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Analysis of Stochastic Resonance in Quadruple-Well Potential

Shintaro ARAI†, Shota OZAKI†, Takaya YAMAZATO‡ and Yoshifumi NISHIO*

†Kagawa National College of Technology
551 Kohda, Takuma-cho, Mitoyo, Kagawa, JAPAN
Phone:+81-875-83-8617
Email: arai@cn.kagawa-nct.ac.jp

‡Nagoya University
Furo-cho, Chikusa-ku, Nagoya, JAPAN
Phone:+81-52-747-6596
Email: yamazato@nuee.nagoya-u.ac.jp

* Tokushima University
2-1 Minami-Josanjima, Tokushima, JAPAN
Phone:+81-88-656-7470
Email: nishio@ee.tokushima-u.ac.jp

Abstract

This paper analyzes a stochastic resonance (SR) in a quadruple-well potential, which extended the SR in a double-well potential, to control of SR. For achieving the control of SR, it is important to analyze the state of the SR by injecting the noise. In this study, we calculate existence probabilities of the SR's state to analyze how the state changes by a noise intensity.

1. Introduction

Stochastic resonance (SR) is a nonlinear phenomenon in which a responsiveness of a system is improved by injecting suitable noise to certain nonlinear systems. Recently, SR has attracted a great deal of attention from a variety of researchers[1]–[7]. In this paper, we consider that SR is applied to a communication system which is one of engineering systems. In standard communication systems, the signal detection becomes difficult according to increase a noise level. By applying SR to the communication systems, we expect that it is possible to detect the signal which is influenced significantly by noise. For robustly detecting the signal in the communication system using SR, a control of SR by noise is very important task. Therefore, we consider that it is necessary to investigate a relationship between SR and noise.

Based on the above research background, this paper analyzes SR with various noise intensities. Especially, we focus on a state of SR depending on the noise intensity. In Fig. 1, we show a mechanism of SR model in a double-well potential which is well-known as a typical bistable SR model. Hear, the ball (particle) in this figure is a state of SR. In this model, the particle stays in the one potential well unless the noise, as shown in Fig. 1(a). However, a height of a potential barrier, which is a center of SR in Fig. 1(a), is changed by injecting the noise to SR model, and the particle hops from one potential well to the other one, as shown in Fig 1(b). Thus, we consider that it is important to analyze the state of SR by injecting the noise for achieving the control of SR.

In this study, we therefore calculate existence probabilities of the state (particle) to analyze how the state changes by the noise intensity. For analyzing the existence probabilities of the particle, this paper uses a SR in a quadruple-well potential which added one dimension in SR model in a double-well potential. By analyzing the existence probabilities of the particle in the SR with various noise intensities, we discuss the capability of the control of SR by the noise.

2. SR in double-well potential

We briefly explain SR in the double-well potential. In Ref. [7], the bistable SR is performed by following equations.

$$\frac{dx}{dt} = f(x) + Dn(t) + s(t), \tag{1}$$

$$s(t) = A\sin(2\pi f_0 t), \tag{2}$$

$$f(x) = -\frac{dU_0(x)}{dx}, \tag{3}$$

$$U_0(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4. \tag{4}$$

Where $U_0(x)$ is a bistable potential having two local minimums (i.e., two potential wells), x is a state variable, $s(t)$ is an input signal, A is an amplitude of the input signal, n is assumed to be the additive white Gaussian noise (AWGN), D is an intensity of noise. An effective potential $U(x,t)$ is described as follows.

