Actual Circuit Experiments and Computer Simulations of Phase-Inversion Waves in In-and-Anti-Phase Synchronization on 2D Lattice Oscillators

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Abstract—We have been observing synchronization phenomena on coupled oscillators systems. A special phenomenon was observed on lattice oscillators by using computer simulations. The special phenomenon is a phenomenon changing phase states between two adjacent oscillators from in-phase synchronization to anti-phase synchronization or from anti-phase synchronization to in-phase synchronization in steady state. We call the phenomenon phase-inversion waves. In this paper, we clarify a mechanism of penetration between two phase-inversion waves in in-and-anti-phase synchronization on the lattice system. Furthermore, we analyze phase-inversion waves on the coupled oscillators systems as a lattice by using actual circuit experiments and computer simulations.

1. Introduction

There are a lot of synchronization phenomena in this world. For example, we can observe biological clocks, the synchronization of many fireflies, synchronization in combustion of candle, and so on. Furthermore, synchronization phenomena play an important role in biological and physiological systems[1]-[2].

In our previous study, we observed particular synchronization phenomena on coupled oscillators systems by using van der Pol oscillators which are coupled by inductors as a lattice[3]-[4]. We clarified regions which the phase-inversion waves can be observed in in-and-anti-phase synchronization when N = 9, and a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization. These studies analyzed by using computer simulations. In other hand, on a ladder system, we analyzed the phase-inversion waves by using actual circuit experiments[5].

In this study, we clarify a mechanism of penetration between two phase-inversion waves in in-and-anti-phase synchronization using instantaneous frequency of each oscillator and phase differences between adjacent oscillators on the lattice system. Furthermore, we analyze the phase-inversion waves on the coupled oscillators systems as a lattice by using actual circuit experiments and computer simulations.

2. Circuit model

The van der Pol oscillators are coupled by inductors $L_0$ as a lattice(see Fig. 1). The numbers of column and row of this system are assumed as "N" respectively. We name each oscillator OSC$(k,l)(0 \leq k, l \leq N - 1)$. A voltage of each oscillator is named $v(k,l)$, and a current of a inductor of each oscillator is named $i(k,l)$ (see Fig. 1).

3. Computer simulations

We observe on the coupled oscillators systems as a lattice by using computer simulations. An equation of the nonlinear negative resistor is shown as Eq. (1). Circuit equations of this circuit are normalized by Eq. (2). The normalized circuit equations are shown as Eqs. (3)-(7).

$$i_t(v(k,l)) = -g_1 v(k,l) + g_3 v^3(k,l).$$  \hspace{1cm} (1)

$$i_t(v(k,l)) = \sqrt{C_3 L_3} v(k,l), \quad v(k,l) = \sqrt{\frac{g_3}{C_3}} v(k,l),$$  \hspace{1cm} (2)

[Corner–top] \hspace{1cm} \text{(left:} \hspace{1cm} (a, b) = (0, 1), \hspace{1cm} \text{right:} \hspace{1cm} (a, b) = (N - 1, N - 2)) \hspace{1cm} \text{.]}

$$\frac{dv_{(0,a)}}{dt} = y_{(0,a)},$$ \hspace{1cm} (3)

$$\frac{dv_{(N,a)}}{dt} = -x_{(0,a)} + x_{(1,a)} - 2x_{(0,a)} + e(y_{(0,a)} - \frac{1}{3}y^3_{(0,a)}).$$

[Corner–bottom] \hspace{1cm} \text{(left:} \hspace{1cm} (a, b) = (0, 1), \hspace{1cm} \text{right:} \hspace{1cm} (a, b) = (N - 1, N - 2), \hspace{1cm} \text{.)}

$$\frac{dv_{(N,a)}}{dt} = y_{(N,a)},$$ \hspace{1cm} (4)

$$\frac{dv_{(N,a)}}{dt} = -x_{(N,a)} + x_{(N-1,a)} - 2x_{(N,a)} + e(y_{(N,a)} - \frac{1}{3}y^3_{(N,a)}).$$
Figure 2: Computer simulation result of the phase-inversion waves in in-and-anti-phase synchronization when $N = 6$.

