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Double-Mode Simultaneous Oscillation in Three Coupled Hard Oscillators

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Abstract

In this study, synchronization phenomena observed from three inductively coupled hard oscillators with two resonators are investigated. Various interesting synchronization phenomena including in-phase and anti-phase synchronizations are generated by giving different initial conditions. Especially, we can confirm that double-mode and simultaneous oscillations are generated at the same time.

1. Introduction

There are a lot of synchronization phenomena in the natural environment. For example, swing of the pendulum, firefly luminescence, cardiac heartbeat, and so on, are well known as synchronization phenomena. Oscillators containing a nonlinear resistor whose $v-i$ characteristics are described by fifth-power nonlinear characteristics are known to exhibit hard excitation [1][2]. Namely, the origin is asymptotically stable and an proper initial condition, which is larger than a critical value, is necessary to generate the oscillation. Such an oscillator is often called as hard oscillator or said to have hard nonlinearity. Datardina and Linkens have investigated two identical oscillators with hard nonlinearities coupled by an inductor [3]. They have confirmed that nonresonant double-mode oscillations, which could not occur for the case of third-power nonlinearity, were stably excited in the coupled system. They have also confirmed that four different modes coexist for some range of parameter values; zero, two single-modes, and a double-mode. In 1954, Schaffner reported that an oscillator with two degrees of freedom could oscillate simultaneously at two different frequencies when the nonlinear characteristics are described by a fifth-power polynomial function [4]. Kuramitsu also investigated the simultaneous oscillations for three or more degrees case theoretically and confirmed the generation of simultaneous oscillation with three frequencies by circuit experiments [5]. The simultaneous oscillations are definitely one of the most common nonlinear phenomena observed in various higher-dimensional systems in the natural science fields. However, after their pioneering works, as far as the authors know, there have not been many researches clarifying the basic mechanism of the simultaneous oscillations except [6][7]. In our past study, we have reported synchronization phenomena observed from two resistively coupled hard oscillators [8] and two inductively coupled hard oscillators [9].

In this study, three inductively coupled hard oscillators including two resonators are investigated. By computer simulaitons, we can confirm that various synchronization phenomena including in-phase and anti-phase synchronizations can be generated in the coupled circuits. Especially, we can observe an interesting phenomenon that double-mode and simultaneous oscillations are generated at the same time. Namely, the neighboring oscillators generate double-mode oscillations, while the above and the below resonators generate simultaneous oscillations.

2. Circuit model

The circuit model is shown in Fig. 1. In the circuit, three hard oscillators with two LC resonators are coupled by inductors L_C and each hard oscillator consists of a nonlinear negative resistor, whose $v-i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \quad (g_1,\ g_3,\ g_5 > 0). \tag{1}$$

and two resonators with different natural frequencies (L_1C_1 and L_2C_2). The equations governing the coupled oscillators are described by the following 12th-order differential equa-

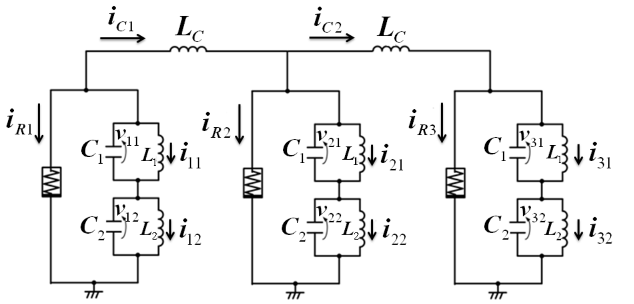

Figure 1: Three coupled hard oscillators with two resonators.

tions including three nonlinear functions i_{R1}, i_{R2} and i_{R3}.

$$\begin{cases} C_1\frac{dv_{11}}{dt} = -i_{11} - i_{R1} - i_{C1}, & L_1\frac{di_{11}}{dt} = v_{11} \\ C_2\frac{dv_{12}}{dt} = -i_{12} - i_{R1} - i_{C1}, & L_2\frac{di_{12}}{dt} = v_{12} \\ C_1\frac{dv_{21}}{dt} = -i_{21} - i_{R2} - i_{C2} + i_{C1}, & L_1\frac{di_{21}}{dt} = v_{21} \\ C_2\frac{dv_{22}}{dt} = -i_{22} - i_{R2} - i_{C2} + i_{C1}, & L_2\frac{di_{22}}{dt} = v_{22} \\ C_1\frac{dv_{31}}{dt} = -i_{31} - i_{R3} + i_{C2}, & L_1\frac{di_{31}}{dt} = v_{31} \\ C_2\frac{dv_{32}}{dt} = -i_{32} - i_{R3} + i_{C2}, & L_2\frac{di_{32}}{dt} = v_{32}, \end{cases} \tag{2}$$

