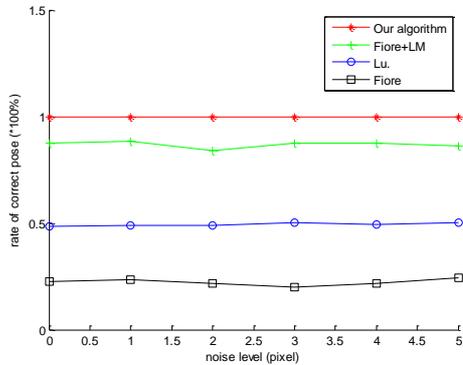


Fig.2. Rate of correct pose for different algorithms.

In Fig.2, we can see that for different noise level, the Lu's algorithm has a correct rate about 50 percent, it is because the Lu's algorithm gets the pose through the minimums of the object space error function  $E_{OS}$ . According to the above analysis, when object points have a nearly coplanar configuration, there are two minimums of  $E_{OS}$ . Therefore, for the case of nearly coplanar configuration, Lu's algorithm chooses the correct pose for two solutions, and the rate of finding the correct pose is just about 50 percent. The Fiore algorithm is a linear pose estimation method, when the object points are nearly

coplanar, the result is extremely unstable. Though analyzing the condition number of the matrix which is constructed by object points, we find that it has a order of magnitude about  $10^4$ , thus the correct rate is extremely low. Fiore + LM algorithm is Fiore algorithm followed with LM optimization, In Fig.2 we find that the rate has a greatly improvement. The slightly less robust results (about 10 percent) can be explained: in a larger range, the result can be optimized to the correct pose, however, in some range it will reach to the other pose, which will lead to pose ambiguity. The rate of our new algorithm is always around 100 percent, these results demonstrate the effectiveness of our new algorithm, even if the object points have nearly coplanar configuration.

### 4.2. Real Image

We select a manual model to validate actual results of each algorithm, as depicted in Fig.3.

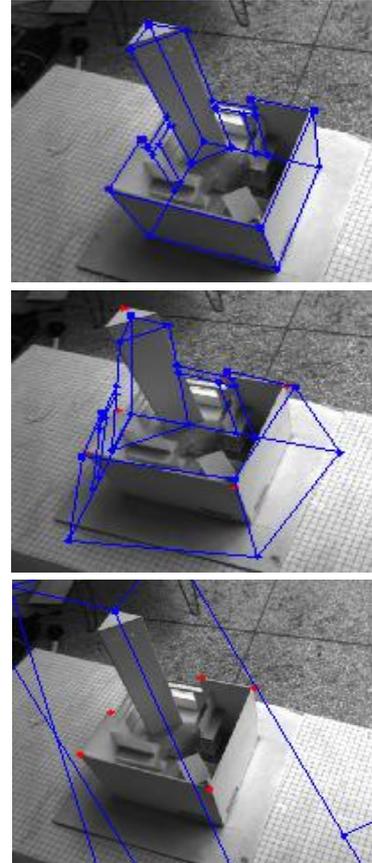


Fig.3. Experiment results: top: our algorithm; middle: Lu's algorithm; bottom: Fiore's algorithm.

We select six points from them to estimate

pose using the algorithm of Lu, Fiore and our new one to calculate pose respectively, and then project the model to the image plane.

The Fig.3 shows that Fiore algorithm has significant deviation; Lu. algorithm has a certain bias, which means this is a wrong minimum and our algorithm coincides with the model very well and then the parameters calculated by our algorithm is the correct pose. The real image results demonstrate the effectiveness of our new algorithm.

## 5. Conclusions

In this paper, we present a thorough analysis of the reason of pose ambiguities. Then, we develop a novel robust method to solve pose ambiguities for nearly coplanar points. According to the analysis, when we need to estimate pose in practice, the distribution of object points should be considered to estimate the possibility of pose ambiguity. When designing the configuration of object points, it would be better for the uniform distribution in all directions, which can decrease the rate of pose ambiguity and then increase the robustness of algorithm. If the configuration is confirmed, our novel algorithm can be used to ensure the robustness and precision of pose parameters. This novel algorithm should be relevant for many applications in AR and autonomous navigation.

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