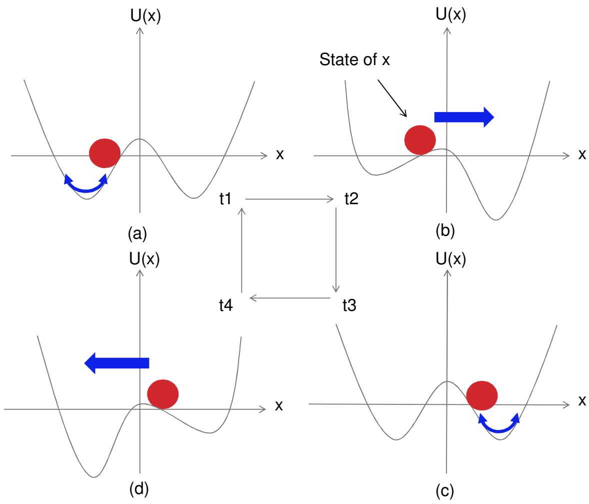


Figure 1: Mechanism of SR in double-well potential.

$$U(x,t) = U_0(x) + xs(t) = -\frac{1}{2}x^2 + \frac{1}{4}x^4 + xA\sin(2\pi f_0 t). \quad (5)$$

The bistable system has two stable states (1 or -1). Thus, we can regard the two stable states as 1bit data in the communication systems.

3. SR in quadruple-well potential

As described in Sec. 1, this paper uses the SR in the quadruple-well potential, which added one dimension in SR model in a double-well potential, for analyzing the existence probabilities of the particle. In this study, we add a new state variable "y" to the bistable system and extend the system to SR in the quadruple-well potential.

Here, we explain equations of SR in the quadruple-well potential. First, based on Eq. (1), the equation of the state x is described as follows.

$$\frac{dx}{dt} = f(x) + D_x n_x(t) + s(t). \quad (6)$$

Where n_x is noise injected to the state x, D_x is an intensity of noise for the state x. Next, the equation of the new state y is described as follows.

$$\frac{dy}{dt} = f(y) + D_y n_y(t) + s(t), \quad (7)$$

$$f(y) = -\frac{dU_0(y)}{dy}, \quad (8)$$

$$U_0(y) = -\frac{1}{2}y^2 + \frac{1}{4}y^4, \quad (9)$$

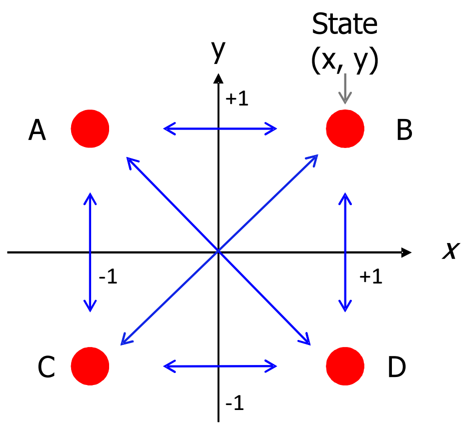


Figure 2: Four-ideal-stable states of SR in quadruple-well potential.

Where n_y is noise injected to the state y, D_y is an intensity of noise for the state y.

SR in the quadruple-well potential has four local minimums, i.e., four potential wells ($(1,1)$, $(-1,1)$, $(1,-1)$ and $(-1,-1)$). Thus, we can regard the two stable states as 2bit data in the communication systems.

In this study, n_x and n_y are assumed to be AWGN. Additionally, n_x and n_y are different from each other. In other words, n_x and n_y are independent of each other.

4. Simulation results and discussions

4.1. Simulation conditions

Using the above system, we carry out computer simulations. In the simulation, the two states (particles) are expressed in (x,y) planes as a coordinate (point (x,y)). As parameters of the simulation, the initial condition of x and y is "1", $t = 10,000$, $dt = 0.01$. In addition, we use two types of the noise parameter for the simulations:

