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Clustering Phenomena Obtained from Coupled Rulkov Maps

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Abstract— **In this study, we investigate clustering phenomena observed in coupled map lattice (CML). A two dimensional map which produces spiking-bursting behavior like real biological neurons is used for CML. By using computer simulations, we confirm that the data are separated to several groups when a different parameter which depends on the data information is set for each map.**

I. Introduction

Coupled map lattice (CML) is well known as mathematical models in discrete-time system. CML has proposed by Kaneko [1], to use as general models for the complex high-dimensional dynamics, such as biological systems, networks in DNA, economic activities, neural networks, and evolutions.

In recent years, we often deal with huge amounts of data. It is useful to partition such data sets into clusters of items that share similar properties. The clustering algorithms using the CML have proposed [2], [3]. Here, the logistic map was chosen as the simple chaotic map.

In this study, we consider a two dimensional map (Rulkov map) which produces spiking-bursting behavior like real biological neurons [4]-[6]. First, we investigate basic synchronization phenomena when two identical Rulkov maps are coupled. In this case, the in-phase synchronization with same amplitude can be observed. Next, the control parameter of one side map is varied. We confirm the in-phase synchronization with different amplitude depending on value of the control parameter. We focus on the amplitude of map for clustering algorithm. By using computer simulations, we confirm that the data are separated to several groups when a different parameter which depends on the data information is set for each map.

II. Rulkov Map

In recent years, the two dimensional map with the interesting phenomena was proposed by Rulkov [4]-[6] (see Fig. 1). This map produces spiking-bursting neural behavior.

The expression of the Rulkov map is described as following:

$$\begin{aligned} x_{n+1} &= f(x_n, x_{n-1}, y_n) \\ y_{n+1} &= y_n - \mu(x_n + 1) + \mu\sigma + \mu\sigma_n, \end{aligned} \quad (1)$$

where x is the fast and y is the slow dynamical variable. $\mu = 10^{-3}$ and α and σ denotes the control parameters of the maps. And parameters g shows the coupling weight between the maps. The nonlinear function $f(x_n, x_{n-1}, y_n)$ is described as following:

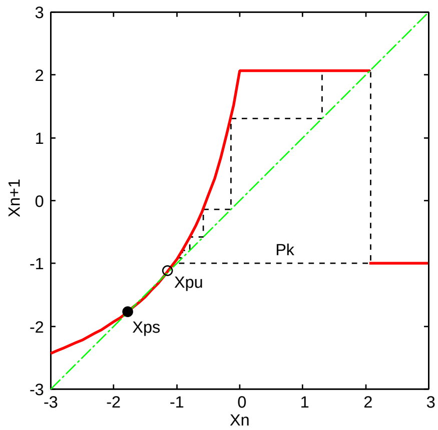


Fig. 1. Rulkov map. The dashed line illustrates a super stable cycle P_k. The stable and unstable fixed points of the map are indicated by x_{ps} and x_{pu}, respectively.

$$f(x_n, x_{n-1}, y_n) = \begin{cases} \alpha/(1 - x_n + u), (x_n \leq 0) \\ \alpha + u, \quad (0 < x_n < \alpha + u \ and \ x_{n-1} \leq 0) \\ -1, \quad (x_n \geq \alpha + u \ or \ x_{n-1} > 0) \end{cases} \quad (2)$$

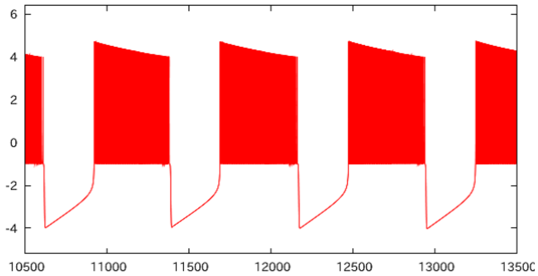
σ is set to random value, and α is the parameter to control the fast variable.

