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Actual Circuit Experiments and Computer Simulations of Phase-Inversion Waves in In-and-Anti-Phase Synchronization on 2D Lattice Oscillators

Seiko Kunihiro†, Ryusei Okamoto†,Hitoshi Aburatani†, Masayuki Yamauchi† and Yoshifumi Nishio‡

†Department of Electronics and Computer Engineering,
Hiroshima Institute of Technology
2-1-1,Miyake, Saeki-ku, Hiroshima , Japan
Email: seiko.kunihiro@gmail.com
‡Department of Electrical and Electronic Engineering,
Tokushima University
2-1, Minami-Josanjima, Tokushima, Japan

Abstract—We have been observing synchronization phenomena on coupled oscillators systems. A special phenomenon was observed on lattice oscillators by using computer simulations. The special phenomenon is a phenomenon changing phase states between two adjacent oscillators from in-phase synchronization to anti-phase synchronization or from anti-phase synchronization to in-phase synchronization in steady state. We call the phenomenon phase-inversion waves. In this paper, we clarify a mechanism of penetration between two phase-inversion waves in in-and-anti-phase synchronization on the lattice system. Furthermore, we analyze phase-inversion waves on the coupled oscillators systems as a lattice by using actual circuit experiments and computer simulations.

1. Introduction

There are a lot of synchronization phenomena in this world. For example, we can observe biological clocks, the synchronization of many fireflies, synchronization in combustion of candle, and so on. Furthermore, synchronization phenomena play an important role in biological and physiological systems[1]-[2].

In our previous study, we observed particular synchronization phenomena on coupled oscillators systems by using van der Pol oscillators which are coupled by inductors as a lattice[3]-[4]. We clarified regions which the phase-inversion waves can be observed in in-and-anti-phase synchronization when $N = 9$, and a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization. These studies analyzed by using computer simulations. In other hand, on a ladder system, we analyzed the phase-inversion waves by using actual circuit experiments[5].

In this study, we clarify a mechanism of penetration between two phase-inversion waves in in-and-anti-phase synchronization using instantaneous frequency of each oscillator and phase differences between adjacent oscillators on the lattice system. Furthermore, we analyze the phase-inversion waves on the coupled oscillators systems as a lattice by using actual circuit experiments and computer simulations.

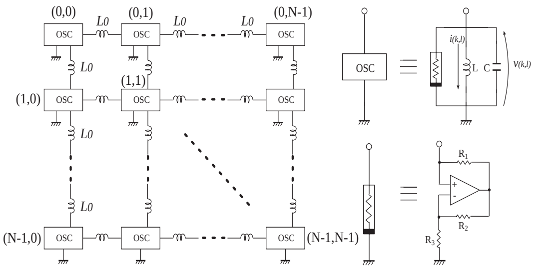

Figure 1: Circuit model

2. Circuit model

The van der Pol oscillators are coupled by inductors L_0 as a lattice(see Fig. 1). The numbers of column and row of this system are assumed as "N" respectively. We name each oscillator OSC(k,l)($0 \le k, l \le N-1$). A voltage of each oscillator is named $v_{(k,l)}$, and a current of a inductor of each oscillator is named $i_{(k,l)}$(see Fig. 1).

3. Computer simulations

We observe on the coupled oscillators systems as a lattice by using computer simulations. An equation of the nonlinear negative resistor is shown as Eq. (1). Circuit equations of this circuit are normalized by Eq. (2). The normalized circuit equations are shown as Eqs. (3)–(7).

$$i_r(v_{(k,l)}) = -g_1 v_{(k,l)} + g_3 v_{(k,l)}^3. \quad (1)$$

$$\begin{aligned} &i_{(k,l)} = \sqrt{\frac{Cg_1}{3Lg_3}}\, x_{(k,l)}, \quad v_{(k,l)} = \sqrt{\frac{g_1}{3g_3}}\, y_{(k,l)}, \\ &t = \sqrt{LC}\tau, \quad \frac{d}{d\tau} = \text{“ } \cdot \text{ ”}, \alpha = \frac{L}{L_0}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}}. \end{aligned} \quad (2)$$

[Corner–top] (left:(a, b)=$(0, 1)$. right:(a, b)=$(N-1, N-2)$.)

$$\begin{aligned} \frac{dx_{(0,a)}}{d\tau} &= y_{(0,a)}, \\ \frac{dy_{(0,a)}}{d\tau} &= -x_{(0,a)} + \alpha(x_{(0,b)} + x_{(1,a)} - 2x_{(0,a)}) \\ &\quad + \varepsilon(y_{(0,a)} - \frac{1}{3}y_{(0,a)}^3). \end{aligned} \quad (3)$$