[Center] $(0 < k < N - 1, 0 < l < N - 1)$

\[
\frac{dx_{a,b}}{dt} = y_{a,b}, \quad (5)
\]

\[
\frac{dy_{a,b}}{dt} = -x_{a,b} + \alpha(x_{a,l+1} + x_{a,l-1} + x_{a+b,1} + x_{a+b,l-1}) + \epsilon(y_{a,b} - \frac{1}{3}y_{a,b}).
\]

[Edge]

(top: $(a, b) = (0, 1)$, bottom: $(a, b) = (N - 1, N - 2)$, both: $0 < l < N - 1$)

\[
\frac{dx_{a,b}}{dt} = y_{a,b}, \quad (6)
\]

\[
\frac{dy_{a,b}}{dt} = -x_{a,b} + \alpha(x_{a,l+1} + x_{a,l-1} + x_{a+b,1} + x_{a+b,l-1}) + \epsilon(y_{a,b} - \frac{1}{3}y_{a,b}).
\]

(left: $(a, b) = (0, 1)$, right: $(a, b) = (N - 1, N - 2)$, both: $0 < k < N - 1$)

\[
\frac{dx_{a,b}}{dt} = y_{a,b}, \quad (7)
\]

\[
\frac{dy_{a,b}}{dt} = -x_{a,b} + \alpha(x_{a,k-1,0} + x_{a,k+1,0} + x_{a,k,l} + x_{a,k,l-1}) + \epsilon(y_{a,b} - \frac{1}{3}y_{a,b}).
\]

The $\alpha$ corresponds to a coupling parameter. The $\epsilon$ corresponds to a nonlinearity of each oscillator. This circuit is simulated by using the fourth order Runge-Kutta method and Eqs. (3)-(7). A computer simulation result of the phase-inversion waves in in-and-anti-phase synchronization when $N = 6$ are shown in Fig. 2. We observe laterally-propagated and longitudinally-propagated phase-inversion waves in in-and-anti-phase synchronization. The Fig. 2-A expresses an attractor of each oscillator(current vs. voltage). The Fig. 2-B expresses itinerary of phase difference by which sum of voltages of adjacent oscillators is shown along the time(sum of voltage vs. time). The coupling parameter $\alpha$ is fixed as 0.05, and nonlinearity $\epsilon$ is fixed as 0.15.

Figure 3 shows the signs of the initial values of the voltages and currents of each oscillator when the Fig. 2 can be observed. We can observe some characteristics of phase-inversion waves in in-and-anti-phase synchronization. These characteristics are a propagation, a penetration, a reflection at an edge, a reflection at a corner and a reflection between two phase-inversion waves(see Fig. 2). These characteristics are shown in Table 1.

3.1. Penetration mechanism

We can observe a phenomenon that a vertical phase-inversion wave and a horizontal phase-inversion wave penetrate each other in in-and-anti-phase synchronization. This mechanism is analyzed by using instantaneous frequency of each oscillator and phase differences between adjacent oscillators when $N = 19$.

An equation of the instantaneous frequency of $OSC(k, l)$ is obtained as Eq. (8).

\[
f_{i,k,l}(a) = \frac{1}{\tau_{i,k,l}(a) - \tau_{i,k,l-1}(a) - 1}. \quad (8)
\]

The instantaneous frequency is named $f_{i,k,l}(a)$ where “a” expresses the number of times of the positive peak value of the voltage. Time of $a$-th positive peak value of the voltage of $OSC(k, l)$ is assumed as $\tau_{i,k,l}(a)$(see Fig. 4). Similarly, $\tau_{i,k,l}(a)$ and $\tau_{i,k,l-1}(a)$ are decided. Three frequencies are observed in this system. To consider of the synchronizations for the vertical direction and for the horizontal direction are needed, because this system is 2 dimensional array. The in-phase synchronization and the anti-phase synchronization exist. Therefore, three types of synchronizations are observed as follows:

1. $OSC(k, l) - OSC(k, l + 1)$, $OSC(k, l) - OSC(k, l - 1)$, $OSC(k, l) - OSC(k + 1, l)$, and $OSC(k, l) - OSC(k - 1, l)$ are the in-phase synchronization.