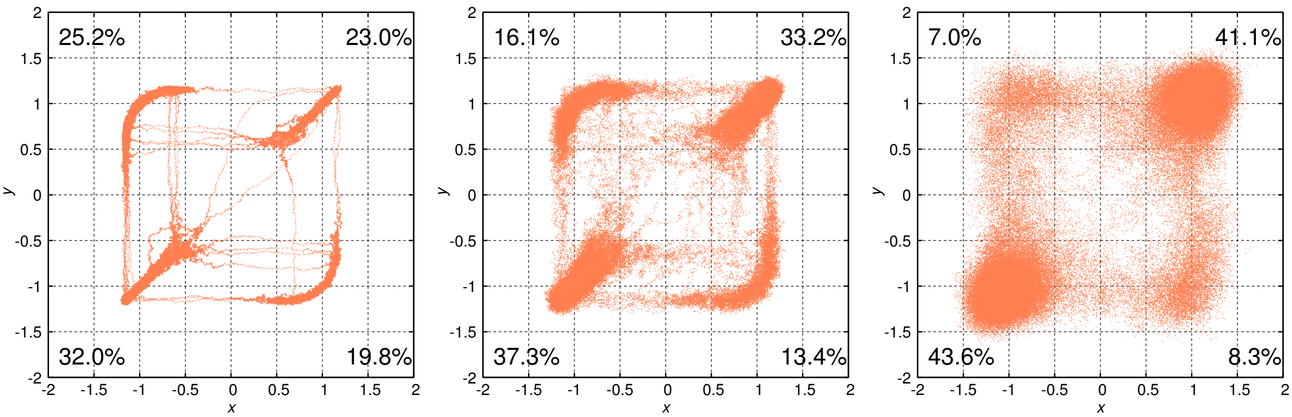
(1) Same noise intensity ($D_x = D_y$),

(2) Different noise intensity ($D_x = 10$ (fixed), D_y: variable).

Figure 2 shows four-ideal-stable states of SR in the quadruple-well potential in (x,y) plane. By changing the noise intensity, we observe various behaviors of the particles and calculate the existence probability of the particle in each potential well.

4.2. Results of same noise intensity ($D_x = D_y$)

Table 1 shows existence probabilities of SR in the quadruple-well potential for $D_x = D_y$. In addition, Fig. 3 shows some outputs of the SR based on Tab. 1. From these



(a) (b) (c)

Figure 3: Outputs of SR in quadruple-well potential ($D_x = D_y$): (a) $D_x = D_y = 3$, (b) $D_x = D_y = 10$, (c) $D_x = D_y = 30$.

Table 1: Existence probabilities of (x, y) ($D_x = D_y$)

$D_x = D_y$	Existence probabilities of (x, y) [%]			
	$(1, 1)$	$(-1, 1)$	$(1, -1)$	$(-1, -1)$
0	100	0	0	0
3	23.0	25.2	19.8	32.0
7	26.2	22.1	17.0	34.7
10	33.2	16.1	13.4	37.3
20	41.1	7.5	8.9	42.5
30	41.1	7.0	8.3	43.6

Table 2: Existence probabilities of (x, y) ($D_x = 10$ (fixed), D_y: variable).

D_y	Existence probabilities of (x, y) [%]			
	$(1, 1)$	$(-1, 1)$	$(1, -1)$	$(-1, -1)$
0	46.6	53.4	0	0
3	25.7	22.5	20.9	30.9
7	29.1	19.2	17.5	34.2
20	35.3	13.3	11.3	40.1
30	31.8	16.3	14.8	37.1