III. Two Coupled Rulkov Maps

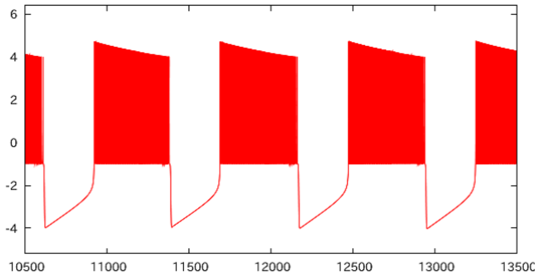
First, we consider the basic synchronization phenomena when two Rulkov maps are coupled as chain topology. The equation of the chain of Rulkov maps is described as:

$$\begin{aligned} x_{i,n+1} &= f(x_{i,n}, x_{i,n-1}, y_{i,n}) \\ &+ \frac{1}{2} g(x_{i+1,n} - 2x_{i,n} + x_{i-1,n}) \\ y_{i,n+1} &= y_{i,n} - \mu(x_{i,n} + 1) + \mu\sigma + \mu\sigma_{i,n} \\ &+ \frac{1}{2} g(x_{i+1,n} - 2x_{i,n} + x_{i-1,n}) \end{aligned} \quad (3)$$

where parameter g denotes the coupling weight of the between the maps. Each parameters of x_1, x_2 sets individually. When α_1 and α_2 are equal. The waveform of each maps are shown in Fig. 2. The in-phase synchronziation with same amplitude is observed.

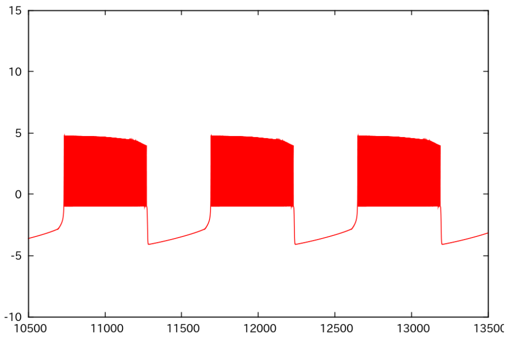


x_1 $(\alpha_1 = 10.0)$.

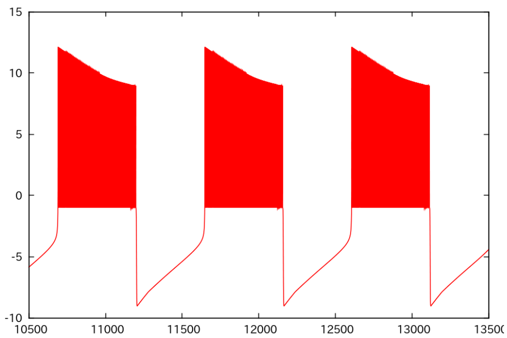


x_2 $(\alpha_2 = 10.0)$.

Fig. 2. Two coupled maps with same parameters.



x_1 $(\alpha_1 = 10.0)$.



x_2 $(\alpha_2 = 20.0)$.

Fig. 3. Two coupled maps with different parameters.

Next, we vary the control parameter as follows; $\alpha_1 = 10.0$ and $\alpha_2 = 20.0$. The waveforms of each maps are shown in Fig. 3. From this figure, we can see that two maps have different amplitude with same phase. Namely, we observe the in-phase synchronization with different amplitude depending on value of the control parameter. We focus on the different amplitude of map for clustering algorithm.

IV. Three Coupled Maps with Distance Information

Next, we consider the case that three coupled maps placed on a two dimension area including the positional information. Furthermore, from synchronization results we investigate the clustering phenomena by using the positional information. The alignment of three maps is shown in Fig. 4.

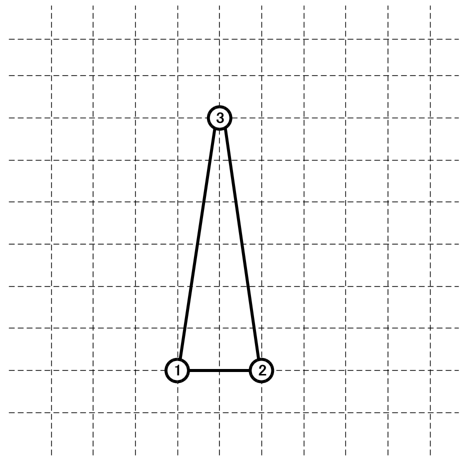


Fig. 4. The alignment of three maps.

