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Switching of Synchronization States in Coupled Maps by Intermittency Chaos

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***Abstract*— In this study, we investigate switching synchronization phenomena which are observed in coupled chaotic maps. Two logistic maps with generating intermittency chaos near three periodic window are coupled as a CML coupling topology. Moreover, we compare obtained synchronization states of coupled maps with coupled chaotic circuits.**

I. Introduction

Synchronization is one of the most typical nonlinear phenomena, there have been many studies until now. In addition, coupled oscillatory system can produce interesting synchronization phenomena. Also, the coupled chaotic circuits systems are suitable model to express the many various high-dimensional nonlinear phenomena. In particular, chaotic synchronization phenomena are paid attention to many researchers in various science fields because those mechanism are clear by piecemeal. Therefor, their investigations of chaotic synchronization phenomena have been done in many research fields including physics, and the biologies. In our previous study, we have investigated the complex behavior in coupled chaotic circuits [1]. Figure 1 shows the circuit model which used our previous study. In this circuit, two identical chaotic circuits are coupled by a coupling resistor R. We could find the three different types of synchronization states when the two circuits generate three periodic attractors in this coupled chaotic circuit. These three synchronization states can be obtained by giving different initial conditions. Next, we vary the bifurcation parameter of each chaotic circuit to generate intermittency chaos near three-periodic window. In this case, we could observe a complex behavior of the three synchronization states. Namely, intermittency bursts disturb the synchronizations and different synchronizations reappear after the bursts settle down.

Does such synchronization phenomenon of chaotic circuits observe in coupled chaotic maps or not? In order to make clear this question, we investigate synchronization phenomena in coupled chaotic map as a Coupled Map Lattice (CML) topology. In particular, CML is well known to as a discrete-time mathematical models and we can observe the various dynamics in these corollary. In the investigations of system such as CML, it is very important to analyze the behavior for the application of the engineering of the nonlinear phenomena, biological networks.

In this study, we investigate synchronization phenomenon when two logistic maps are coupled. We observe the difference of behavior between adjacent maps by changing the parameter of the logistic map, and make a comparison the difference of coupled maps and coupled chaotic circuits. Also, we pay attention to the bursting part, and investigate the behavior of the maps before and after the burst occurs.

II. Coupled Maps Lattice

A CML is known as suitable models to describe chaos and pattern dynamics. CML is a network of grid to generate the interaction between oneself and adjacent maps. Figure 2 shows an example of CML when two maps are coupled. In the CML, the each map connects to only adjacent maps. In this study, we use the logistic maps as chaos maps. A logistic map is expressed as following equation:

$$f_{i,n} = \alpha x_{i,n}(1 - x_{i,n}) \tag{1}$$

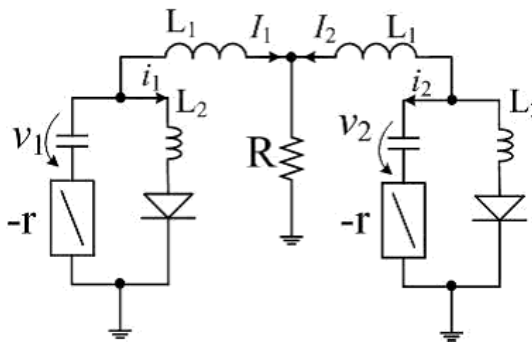

Fig. 1. Two coupled chaotic circuits model [1].

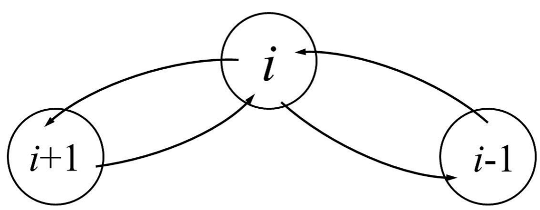

Fig. 2. Coupled map lattice.

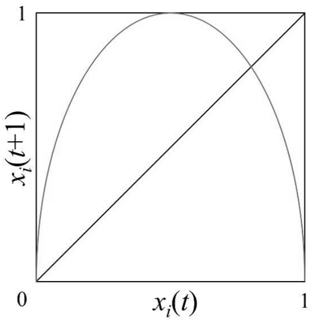

Fig. 3. Logistic map.

where, α is a bifurcation parameter. The time series obtained form the logistic map for $\alpha = 4.0$ is shown in Fig. 3.

A general coupled map lattice is described as following equation.

$$x_{i,n+1} = (1-\varepsilon)f_{i,n} + \frac{\varepsilon}{2}(f_{i-1,n} + f_{i+1,n}) \quad (2)$$

III. Simulation Result

A. Three Types of Synchronization

First, we consider simple model of CML. The parameters are fixed as $\alpha = 3.835$ and $\varepsilon = 0.000015$ which produce three-periodic solution. We can obtain the three different types of synchronization states by differential initial values as shown in Fig. 4.