2. $\ominus$ ($OSC(k, l) - OSC(k, l - 1)$, and $OSC(k, l) - OSC(k, l + 1)$ are a same phase synchronization state, and $OSC(k, l) - OSC(k - l)$, and $OSC(k, l) - OSC(k + 1, l)$ are another phase synchronization state.) $\ominus$ ($OSC(k, l) - OSC(k - 1, l)$, and $OSC(k, l) - OSC(k + 1, l)$ are a same phase synchronization state, and $OSC(k, l) - OSC(k - l)$, and $OSC(k, l) - OSC(k + 1, l)$ are another phase synchronization state.) $\ominus$ ($OSC(k, l) - OSC(k - 1, l)$, and $OSC(k, l) - OSC(k + 1, l)$ are a same phase synchronization state, and $OSC(k, l) - OSC(k - l)$, and $OSC(k, l) - OSC(k + 1, l)$ are another phase synchronization state.)

3. $OSC(k, l) - OSC(k, l + 1)$, $OSC(k, l) - OSC(k, l - 1)$, $OSC(k, l) - OSC(k + 1, l)$, and $OSC(k, l) - OSC(k - 1, l)$ are the anti-phase synchronization.
Table 1: Characteristics of the phase-inversion waves in in-and-anti-phase synchronization.

<table>
<thead>
<tr>
<th>Names of characteristics</th>
<th>Phenomena</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagations</td>
<td>The phase-inversion waves propagate for vertical direction or horizontal direction. The vertical phase-inversion waves independently move from the horizontal phase-inversion waves.</td>
</tr>
<tr>
<td>Penetrations</td>
<td>Two phase-inversion waves arrive at an oscillator from different direction and horizontal direction, and each phase-inversion wave penetrates each other.</td>
</tr>
<tr>
<td>Reflections at an edge</td>
<td>When a phase-inversion wave arrives at an edge, the phase-inversion wave reflects and propagates to where they came from. Sometime this phenomenon is happened with penetration.</td>
</tr>
<tr>
<td>Reflections at a corner</td>
<td>When two phase-inversion waves coming from the vertical direction and the horizontal direction arrive at a corner oscillator at the same time, the phase-inversion waves reflect and propagate to where they came from.</td>
</tr>
<tr>
<td>Reflections between two phase-inversion waves</td>
<td>When two phase-inversion waves coming from the opposite directions arrive to two adjacent oscillator at same time, the phase-inversion waves reflect and propagate to where they came from.</td>
</tr>
</tbody>
</table>

Figure 4: The detection method of frequencies and the phase differences.

OSC($k$, $l$) and OSC($k$, $l + 1$) are obtained. The phase differences are assumed as $\Phi_{(k,l)(k+1,l)}(a)$ and $\Phi_{(k,l)(k+1,l+1)}(a)$ respectively. The $\Phi_{(k,l)(k+1,l)}(a)$ and $\Phi_{(k,l)(k+1,l+1)}(a)$ are obtained by Eq. (9)(see Fig. 4).

$$\Phi_{(k,l)(k+1,l)}(a) = \frac{\tau_{(k+1,l)}(a) - \tau_{(k,l)}(a)}{\tau_{(k,l)}(a) - \tau_{(k,l-1)}(a)} \times 360 \text{ [degree]}$$

$$\Phi_{(k,l)(k+1,l+1)}(a) = \frac{\tau_{(k+1,l+1)}(a) - \tau_{(k,l)}(a)}{\tau_{(k,l)}(a) - \tau_{(k,l-1)}(a)} \times 360 \text{ [degree]}$$

Penetration mechanism is shown in Table 2(see Fig. 5). In Fig. 5(a), the vertical axis is instantaneous frequency, and horizontal axis is time. In Fig. 5(b), the vertical axis is the phase difference, and the horizontal axis is time.