where i_{C1} and i_{C2} are the current through the coupling inductors and are given as

$$i_{C1} = \frac{L_1(i_{11} - i_{21}) + L_2(i_{12} - i_{22})}{L_C} \tag{3}$$

$$i_{C2} = \frac{L_1(i_{21} - i_{31}) + L_2(i_{22} - i_{32})}{L_C}. \tag{4}$$

The currents through the nonlinear resistors i_{R1}, i_{R2} and i_{R3} are given as

$$\begin{cases} i_{R1} = i_R(v_{11} + v_{12}) \\ i_{R2} = i_R(v_{21} + v_{22}) \\ i_{R3} = i_R(v_{31} + v_{32}). \end{cases} \tag{5}$$

By using the following variables and parameters,

$$v_{mn} = \sqrt[4]{\frac{g_1}{5g_5}}x_{mn}, \quad i_{mn} = \sqrt[4]{\frac{g_1}{5g_5}}\sqrt{\frac{C_1}{L_1}}y_{mn},$$
$$\alpha_C = \frac{C_1}{C_2}, \quad \alpha_L = \frac{L_1}{L_2}, \quad \gamma = \frac{L_1}{L_C}, \tag{6}$$
$$\varepsilon = g_1\sqrt{\frac{L_1}{C_1}}, \quad \beta = \frac{3g_3}{g_1}\sqrt{\frac{g_1}{5g_5}}, \quad t = \sqrt{L_1C_1}\tau \ .$$

the normalized circuit equations are given as follows.

$$\begin{cases} \frac{dx_{11}}{d\tau} = -y_{11} - f(x_{11} + x_{12}) - y_{C1} \\ \frac{dx_{12}}{d\tau} = \alpha_C\{-y_{12} - f(x_{11} + x_{12}) - y_{C1}\} \\ \frac{dx_{21}}{d\tau} = -y_{21} - f(x_{21} + x_{22}) + y_{C1} - y_{C2} \\ \frac{dx_{22}}{d\tau} = \alpha_C\{-y_{21} - f(x_{21} + x_{22}) + y_{C1} - y_{C2}\} \\ \frac{dx_{31}}{d\tau} = -y_{31} - f(x_{31} + x_{32}) + y_{C2} \\ \frac{dx_{32}}{d\tau} = \alpha_C\{-y_{31} - f(x_{31} + x_{32}) + y_{C2}\} \\ \frac{dy_{m1}}{d\tau} = x_{m1} \qquad (m = 1, 2, 3) \\ \frac{dy_{m2}}{d\tau} = \alpha_L x_{m2} \qquad (m = 1, 2, 3) \end{cases} \tag{7}$$

where y_C corresponds to i_C and is given as

$$y_{C1} = \gamma\left(y_{11} - y_{21} + \frac{y_{12} - y_{22}}{\alpha_L}\right) \tag{8}$$

$$y_{C2} = \gamma\left(y_{21} - y_{31} + \frac{y_{22} - y_{32}}{\alpha_L}\right), \tag{9}$$

and the nonlinear function $f(\cdot)$ which corresponds to the $v-i$ characteristics of the nonlinear resistors is given as

$$f(x) = \varepsilon\left(x - \frac{\beta}{3}x^3 + \frac{1}{5}x^5\right) \ . \tag{10}$$

3. Synchronization Phenomena

The proposed circuits exhibit various types of synchronizations including double-mode and oscillation death. In this article, we show only several comupter simulated results obtaind by giving different initial conditions for the fixed parameters as $\alpha_C = 0.64$, $\alpha_L = 0.6$, $\gamma = 0.01$, $\varepsilon = 0.005$ and $\beta = 3.2$. Namely, all the following results coexist as well as some more unshown synchronization modes. The detailed explanation of all phenomena and their analysis will be presented elsewhere.

Figures 2, 3, 4, and 5 show four different kinds of synchronization modes observed from the circuit. In these figures, (a) show the Lissajous figures of six resonators. The positions of the six figures correspond to the positions of the resonators in Fig. 1. The horizontal axes are x_{mn} (voltages across capacitors) and the vertical axes are y_{mn} (currents through inductors). Figures (b) show the phase differences between resonators. The left four figures show the relationships between horizontal neighbors, while the right three figures show those between vertical neighbors. Both horizontal and vertical axes are x_{mn} (voltages across capacitors). Figures (c) are the time waveforms of six x_{mn} (voltages across capacitors).

