2011 International Workshop on Nonlinear Maps and their Applications
NOMA'11, Evora, Portugal, September 15-16, 2011

Hopfield Neural Networks Operated by Template of Cellular Neural Networks

Yasuhiro Ueda, Yoko Uwate and Yoshifumi Nishio
Dept. Electrical and Electronic Eng.,
Tokushima University
Email: { yasuhiro, uwate, nishio } @ee.tokushima-u.ac.jp

***Abstract*— In this study, we propose the hybrid system that consists of Cellular Neural Networks (CNN) and the Hopfield Neural Networks (HNN). This system has a part of structure, characteristics and property in CNN and HNN. The system has local connectivity property and toroidally-linked boundary condition. Neurons are updated by the update equation of HNN. Output is controlled between 1 and -1 by the piece-wise linear function of CNN. Also the wiring weights are fixed to obtain the local information like a template in CNN. By the computer simulations, we investigate the behavior of the proposed system.**

I. Introduction

Cellular Neural Networks (CNN) [1] were introduced by Chua and Yang in 1988. The idea of CNN was inspired from the architecture of the cellular automata and the neural networks. A different point from the conventional neural networks is that CNN has local connectivity property. Since the structure of CNN resembles the structure of animals' retina, CNN can be used for various image processing applications [2]-[5]. CNN is composed of the basic circuit units called cells. The cell contains linear and nonlinear circuit elements which are typically linear capacitors, linear resistors, linear and nonlinear controlled sources. All cells are connected neighborhood 8 cells.

In addition, Hopfield Neural Networks (HNN) [6] were introduced by J. J. Hopfield in 1984. HNN is one of the mutual connection neural network and studied chiefly about the associative memory and optimization problem. A neuron is connected all other neurons. The neuron updates itself by the value of other neurons and wiring weights in discrete-time. The updated neuron is decided randomly and the wiring weights are set up depending on the intended purpose. This network continue to update a neuron and the energy function of network becomes 0.

In this study, we propose the hybrid system that consists of CNN and HNN. This system has a part of structure, characteristics and property in CNN and HNN. In each conventional system, CNN carry out parallel signal processing in continuous time. CNN has local connectivity property and output is controlled between 1 and -1 by the piece-wise linear function. Also HNN is one of the mutual connection neural networks. The neurons are updated by other neurons and the wiring weights. The wiring weights of the HNN are set up according to the intended purpose. In the proposed system, the system has local connectivity property and toroidally-linked boundary condition. Neurons are updated by the update equation of HNN. Output is controlled between 1 and -1 by the piece-wise linear function of CNN. Also the wiring weights are fixed to obtain the local information like a template in CNN. By computer simulations, we investigate the behavior of the proposed system. In Sec. II, we explain the basic of the standard CNN. In Sec. III, we explain the basic of the standard HNN. In Sec. IV, we describe the structure of the proposed system. In Sec. V, simulation results are shown and compare the conventional CNN with the proposed system. Section VI concludes this article.

II. Cellular Neural Networks

In this section, we describe the basic structure of CNN. CNN has M by N processing unit circuits called cells. Cells are arranged in a reticular pattern to M line by N row. We represent a cell $C(i,j)$ using a variable i which denotes vertical position and a variable j which denotes horizontal position. The cell contains linear and nonlinear circuit elements. CNN is an array of cells. Each cell is connected to its neighborhood cells according to a template. Usually, the same template is used for all the cells. The boundary condition is changed according to a kind of the template. CNN has the features of time continuity, spatial discreteness, nonlinearity and parallel processing capability.

The state equation and the output equation of the cell $C(i,j)$ are shown as follows.
State equation:

$$\begin{aligned}\frac{dv_{xij}}{dt} &= -v_{xij} + \sum_{k=i-r}^{i+r}\sum_{l=j-r}^{j+r} A_{(i,j;k,l)}v_{ykl}(t) \\ &\quad + \sum_{k=i-r}^{i+r}\sum_{l=j-r}^{j+r} B_{(i,j;k,l)}v_{ukl}(t) + I. \qquad (1)\end{aligned}$$

Output equation:

$$v_{yij}(t) = \frac{1}{2}(|v_{xij}(t)+1| - |v_{xij}(t)-1|). \qquad (2)$$

where v_x, v_y and v_u represent a state, an output and an input of the cell, respectively. In the Eq. (1), A is the feedback template and B is the control template. These templates and threshold value I are collectively called the general template. The performance of CNN decided by the template.