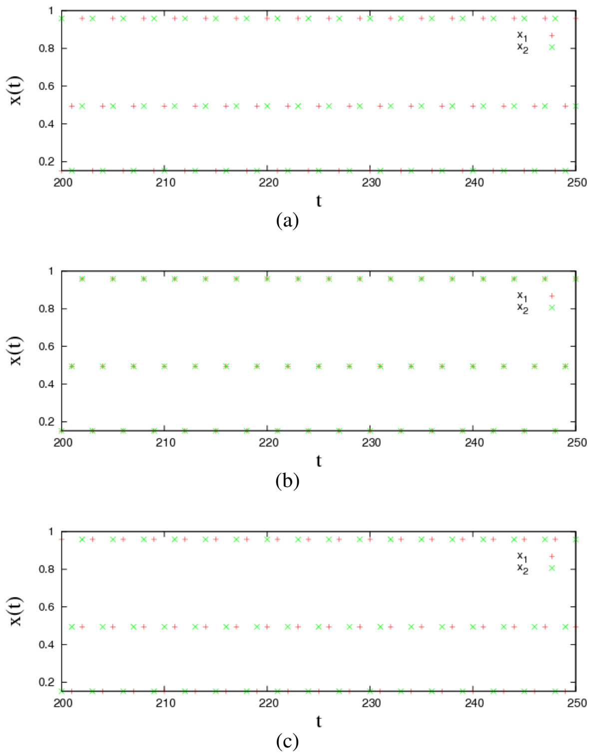

Fig. 4. Three synchronization states depending on initial values. (a) $x_1 = 0.20$, $x_2 = 0.60$. (b) $x_1 = 0.10$, $x_2 = 0.90$. (c) $x_1 = 0.40$, $x_2 = 0.70$.

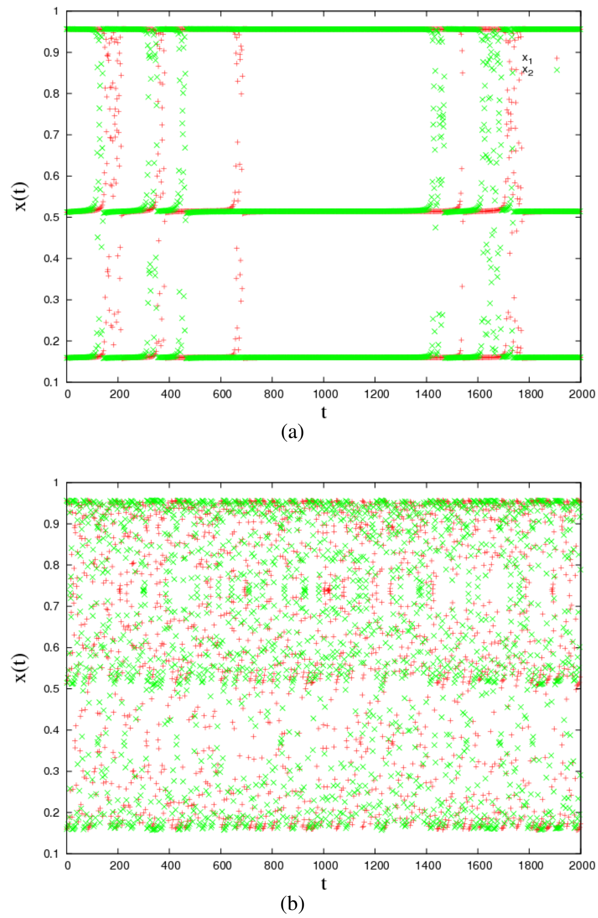

Fig. 5. Switching of three types of synchronization. (a) $\varepsilon = 0.000015$. (b) $\varepsilon = 0.0015$.

B. Switching of Three Types of Synchronization

Next, we vary the bifurcation parameter from $\alpha = 3.835$ to 3.828427. Under this parameter α, the intermittency chaos near the three-periodic window is obtained. In Fig. 5, we show the simulation result when the bifurcation parameters is fixed as $\alpha = 3.828427$, and the initial conditions of each logistic map is set to $x_{1,0} = 0.159126$, $x_{2,0} = 0.956548$. In these figures, we confirm that the three types of synchronization states are switching via bursting part. Furthermore, the switching frequency becomes fast by increasing the value of the coupling strength ε.

In order to make clear the switching mechanism, we partition the obtained synchronization state (Fig. 5(a)) into four patterns ((a):$50 \le t \le 100$, (b):$250 \le t \le 300$, (c):$1000 \le t \le 1050$, (d):$75 \le t \le 500$) as shown in Fig. 6.

Figure 6 (a),(b) and (c) are corresponding the three types of synchronization. In Fig. 6(d), we show the switching of synchronization part, and we can observe that after a burst of x_2 is occurred, a burst of x_1 is also occurred.