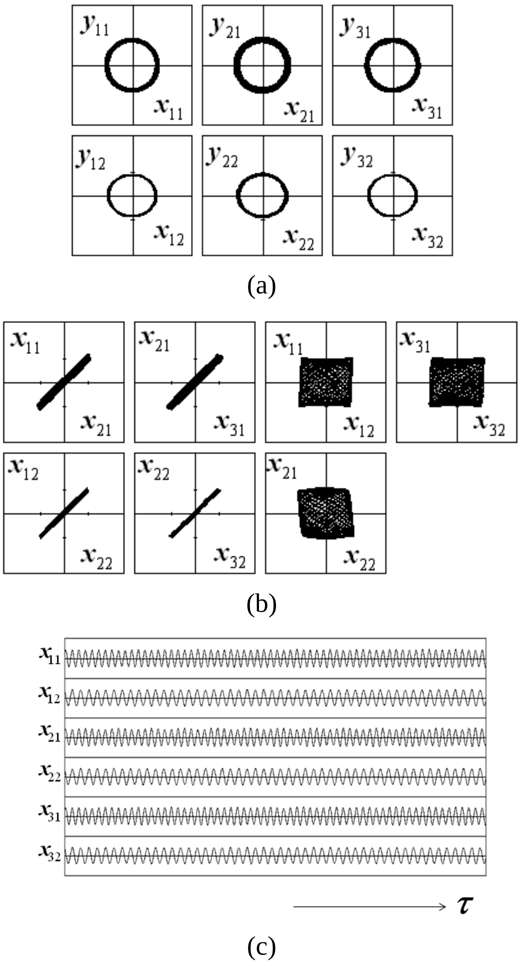

Figure 2: In-phase and in-phase simultaneous oscillation. (a) Lissajous figures of six resonators. (b) Phase differences between six resonators. (c) Time waveforms of six resonators.

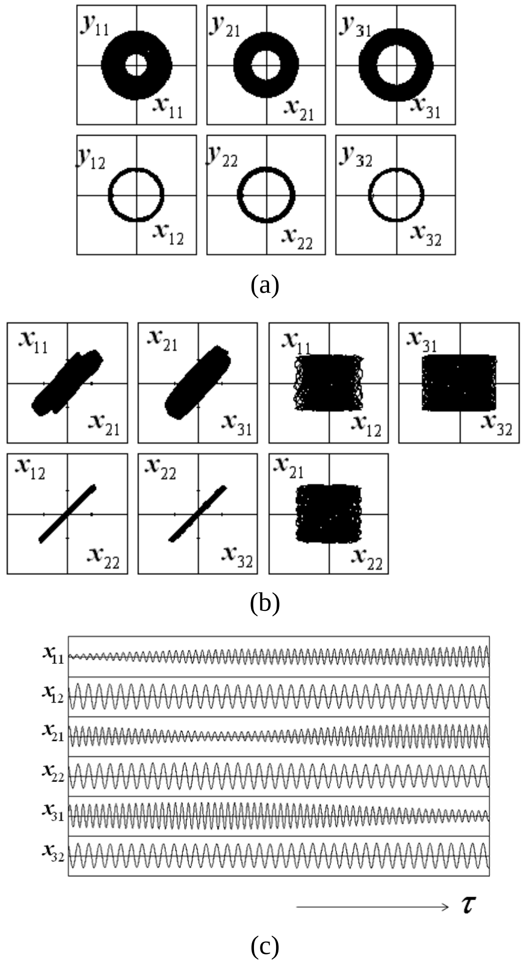

Figure 3: Double-mode and in-phase simultaneous oscillation. (a) Lissajous figures of six resonators. (b) Phase differences between six resonators. (c) Time waveforms of six resonators.

From these three types of figures, we can say that simultaneous oscillations are generated in Figs. 2, 3, and 5. Also, the upper resonators in Figs. 3 and 5 and the lower resonators in Figs. 4 and 5 exhibit double-mode oscillations. On the othr hand, the upper resonators in Fig. 4 stops their oscillations, although their lower pairs oscillate in double-mode. From these observations, we can conclude these four oscillation modes as follows:
Figure 2: Simultaneous oscillation of in-phase (upper) and in-phase (lower).
Figure 3: Simultaneous oscillation of double-mode (upper) and in-phase (lower).
Figure 4: Single oscillation of oscillation-death (upper) and double-mode (lower).
Figure 5: Simultaneous oscillation of double-mode (upper) and double-mode (lower).