The r-*neighborhood* of $C(i,j)$ in CNN is defined by

$$Nr(i,j) = \{C(k,l) \mid \max\{|k-i|, |l-j|\} \leq r, \quad 1 \leq k \leq M; \quad 1 \leq l \leq N\}. \tag{3}$$

where r is a positive integer number.

III. Hopfield Neural Networks

In this section, we describe the basic structure of Hopfield Neural Network (HNN). This neural network is one of the mutual connection neural network. A neuron is connected all other neurons. The neuron updates itself by the value of other neurons and wiring weights in discrete-time. The wiring weights are set up depending on the intended purpose. This network continue to update a neuron and the energy function of network becomes 0. The energy function of the HNN with N neurons is defined by the following equation.

Energy function:

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}x_i x_j + \sum_{i=1}^{N}\theta_i x_i, \tag{4}$$

where w_{ij} is the weight between i-th neuron and j-th neuron, and θ_i is the threshold of the i-th neuron. The weight is 0 when $i = j$, because all the units have combined with all the units of the others except themselves. The states of the neurons are asynchronously updated due to the following difference equation.

Update equation:

$$x_i(t+1) = sgn(\sum_{i \neq j} w_{ij}x_j(t)), \tag{5}$$

where sgn is an output function as follows.

Output equation:

$$sgn(a) = \begin{cases} 1 & (a \geq 0) \\ -1. & (a < 0) \end{cases} \tag{6}$$

IV. Hopfield Neural Networks Operated by Template of Cellular Neural Networks

In this section, we explain the proposed system that consists of CNN and HNN. (Hereinafter called "Cellular Hopfield Networks (CHN)") CHN has M by N neurons which show the gray-scale value. The structure of CHN is same with CNN like Fig. 1(a). One neuron is connected with around neighbor neurons. The boundary condition is toroidally-linked like Fig. 1(b).

The neurons are asynchronously updated due to the following equation.

Update equation:

$$x_{ij}(t+1) = sgn(\sum_{k=i-r}^{i+r}\sum_{l=j-r}^{j+r} T_{(i,j;k,l)}x_{ij,kl}(t)), \tag{7}$$

where sgn is an output function which is same with the output function of CNN as follows.

Output equation:

$$sgn(a) = \frac{1}{2}(|a+1| - |a-1|). \tag{8}$$

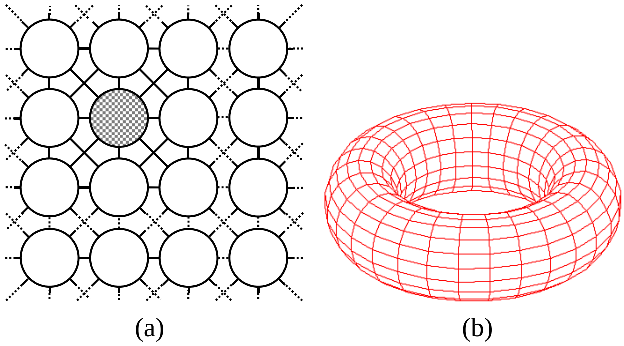

Fig. 1. CHN model. (a) Structure of CHN. (b) Toroidally-linked boundary condition.

The r-*neighborhood* of the neuron in CHN is defined by

$$Nr(i,j) = \{Neuron(k,l) \mid \max\{|k-i|, |l-j|\} \leq r, \quad 1 \leq k \leq M; \quad 1 \leq l \leq N\}, \tag{9}$$

where r is a positive integer number. In our study, we fix the value of r as 1. Therefore, the size of the template is fixed as 3×3 matrix. Also x_{ij} is the state value of the neuron and $T_{(i,j;k,l)}$ is the wiring weights of this network in the update equation to obtain the local information around neighbor neurons.

V. Simulation Results

In this section, we show some simulation results and compare our proposed system with CNN. The results are composed by two tasks. One is a pattern formation and another is the performance of the "Heat Diffusion" template. The output image is composed by 2500 pixels(50×50 pix.). Therefore each network is consisting of by 2500 neurons.