4. Actual circuit experiments

We confirm which the phase-inversion waves can be stable and actually existing in in-and-anti-phase synchronization on the coupled oscillators systems as a lattice by using actual circuit experiments. The actual circuit experimental result of $N = 6$ shows in Fig. 6. We can observe same phenomenon by using the computer simulation(See Fig 7). Figures 8 and 9 are extracted from the Figs. 6 and 7. The phase-inversion waves of horizontal direction exist in in-and-anti-phase synchronization. We can consider that the small difference between these two results differ from the resistances in real inductors and variations on electric elements.

5. Conclusion

We clarified a mechanism of penetration between two phase-inversion waves in in-and-anti-phase synchronization on the lattice system. Furthermore, we analyzed phase-inversion waves on the coupled oscillators systems as a lattice by using actual circuit experiments and computer simulations. We observed same phenomenon in the actual circuit experimental results and the computer simulation results(see Figs. 6 and 7). We proved that the phase-inversion waves can be stable and actually existing on actual circuit.

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References


A horizontal phase-inversion wave and a vertical phase-inversion wave arrives at OSC(5,5).

Phase-inversion waves, which changes synchronized state, are arrived at the number of row is 7 from the number of row is 8.

1 A phase difference $\Phi_{5,5}(7,5)$ starts to change from the anti-phase synchronization toward the in-phase synchronization by a phase-inversion wave.

2 A instantaneous frequency $f_{4,5}$ starts to change from $f_{\text{fin}}$ toward $f_{\text{fin}}$, because $\Phi_{4,5}(5,5)$ starts to change from the anti-phase synchronization toward the in-phase synchronization.

3 $\Phi_{5,5}(6,5)$ and $\Phi_{4,5}(6,4)$ start to from the in-phase synchronization toward the anti-phase synchronization, because $f_{4,6}$ and $f_{5,6}$ start to change from $f_{\text{fin}}$ toward $f_{\text{fin}}$.

4 A horizontal phase-inversion wave and a vertical phase-inversion wave arrives at OSC(5,5). $f_{5,5}$ starts to change from $f_{\text{fin}}$ toward $f_{\text{anti-anti}}$.

5 $\Phi_{5,5}(5,5)$ starts to change from the anti-phase synchronization toward the in-phase synchronization, because $f_{5,5}$ starts to change from $f_{\text{anti-anti}}$ toward $f_{\text{anti-anti}}$.

The phase-inversion waves penetrate each other by above mechanism.

Table 2: Penetration mechanism of two phase-inversion waves (see Fig. 5).

<table>
<thead>
<tr>
<th>no.</th>
<th>vertical direction</th>
<th>horizontal direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>At this time, $\Phi_{4,5}(5,5)$ and $\Phi_{6,5}(7,5)$ are fixed the anti-phase synchronization. $\Phi_{5,5}(6,5)$ and $\Phi_{4,5}(6,4)$ are fixed the in-phase synchronization. Phase-inversion waves, which changes synchronized state, are arrived at the number of row is 7 from the number of row is 8.</td>
<td>At this time, $\Phi_{5,5}(5,5)$ and $\Phi_{5,6}(6,5)$ are fixed the anti-phase synchronization. $\Phi_{5,5}(6,5)$ and $\Phi_{4,5}(4,6)$ are fixed the in-phase synchronization. Phase-inversion waves, which changes synchronized state, are arrived at the number of column is 7 from the number of column is 8.</td>
</tr>
</tbody>
</table>
