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Synchronization of Switching Phenomena in Star-Coupled Chaotic System

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Abstract

Some chaotic circuits have coexisting attractors. Switching phenomena of coexisting attractors are observed in these circuits. However, there are a few of study that is focused on switching phenomenon. In our past study, synchronization of switching phenomena are confirmed in coupled chaotic systems which have some kinds of topologies.

In this study, we investigate an existence of the phenomena in the case of a star-coupled system.

1. Introduction

Many kinds of complex phenomena can be observed on the large-scale coupled chaotic circuits. Investigations of these phenomena are very important works in order to declare nonlinear phenomena in the natural world. An electric circuit is a models suitable for large-scale coupled nonlinear systems. The reason of why is follows. Getting electric parts is easy and inexpensive, experiment time is very short, repeatability of experiments is high and an electric circuit is a real physical system. Therefore, there are many studies of large-scale coupled circuit systems. In these studies, synchronization phenomena are attracted researcher's attentions.

On the other hand, some chaotic circuits have coexisting attractors. In these circuits, switching phenomena of attractors can be observed. Normally, in the case of a synchronization state, the switching of attractors is also synchronized. And in the case of asynchronous states, the switching of attractors is also asynchronized. However, synchronization of self-switching phenomena on full-coupled chaotic oscillators is reported by [1]. Additionally, we could observe similar phenomena in an other system [2][3].

In this study, synchronization of switching phenomena is investigated in a star-coupled chaotic circuits.

2. System Model

Figure 1 shows a circuit model used in this study. This circuit consists of a linear negative resister, two inductors, a capacitor and a bi-directionally coupled diodes. Figure 2 shows circuit experimental results. By increasing a value of the negative resistance, a periodic orbit (a), two types periodic orbits (b) and (c) which depending on initial values, two types chaotic orbits (d) and (e) which depending on initial values, chaotic orbit (f) which coexists of (d) and (e) are observed. Switching phenomena can be observed between two types chaotic orbits and the coexistence of two chaotic orbits.

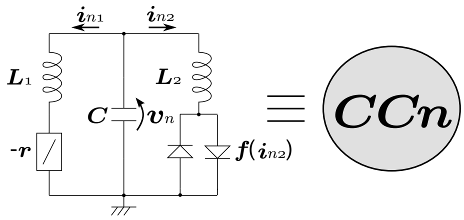

Figure 1: A circuit model

In order to carry out the computer simulations, a bi-directionally coupled diodes models as follows. The *i*-*v* characteristic of a bi-directionally coupled diodes is described as: $f(i_d) = 2Vu(i_d) - V$ and shown in Fig. 3.

By using this model, computer simulation results corresponding to Fig. 2 are obtained as shown in Fig. 4 The coexisting attractors are observed in Fig. 4 (c) which is corresponding to Fig. 2 (f). The attractors are classified by colors according to following definition. When the solution hits $y \geq 0.675$, $z = 1$ and $\dot{y} < 0$, then the color is set as blue. The other case, namely, the case of $y < 0.675$, $z = 1$ and $\dot{y} < 0$, the color is set as red. This definition is determined from our previous study [2]. This is applied to all simulation results of this study.

In our previous study, full-coupled, ladder and ring topology were applied to investigated system. In these system, synchronizations of switching phenomena are observed. However, all of them are equivalent. In this study, star-coupled chaotic system is investigated. This system is one-to-many coupling topology. One center element is not equivalent to others. It is very interesting to observe the synchronization of switching phenomena.

Figure 5 shows the star-coupled chaotic system. The system equation is described as follows:

$$\begin{cases} L_1 \dfrac{di_{11}}{dt} &= v_1 + ri_{11}, \\ L_2 \dfrac{di_{12}}{dt} &= v_1 - V\left(2u(i_{12}) - 1\right), \\ C\dfrac{dv_1}{dt} &= -\left(i_{11} + i_{12}\right) - G\left((N-1)v_1 - \displaystyle\sum_{k=2}^{N} v_k\right). \end{cases} \tag{1}$$

$$\begin{cases} L_1 \dfrac{di_{n1}}{dt} &= v_n + ri_{n1}, \\ L_2 \dfrac{di_{n2}}{dt} &= v_n - V\left(2u(i_{n2}) - 1\right), \\ C\dfrac{dv_n}{dt} &= -\left(i_{n1} + i_{n2}\right) - G\left(v_n - v_1\right). \\ & \quad (n = 2, 3, \cdots, N) \end{cases} \tag{2}$$

By changing parameters and variables as follows,

$$\begin{aligned} &t = \sqrt{L_1 C}\tau, \;\; i_{n1} = V\sqrt{\frac{C}{L_1}}x_n, \;\; i_{n2} = y_n, \\ &v_n = Vz_n, \;\; \text{“}\cdot\text{”} = \frac{d}{d\tau}, \;\; \alpha = r\sqrt{\frac{C}{L_1}}, \\ &\beta = \frac{\sqrt{L_1 C}}{L_2} \;\; \text{and} \;\; \delta = G\sqrt{\frac{L_1}{C}}, \end{aligned} \tag{3}$$

the system equation is described as follows:
A center circuit

$$\begin{cases} \dot{x}_1 &= \alpha x_1 + z_1 \\ \dot{y}_1 &= \beta\left\{z_1 - \left(2u(y_1) - 1\right)\right\} \\ \dot{z}_1 &= -x_1 - y_1 - \delta\left((N-1)z_1 - \displaystyle\sum_{k=2}^{N} z_k\right) \end{cases} \tag{4}$$