A. Pattern formation

In this task, we show the simulation results about a pattern formation. The CNN is known to be able to generate periodic patterns [7]-[9]. The template for pattern formation has designed in CNN [10][11]. We investigate the pattern formation in CHN and compare CHN with CNN. In this task, we apply following template.

$$T = \begin{bmatrix} 0.15 & 0 & 0.15 \\ 0 & 0.6 & 0 \\ 0.15 & 0 & 0.15 \end{bmatrix} \tag{10}$$

Figure 2(a) is an initial image constructed by 1246 black pixels and 1254 white pixels. Other images in Fig. 2 are output images. These output images are obtained by the initial condition and the asynchronously random update of the neuron Figure 2(b) is an output image that having uniform checkerboard. Figure 2(c) and (d) is uniform black and white output image, respectively. Figs. 2(b), (c) and (d) are obtained in our simulation with a high rate. Additionally, we show distinctive results like Figs. 2(e) and (f). Usually, only one particular pattern is generated depending on the initial value and the processing in past study of pattern formation. However,

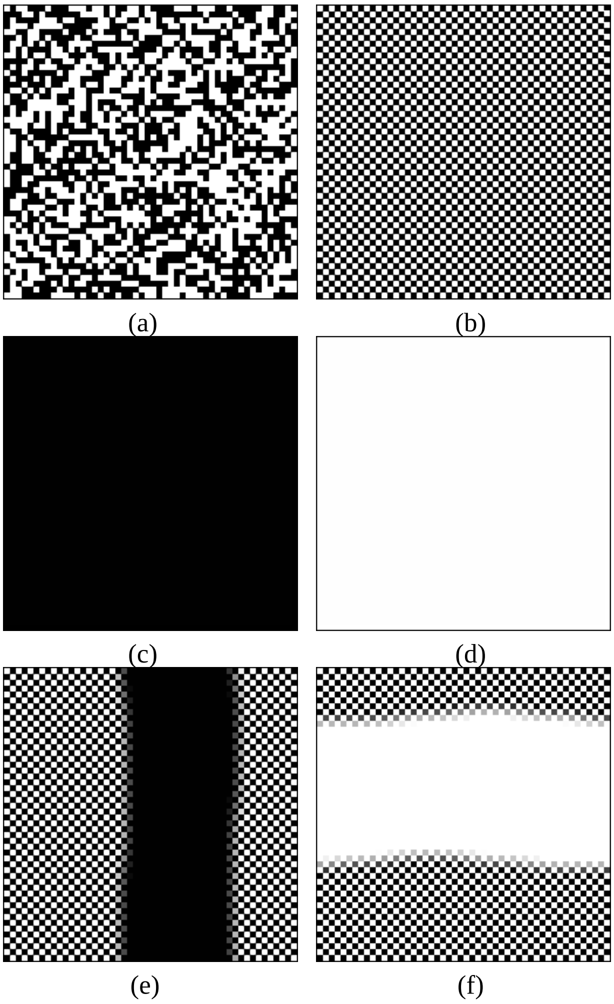

Fig. 2. Simulation results of CHN for pattern formation. (a) “Random noise” image as initial image. (b) Checkerboard as output image. (c) Uniform black as output image. (d) Uniform White as output image. (e) Checkerboard and a black region are mixed in an output image. (f) Checkerboard and a white region are mixed in an output image.

two kinds of patterns are coexisted in one output image. This system holds the promise of generating more complicated patterns.

Also in CNN, we obtain following output image by T. We substitute the T in the feedback template A and the control template B, respectively. For stability of CNN, the center component of the feedback template A is 1 when the T is substituted in the control template B.

In CNN, the input image is Fig. 2(a). Figures 3(a) and (b) are output image by substituting the T in the feedback template A and the control template B, respectively. In Fig 3(a), the output image is uniform checkerboard. On the other hand, Fig 3(b) is same with input image.

From these simulation results, we show clear distinction of the performance between CNN and CHN. Also, the generated patterns are a uniquely-output that coexists two kinds of pattern.

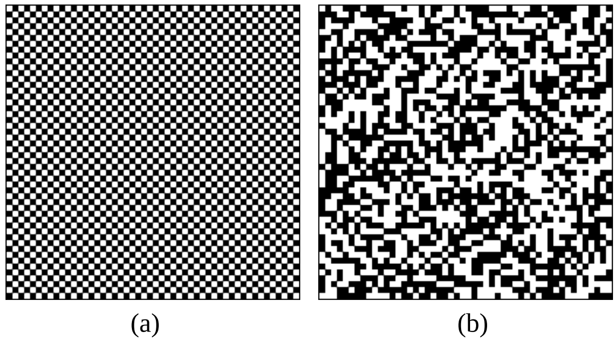

Fig. 3. Performance of the “Pattern formation” template in CNN. (a) Output image by substituting the T in the feedback template A. (b) Output image by substituting the T in the control template B.