[Corner–bottom] (left:(a, b)=$(0, 1)$. right:(a, b)=$(N-1, N-2)$.)

$$\begin{aligned} \frac{dx_{(N,a)}}{d\tau} &= y_{(N,a)}, \\ \frac{dy_{(N,a)}}{d\tau} &= -x_{(N,a)} + \alpha(x_{(N-1,a)} + x_{(N,b)} - 2x_{(N,a)}) \\ &\quad + \varepsilon(y_{(N,a)} - \tfrac{1}{3}y_{(N,a)}^3). \end{aligned} \quad (4)$$

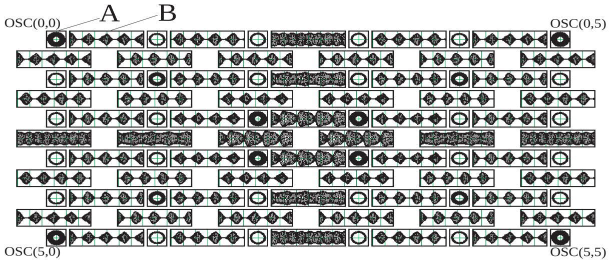

Figure 2: Computer simulation result of the phase-inversion waves in in-and-anti-phase synchronization when $N = 6$.

[Center] $(0 < k < N-1.\ 0 < l < N-1.)$

$$\frac{dx_{(k,l)}}{d\tau} = y_{(k,l)}, \tag{5}$$

$$\frac{dy_{(k,l)}}{d\tau} = -x_{(k,l)} + \alpha(x_{(k+1,l)} + x_{(k-1,l)} + x_{(k,l+1)} + x_{(k,l-1)} - 4x_{(k,l)}) + \varepsilon(y_{(k,l)} - \tfrac{1}{3}y^3_{(k,l)}).$$

[Edge]

(top:(a,b)=$(0,1)$.bottom:(a,b)=$(N-1,N-2)$.both:$0 < l < N-1$.)

$$\frac{dx_{(a,l)}}{d\tau} = y_{(a,l)}, \tag{6}$$

$$\frac{dy_{(a,l)}}{d\tau} = -x_{(a,l)} + \alpha(x_{(a,l-1)} + x_{(a,l+1)} + x_{(b,l)} - 3x_{(a,l)}) + \varepsilon(y_{(a,l)} - \tfrac{1}{3}y^3_{(a,l)}).$$

(left:(a,b)=$(0,1)$. right:(a,b)=$(N-1,N-2)$. both:$0 < k < N-1$.)

$$\frac{dx_{(k,a)}}{d\tau} = y_{(k,a)}, \tag{7}$$

$$\frac{dy_{(k,a)}}{d\tau} = -x_{(k,a)} + \alpha(x_{(k-1,a)} + x_{(k+1,a)} + x_{(k,b)} - 3x_{(k,a)}) + \varepsilon(y_{(k,a)} - \tfrac{1}{3}y^3_{(k,a)}).$$

The α corresponds to a coupling parameter. The ε corresponds to a nonlinearity of each oscillator. This circuit is simulated by using the fourth order Runge-Kutta method and Eqs. (3)-(7). A computer simulation result of the phase-inversion waves in in-and-anti-phase synchronization when $N = 6$ are shown in Fig. 2. We observe laterally-propagated and longitudinally-propagated phase-inversion waves in in-and-anti-phase synchronization. The Fig. 2–A expresses an attractor of each oscillator(current vs. voltage). The Fig. 2–B expresses itinerancy of phase difference by which sum of voltages of adjacent oscillators is shown along the time(sum of voltage vs. time). The coupling parameter α is fixed as 0.05, and nonlinearity ε is fixed as 0.15.

Figure 3 shows the signs of the initial values of the voltages and currents of each oscillator when the Fig. 2 can be observed. We can observe some characteristics of phase-inversion waves in in-and-anti-phase synchronization. These characteristics are a propagation, a penetration, a reflection at an edge, a reflection at a corner and a reflection between two phase-inversion waves(see Fig. 2). These characteristics are shown in Table 1.

	0	1	2	3	4	5
0	+	+	−	−	+	+
1	+	+	−	−	+	+
2	−	−	+	+	−	−
3	−	−	+	+	−	−
4	+	+	−	−	+	+
5	+	+	−	−	+	+

Figure 3: Sign of initial value of each oscillator when the Fig. 2 can be observed.

3.1. Penetration mechanism

We can observe a phenomenon that a vertical phase-inversion wave and a horizontal phase-inversion wave penetrate each other in in-and-anti-phase synchronization. This mechanism is analyzed by using instantaneous frequency of each oscillator and phase differences between adjacent oscillators when $N = 19$.