Other circuits

$$\begin{cases} \dot{x}_n &= \alpha x_n + z_n \\ \dot{y}_n &= \beta\left\{z_n - \left(2u(y_n) - 1\right)\right\} \\ \dot{z}_n &= -x_n - y_n - \delta\left(z_n - z_1\right) \\ & \quad (n = 2, 3, \cdots, N) \end{cases} \tag{5}$$

where N is a number of circuits, $u()$ is a step function and x_n, y_n and z_n are corresponding to i_{n1}, i_{n2} and v_n, respectively.

3. Computer Simulations

3.1. Case of $N = 2$

Figure 6 shows simulation result of $N = 2$. The horizontals show the time, the vertical show the z_n, y_n and $z_1 - z_n$. Red and blue waves are corresponding to the attractor states as shown in Fig. 4(a) and (b). Green lines show differences of attractor state. The synchronization of switching phenomena is observed in a black frame. Note that switching phenomena means transition of attractors in this paper. This system keeps asynchronous state at all times by reason that an amplitude of $z_1 - z_2$ is observed. Though asynchronous state is kept at this system, switching phenomenon is generated in both circuit at the same time. This phenomenon is very interesting at the point of the simultaneity in asynchronous behavior. We also confirmed this phenomena in circuit experiment [2].

We guess a mechanism of the phenomena qualitatively as follows. Basically, two circuits want to be synchronized with each other. Since the coupling elements are resistors. In order to be synchronized, a current flows through coupling elements. However, the current also flows diodes as shown in Fig. 6. Because both circuits have two symmetrical equilibrium points which are caused by $i-v$ characteristic of diodes, two attractors are attracted to reverse equilibrium points each other. Therefore, two attractors can not be synchronized.

3.2. Case of $N \geq 3$

Figure 7 shows the simulation result case of $N = 3$. The horizontals show the time, the vertical show the z_n, y_n and $z_1 - z_n$. The synchronization of switching phenomena is observed in a black frame. It seems that the switching phenomenon of CC_2 and CC_3 have occurred in accordance with CC_1 of it. We can also confirm the reverse currents with y_1 and y_2, y_1 and y_3 respectively. However there is a lag with the switching in each circuit.

We also investigate case of $N = 4, 5, 6$. Observation of synchronization of switching phenomena is a bit difficult in these case. One of the simulation results is shown in Fig. 8. The horizontal axes show the time, the vertical axes show the z_n, y_n and $z_1 - z_n$. Switching of CC_1 can be clearly observed. However, Switching of CC_2, CC_3 and CC_4 are not clear. We consider that the reason is as follows. The effect of the current from CC_1 to CC_n is decreased by increasing the number of the circuits because currents from CC_1 to each circuits are divided into $N-1$. In the case of $N \geq 7$, synchronization of switching phenomena could not be observed.

4. Conclusions

In this study, we have investigated synchronization of switching phenomena in star-coupled system. As results,

- Synchronization of switching phenomena can be observed in the cases of the number of circuits are from two to six.
- The phenomena can be observed clearly in the case of two and three.

- The observation of synchronization of switching phenomena becomes difficult by increasing the number of the circuits.

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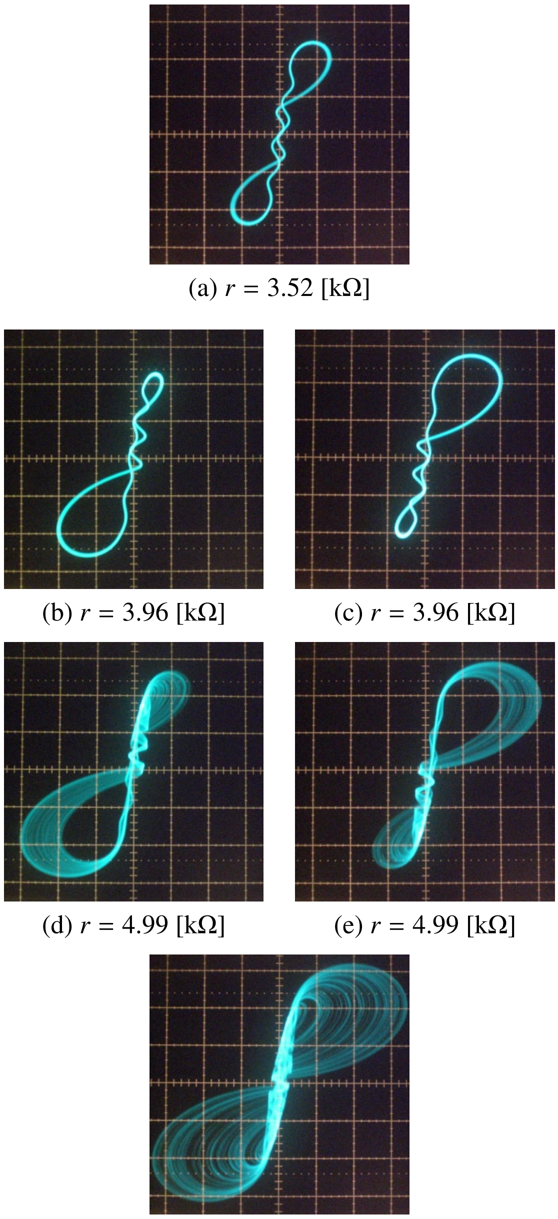