B. Performance of the “Heat Diffusion” template

In this task, we investigate the performance of CHN for “Heat Diffusion” template by comparing CNN. “Heat Diffusion” template carries out diffusing the input image in CNN. The boundary condition is fixed as 0 in CNN [7]. This template gradates the objects to uniform gray scale. The output value is uniform 0.

“Heat Diffusion” template:

$$A = \begin{bmatrix} 0.1 & 0.15 & 0.1 \\ 0.15 & 0 & 0.15 \\ 0.1 & 0.15 & 0.1 \end{bmatrix}, \ B = 0, \ I = 0. \tag{11}$$

In the proposed system, the wiring weights $T_{(i,j;k,l)}$ is same with the feedback template A of “Heat Diffusion” template as follows.

$$T = \begin{bmatrix} 0.1 & 0.15 & 0.1 \\ 0.15 & 0 & 0.15 \\ 0.1 & 0.15 & 0.1 \end{bmatrix} \tag{12}$$

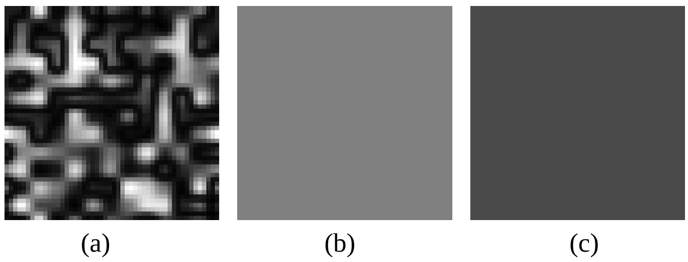

Fig. 4. Performance of the “Heat Diffusion” template in CNN and CHN. (a) “Texture” image as initial image. (b) Output image in CNN. (c) Output image in CHN.

In Fig. 4(a), the average of the initial image is 0.408. By “Heat Diffusion” template, the output image is uniform 0 like Fig. 4(b). On the other hand, the different result is obtained by Eq. (12) in CHN. In Fig. 4(c), the output image is uniform value which is 0.406. We show other simulation results as follows.

In Fig. 5(a), the average of the initial image is -0.467. Also the output image is uniform value which is -0.469 like

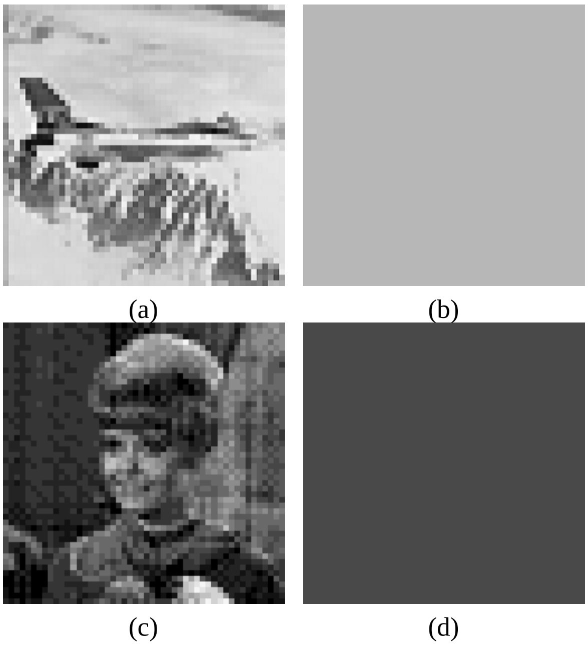

Fig. 5. Other simulation results. (a) “Airplane” image as initial image. (b) Output image for “Airplane” image in CHN. (c) “Girl” image as initial image. (d) Output image for “Girl” image in CHN.

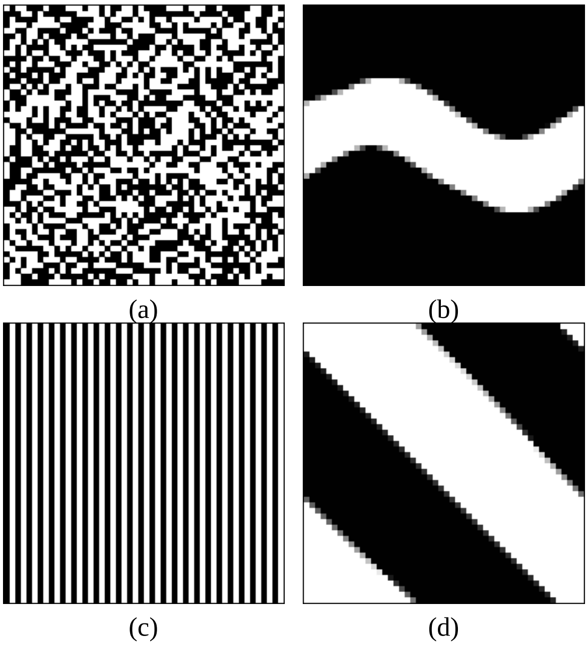

Fig. 6. Simulation results by Eq. (13). (a) “Random noise” image as initial image. (b) Wave form as output image. (c) “Stripe” image as initial image. (d) Band form as output image.