An equation of the instantaneous frequency of OSC(k, l) is obtained as Eq. (8).

$$f_{(k,l)}(a) = \frac{1}{\tau_{(k,l)}(a) - \tau_{(k,l)}(a-1)}. \tag{8}$$

The instantaneous frequency is named $f_{(k,l)}(a)$ where "a" expresses the number of times of the positive peak value of the voltage. Time of a-th positive peak value of the voltage of OSC(k, l) is assumed as $\tau_{(k,l)}(a)$(see Fig. 4). Similarly, $\tau_{(k+1,l)}(a)$ and $\tau_{(k,l+1)}(a)$ are decided. Three frequencies are observed in this system. To consider of the synchronizations for the vertical direction and for the horizontal direction are needed, because this system is 2 dimensional array. The in-phase synchronization and the anti-phase synchronization exist. Therefore, three types of synchronizations are observed as follows:

1. OSC(k, l)–OSC(k, $l+1$), OSC(k, l)–OSC(k, $l-1$), OSC(k, l)–OSC($k+1$, l), and OSC(k, l)–OSC($k-1$, l) are the in-phase synchronization.
2. {① (OSC(k, l)–OSC(k, $l-1$), and OSC(k, l)–OSC(k, l+1) are a same phase synchronization state, and OSC(k, l)–OSC($k-1$, l), and OSC(k, l)–OSC($k+1$, l) are another phase synchronization state.} , {② OSC(k, l)–OSC($k-1$, l), and OSC(k, l)–OSC(k, $l-1$) are a same phase synchronization state, and OSC(k, l)–OSC($k+1$, l), and OSC(k, l)–OSC(k, $l+1$) are another phase synchronization state.} , {③ OSC(k, l)–OSC($k-1$, l), and OSC(k, l)–OSC(k, $l+1$) are a same phase synchronization state, and OSC(k, l)–OSC(k, $l-1$), and OSC(k, l)–OSC($k+1$, l) are another phase synchronization state.}
3. OSC(k, l)–OSC(k, $l+1$), OSC(k, l)–OSC(k, $l-1$), OSC(k, l)–OSC($k+1$, l), and OSC(k, l)–OSC($k+1$, l) are the anti-phase synchronization.

An instantaneous frequency $f_{(k,l)}$ of OSC(k, l) is obtained in each synchronization-type. The 1st situational synchronization frequency is called f_{in-in}. The 2nd situational synchronization frequency is called $f_{in-anti}$. The 3rd situational synchronization frequency is called $f_{anti-anti}$. The phase difference is calculated as follows. A phase difference between OSC(k, l) and OSC($k+1$, l) and a phase difference between

Table 1: Characteristics of the phase-inversion waves in in-and-anti-phase synchronization.

Names of characteristics	*Phenomena*
Propagations	The phase-inversion waves propagate for vertical direction or horizontal direction. The vertical phase-inversion waves independently move from the horizontal phase-inversion waves.
Penetrations	Two phase-inversion waves arrive at an oscillator from vertical direction and horizontal direction, and each phase-inversion wave penetrates each other.
Reflections at an edge	When a phase-inversion wave arrives at an edge, the phase-inversion wave reflects and propagates to where they came from. Sometime this phenomenon is happened with penetration.
Reflections at a corner	When two phase-inversion waves coming from the vertical direction and the horizontal direction arrive at a corner oscillator at the same time, the phase-inversion waves reflect and propagate to where they came from.
Reflections between two phase-inversion waves	When two phase-inversion waves coming from the opposite directions arrive to two adjacent oscillator at same time, the phase-inversion waves reflect and propagate to where they came from.

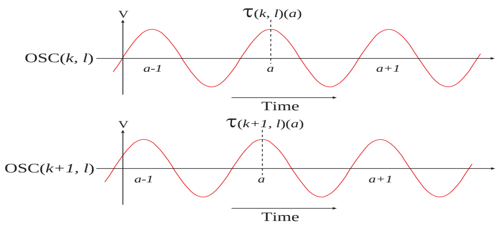

Figure 4: The detection method of frequencies and the phase differences.