(a) r = 3.52 [kΩ]

(b) r = 3.96 [kΩ] (c) r = 3.96 [kΩ]

(d) r = 4.99 [kΩ] (e) r = 4.99 [kΩ]

(f) r = 5.29 [kΩ]

Figure 2: Circuit experimental results. Vertical axes: v_1 [V/div]. Horizontal axes: y_1 [0.1mA/div]. L_1 = 642.8[mH], L_2 = 211.1[mH] and C = 7.06[nF].

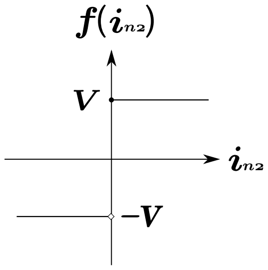
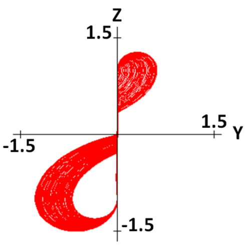
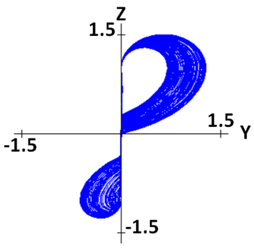


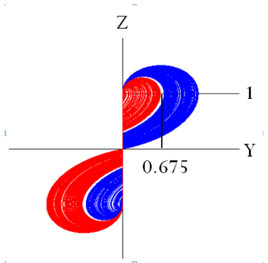
Figure 3: *i*-*v* characteristic



(a) $\alpha = 0.35$



(b) $\alpha = 0.35$



(c) $\alpha = 0.36$

Figure 4: Computer simulation results. $\beta = 3.0$.

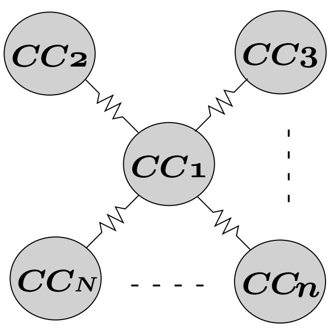


Figure 5: System model

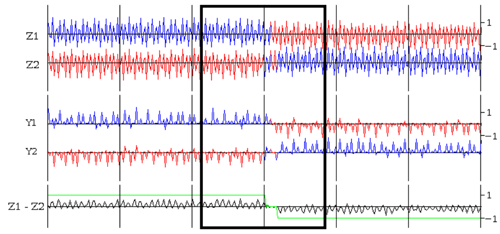


Figure 6: A computer simulation result. $N = 2$, $\alpha = 0.41$, $\beta = 3.0$ and $\delta = 0.26$.

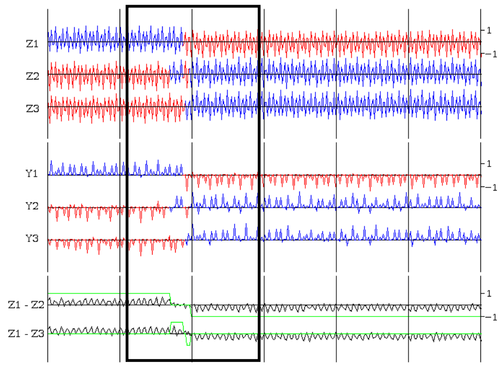


Figure 7: A computer simulation result. $N = 3$, $\alpha = 0.41$, $\beta = 3.0$ and $\delta = 0.26$.

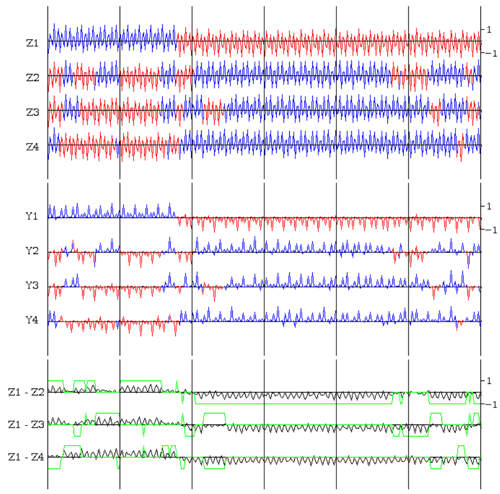


Figure 8: A computer simulation result. $N = 4$, $\alpha = 0.41$, $\beta = 3.0$ and $\delta = 0.24$.