Especially, we are interested in the last synchronization mode; double-mode and double-mode simultaneous oscillation, where the relationships between the horizontal neighbors are double-mode, while the relationships between the vertical neighbors are asynchronous. Because we believe that such a phenomenon exists in natural nonlinear systems with a large number of degrees of freedom and that its analysis contributes to unveil various unknown mysterious phenomena in the nature.

4. Conclusions

In this study, we have investigated the generation of various synchronization phenomena observed from three inductively coupled hard oscillators with two resonators. We could confirm the phenomenon that double-mode and simultaneous oscillation have been generated at the same time. Namely, the

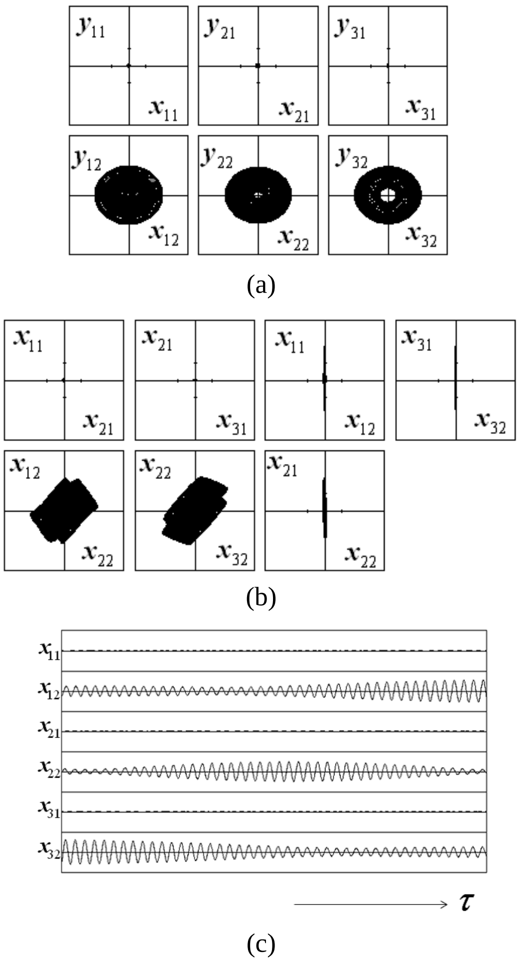

Figure 4: Oscillation-death and double-mode oscillation. (a) Lissajous figures of six resonators. (b) Phase differences between six resonators. (c) Time waveforms of six resonators.

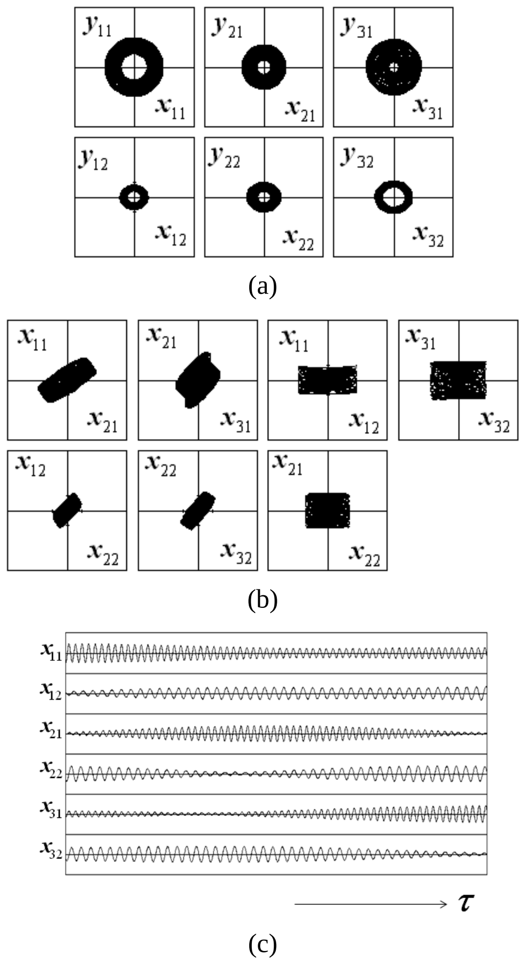

Figure 5: Double-mode and double-mode simultaneous oscillation. (a) Lissajous figures of six resonators. (b) Phase differences between six resonators. (c) Time waveforms of six resonators.

relationships between the horizontal neighbors are double-mode, while the relationships between the vertical neighbors are asynchronous.

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