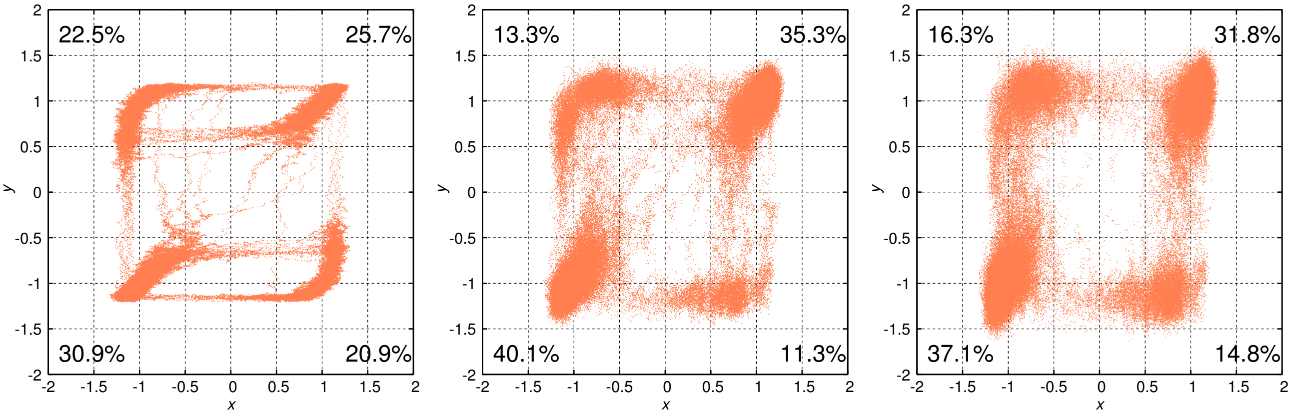
results, we can see that the particles (points (x, y)) are almost evenly distributed in four potential wells when D_x and D_y are small. However, the existence probability is biased according to increasing the noise intensity. Especially, the particles converge on two potential wells ($(1, 1)$ and $(-1, -1)$) , as shown in Fig. 3(c). In other words, the particles only pass through the wells of $(-1, 1)$ and $(1, -1)$ when D_x and D_y are large. Next, we observe the motion of the particles from Fig. 3. As one can see, although the motion of the particles is small in each potential well when D_x and D_y are small (Fig. 3(a)), the particles actively move in the wells of $(1, 1)$ and $(-1, -1)$ according to increasing the noise intensity (Fig. 3(c)). Therefore, it can be said that the particles do not stay for long term in one potential well when D_x and D_y are large, and the state transition occurs frequently in the wells.

4.3. Results of different noise intensity ($D_x = 10$ (fixed), D_y: variable)

Table 2 shows existence probabilities of SR in the quadruple-well potential when $D_x = 10$ and D_y is variable. In addition, Fig. 4 shows some outputs of the SR based on Tab. 2. As a matter of course, it can be observed that the existence probabilities of Tab. 2 are different from that of Tab. 1. Especially, we focus on the larger D_y ($D_y = 20$ and 30). The existence probabilities of $(-1, 1)$ and $(1, -1)$ when $D_y = 20$ and 30 in Tab. 2 are larger than that of $D_x = D_y = 20$ and $D_x = D_y = 30$ in Tab. 1. Since the influence of the noise that the particle of x-component receives is smaller than that of y-component, the probability that the two-state transition occurs simultaneously becomes small. Thus, we consider that it is easy to change the existence probabilities using the different noise intensity. In other words, it is possible that to use the different noise intensity is effective for controlling the state of the SR in the quadruple-well potential. However, it can be also confirmed that the existence probability in Tab. 2 is biased in two potential wells ($(1, 1)$ and $(-1, -1)$) according to increasing the noise intensity in common with $D_x = D_y$. We discuss the reason for this in Sec. 4.4.

4.4. Discussions

Here, we discuss that the reason for the biased existence probability according to increasing the noise intensity. The



(a) (b) (c)

Figure 4: Output of SR in four-stable state ($D_x = 10$ (fixed), D_y: variable): (a) $D_y = 3$, (b) $D_y = 20$, (c) $D_y = 30$.

major reason for this is that $s(t)$ dose not depend on the state of x and y, as shown in Eq. 2. The states of x and y in SR change independently of each other depending on each input signal and each noise. Thus, the particles are biased to the wells of $(1, 1)$ and $(-1, -1)$. Therefore, we consider that to analyze the SR using the input signal which x and y influence mutually is our future work.

5. Conclusions

In this study, we have analyzed existence probabilities of the particle in SR in the quadruple-well potential with various noise intensities. As results of computer simulations, we have found that it is possible that to use the different noise intensity is effective for controlling the state of the SR in the quadruple-well potential. In addition, it has been confirmed that the existence probability is biased in the wells of $(1, 1)$ and $(-1, -1)$ due to $s(t)$ which dose not depend on the state of x and y.

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