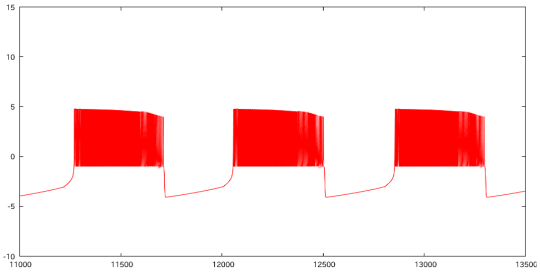
The control parameter α is fixed with distance information. We determine the parameters according to the distance from the reference point. x_1 and x_2 are placed with close distance, then the control parameter α is fixed as α_1=α_2=10.0. And x_3 is placed with far distance form x_1 and x_2, then the control parameter α_3 is set to α_3=20.0. The time waveforms obtained form each map are shown in Fig. 5.

From these results, we can see that the amplitude of third map has different value from first and second maps. The maximum values of the amplitude of each map are $x_1 = 4.78$, $x_2 = 4.78$ and $x_3 = 12.15$. In this case, we can cluster three coupled maps into two groups between x_1, x_2 and x_3 by using the value of the amplitude (see Tab. 1).

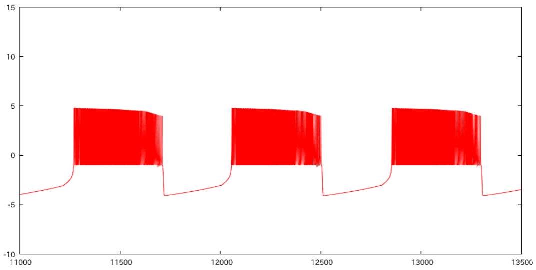
TABLE I

Group of three coupled maps.

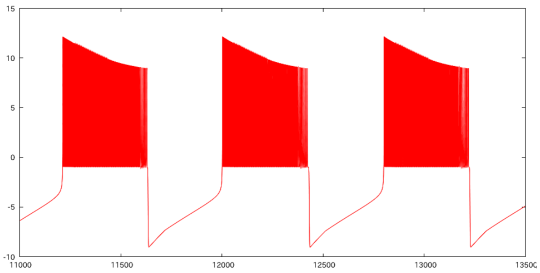
Map	Amplitude	Group
x_1	4.78	1
x_2	4.78	1
x_3	12.15	2



x_1 (α_1=10.0).



x_2 (α_2=10.0).



x_3 (α_3=20.0).

Fig. 5. Three coupled maps

V. Application for General Networks

Finally, we consider the case of general networks. Ten maps are placed randomly on the two-dimensional are including the positional information. k nearest neighbors is used as the way of coupling between the maps. In this coupling method, the number of k maps which are placed with k-th closest distance from the reference map are connected. In this study, the parameter of k is set to $k = 2$. The figure of alignment of coupled ten maps is shown in Fig. 6.

We fix first map (x_1) as the standard map and the parameter α_1 is set to $\alpha_1 = 10$. The other parameters of α sets by using the following equation.

$$\alpha_i = \frac{(20 - 10) \cdot d_i}{d_{max}} \tag{4}$$

The parameter of d_i is the distance from the standard point and d_{max} is the most farthest distance from the standard point. So the other parameters α of the maps are set to $\alpha_2 = 10.67$, $\alpha_3 = 10$, $\alpha_4 = 12.33$, $\alpha_5 = 10$, $\alpha_6 = 15.66$, $\alpha_7 = 15.33$, $\alpha_8 = 16.34$, $\alpha_9 = 20$, and $\alpha_{10} = 19$. Simulation results are shown in Fig. 7.