Finally, as an example, the long span of switching synchronization is shown in Fig. 7. We can see that the three types

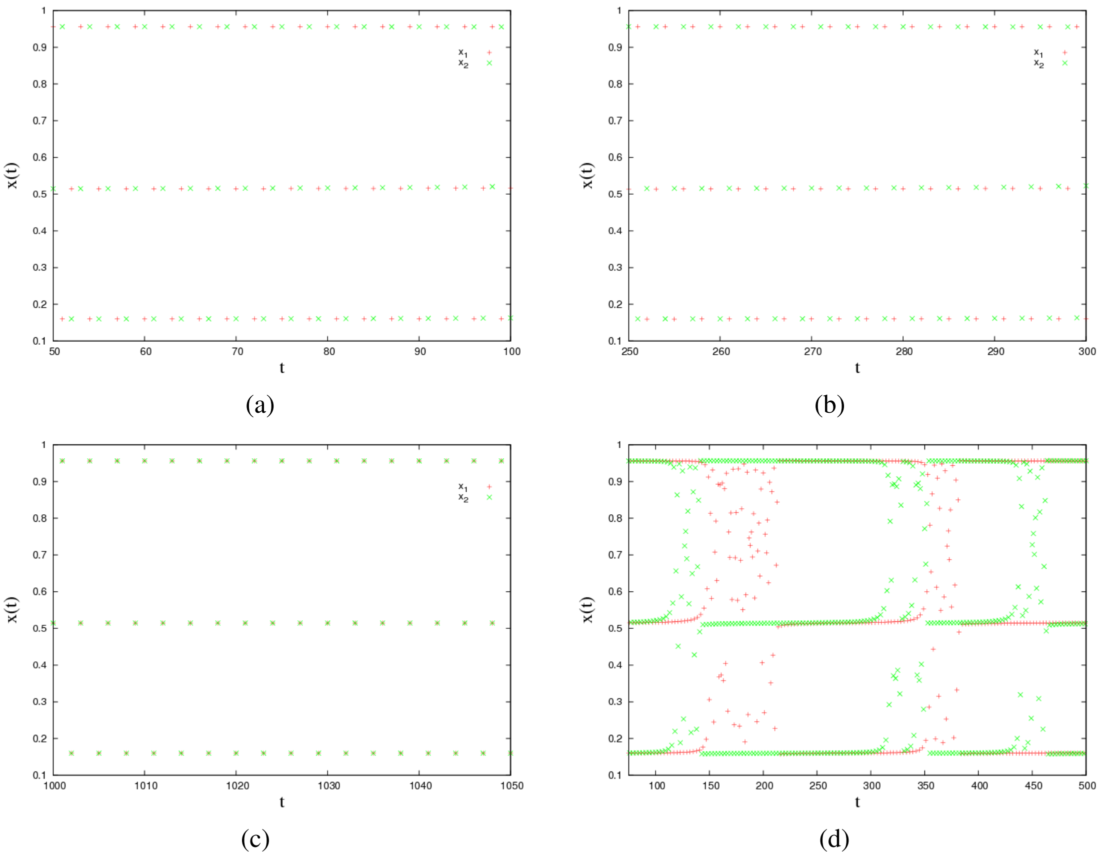

Fig. 6. Detailed drawing of the switching synchronization. (a) $50 \le t \le 100$. (b) $250 \le t \le 300$. (c) $1000 \le t \le 1050$. (d) $75 \le t \le 500$.

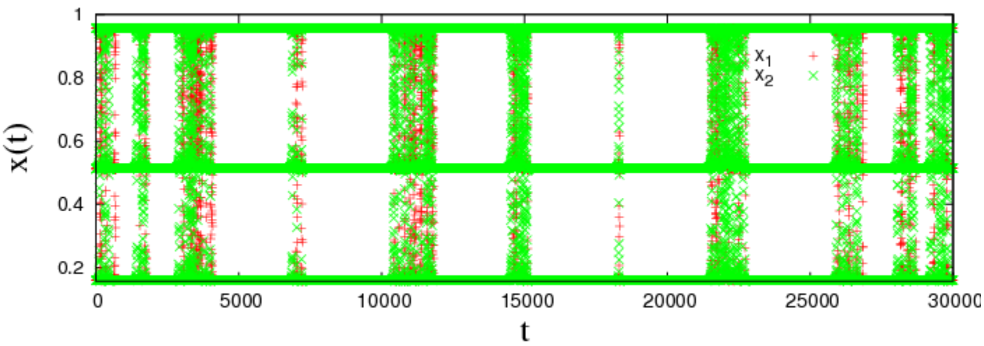

Fig. 7. Long span of switching synchronization.

of synchronization states appear and disappear via bursting behavior.

IV. Conclusions

In this study, we have investigated synchronization phenomena observed in two coupled chaotic maps. By setting the certain parameters we could confirm the interesting phenomena for burst. Three types of synchronization states changed by bursting behavior.

In our future works, we would like to study the synchronization phenomena when the number of the couped maps are increased.

References

[1] Y. Uwate, Y.Nishio, "Modeling Using 1-D Map of Complex Behavior in Coupled Chaotic Circuits with Intermittency" Proceedings of European Conference on Circuit Theory and Design (ECCTD'05), vol. 1, pp. 229-232, Aug. 2005.