Fig. 5(b). In Fig. 5(c), the average of the initial image is 0.436. Also the output image is uniform value which is 0.414 like Fig. 5(d). From these simulation results, we confirm the output value is similar to the value of the average of initial image. In fact the output value is depended on the initial image.

Additionally, we change the value of template in same size and investigate its effect. We change the template that is five times as large as Eq. (12) as follows.

$$T = \begin{bmatrix} 0.5 & 0.75 & 0.5 \\ 0.75 & 0 & 0.75 \\ 0.5 & 0.75 & 0.5 \end{bmatrix} \tag{13}$$

For this simulation, the initial image Fig. 6(a) is same with Fig. 2(a). Figure 6(b) is output image like a wave form. The width of the waves is changed in each time. Also we obtain another simulation results by another initial image like Fig. 6(c). In Fig. 6(d), a diagonal band form is generated. All output images are converged on 1 or -1. Moreover, the convergence time of Eq. (13) is shorter than that of Eq. (12). From these simulation results, the output value, the convergence time and the generated patterns are depends on the value of template. Additionally, we consider that the output image has something to do with the initial image.

VI. Conclusions

In this study, we have proposed the hybrid system that consists of CNN and HNN. This system has a part of structure, characteristics and property in CNN and HNN. From the simulation results, we have confirmed that this system output some interesting results. In diffusion template, we have confirmed that the output depends on the initial. Also in pattern formation, the value of the template and asynchronously random update influences to the output. As a future work, we have to investigate the output characteristics with various templates and images.

References

[1] L.O. Chua and L. Yang, “Cellular Neural Networks:Theory,” IEEE Trans. Circuits Syst., vol. 32, pp. 1257-1272, Oct. 1988.

[2] F. Dirk and T. Ronald, “Coding of Binary Image Data using Cellular Neural Networks and Iterative Annealing,” Proc. of ECCTD’03, vol. 1, pp. 229-232, Sep. 2003.

[3] M. Namba and Z. Zhang, “Cellular Neural Network for Associative Memory and Its Application to Braille Image Recognition,” Proc. of IJCNN’06, pp. 4716-4721, Jul. 2006.

[4] H. Koeppl and L.O. Chua, “An Adaptive Cellular Nonlinear Network and its Application,” Proc. of NOLTA’07, pp. 15-18, Sep. 2007.

[5] T. Kozek, K.R. Crounse, T. Roska and L.O. Chua, “Smart Image Scanning Algorithms for the CNN Universal Machine,” Proc. of NOLTA’95, vol. 2, pp. 707-712, 1995.

[6] J. J. Hopfield, “Neurons with Graded Response Have Collective Computational Properties like Those of Two-State Neurons,” Proc. Natl. Acad. Sci. USA, vol. 81, pp. 3088-3092, 1984.

[7] K. R. Crounse, L.O. Chua, P.Thiran and G.Setti, “Characterization and Dynamics of Pattern Formation in Cellular Neural Networks, ”International Journal of Bifurcation and Chaos, vol.6, no.9, pp.1703-1724, 1996.

[8] R. Neubecker, A. Zimmermann, O. Jakoby, “Utilizing nonlinear optical pattern formation for a simple image-processing task.” Applied Physics B: Lasers and Optics Volume 76, Number 4, 383-392.

[9] K. R. Crounse and L.O. Chua, “Methods for Image Processing and Pattern Formation in Cellular Neural Networks: A Tutorial,” IEEE Trans. Circuits Syst., vol. 42, pp. 583-601, Oct. 1995.

[10] Cellular Sensory Wave Computers Laboratory Computer and Automation Research Institute Hungarian Academy of Sciences, “Cellular Wave Computing Library (Template, Algorithms, and Programs) Version 2.1”.

[11] Taisuke Nishio and Yoshifumi Nishio, “Periodic Pattern Formation and Its Applications in Cellular Neural Networks ” IEEE Trans. Circuits Syst., vol. 55, pp. 2736-2742, Oct. 2008.