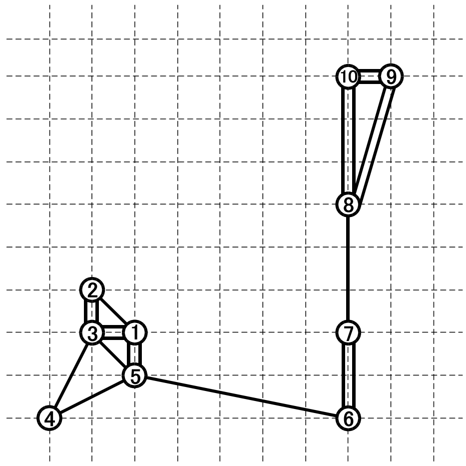
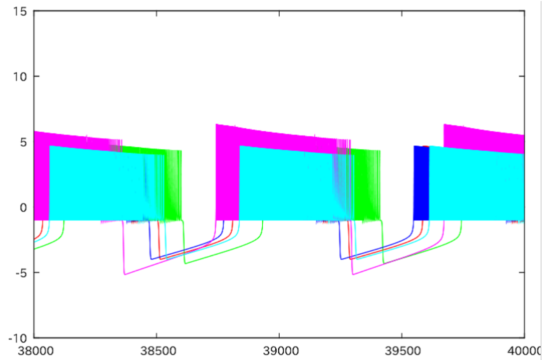
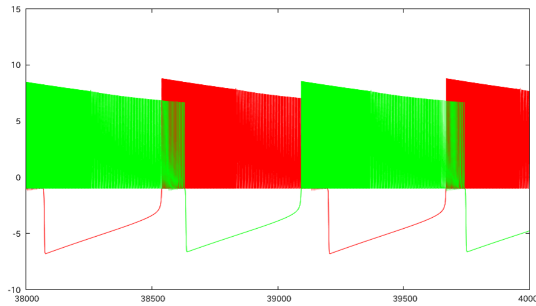


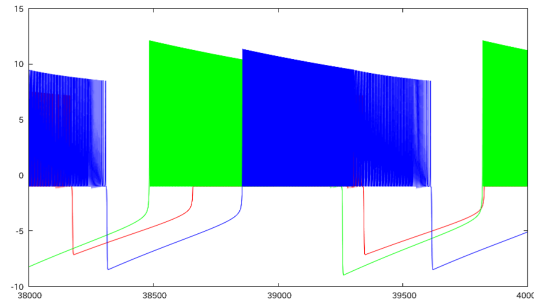
Fig. 6. The alignment of ten maps.



(a) Group 1 (x_1, x_2, x_3, x_4, x_5)



(b) Group 2 (x_6, x_7)



(c) Group 3 (x_8, x_9, x_{10})

Fig. 7. General Networks.

The maximum values of the time waveforms of ten maps are x_1 = 4.71, x_2 = 5.18, x_3 = 4.71, x_4 = 6.35, x_5 = 4.71, x_6 = 8.80, x_7 = 8.55, x_8 = 9.31, x_9 = 12.12, and x_{10} = 11.35. In this case, we can cluster into three groups, group 1 is x_1, x_2, x_3, x_4 and x_5, group 2 is x_6 and x_7, group 3 is x_8, x_9 and x_{10} (see Tab. 2).

TABLE II
GROUP OF TEN COUPLED MAPS

Map	Amplitude	Group
x_1	4.71	1
x_2	5.18	1
x_3	4.71	1
x_4	6.35	1
x_5	4.71	1
x_6	8.80	2
x_7	8.55	2
x_8	9.31	2
x_9	12.12	3
$x_1 0$	11.35	3

VI. Conclusions

In this study, we have investigated clustering phenomena observed in CML. We used the Rulkov map which produces spiking-bursting behavior like real biological neurons. By using computer simulations, we confirmed that the data are separated to several groups when a different parameter which depends on the data information is set for each map. In our future works, we study large-scale CML, and propose efficient algorithm for clustering by using synchronization mechanism.

References

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