OSC(k, l) and OSC(k, $l+1$) are obtained. The phase differences are assumed as $\Phi_{(k,l)(k+1,l)}(a)$ and $\Phi_{(k,l)(k,l+1)}(a)$ respectively. The $\Phi_{(k,l)(k+1,l)}(a)$ and $\Phi_{(k,l)(k,l+1)}(a)$ are obtained by Eq. (9)(see Fig. 4).

$$\Phi_{(k,l)(k+1,l)}(a) = \frac{\tau_{(k,l)}(a) - \tau_{(k+1,l)}(a)}{\tau_{(k,l)}(a) - \tau_{(k,l)}(a-1)} \times 360 \text{ [degree]}$$
$$\Phi_{(k,l)(k,l+1)}(a) = \frac{\tau_{(k,l)}(a) - \tau_{(k,l+1)}(a)}{\tau_{(k,l)}(a) - \tau_{(k,l)}(a-1)} \times 360 \text{ [degree]}. \quad (9)$$

Penetration mechanism is shown in Table 2(see Fig. 5). In Fig. 5(a), the vertical axis is instantaneous frequency, and horizontal axis is time. In Fig. 5(b), the vertical axis is the phase difference, and the horizontal axis is time.

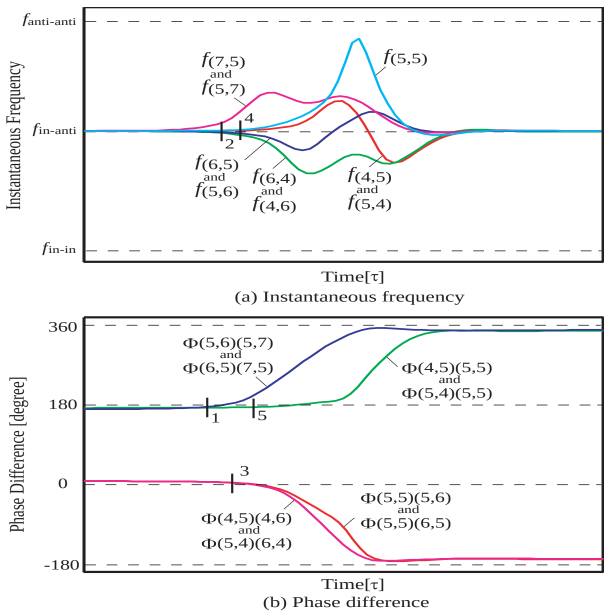

Figure 5: Transitions of phase difference and frequencies by penetration between two phase-inversion waves on in-and-anti-phase synchronization.

4. Actual circuit experiments

We confirm which the phase-inversion waves can be stable and actually existing in in-and-anti-phase synchronization on the coupled oscillators systems as a lattice by using actual circuit experiments. The actual circuit experimental result of $N = 6$ shows in Fig. 6. We can observe same phenomenon by using the computer simulation(See Fig 7). Figures 8 and 9 are extracted from the Figs. 6 and 7. The phase-inversion waves of horizontal direction exist in in-and-anti-phase synchronization. We can consider that the small difference between these two results differ from the resistances in real inductors and variations on electric elements.

5. Conclusion

We clarified a mechanism of penetration between two phase-inversion waves in in-and-anti-phase synchronization on the lattice system. Furthermore, we analyzed of phase-inversion waves on the coupled oscillators systems as a lattice by using actual circuit experiments and computer simulations. We observed same phenomenon in the actual circuit experimental results and the computer simulation results(see Figs. 6 and 7). We proved that the phase-inversion waves can be stable and actually existing on actual circuit.

Acknowledgements

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Table 2: Penetration mechanism of two phase-inversion waves(see Fig. 5).

no.	*vertical direction*	*horizontal direction*
0	At this time, $\Phi_{(4,5)(5,5)}$ and $\Phi_{(6,5)(7,5)}$ are fixed the anti-phase synchronization. $\Phi_{(5,5)(6,5)}$ and $\Phi_{(5,4)(6,4)}$ are fixed the in-phase synchronization. Phase-inversion waves, which changes synchronized state, are arrived at the number of row is 7 from the number of row is 8.	At this time, $\Phi_{(5,4)(5,5)}$ and $\Phi_{(5,6)(5,7)}$ are fixed the anti-phase synchronization. $\Phi_{(5,5)(5,6)}$ and $\Phi_{(4,5)(4,6)}$ are fixed the in-phase synchronization. Phase-inversion waves, which changes synchronized state, are arrived at the number of column is 7 from the number of column is 8.
1	A phase difference $\Phi_{(6,5)(7,5)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization by a phase-inversion wave.	A phase difference $\Phi_{(5,6)(5,7)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization by a phase-inversion wave.
2	A instantaneous frequency $f_{(6,5)}$ starts to change from $f_{in-anti}$ toward f_{in-in}, because $\Phi_{(6,5)(7,5)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization. $f_{(6,4)}$ also starts to change from $f_{in-anti}$ toward f_{in-in}, because a phase-inversion wave of adjacent vertical direction arrives at 6th row line at the same time.	A instantaneous frequency $f_{(5,6)}$ starts to change from $f_{in-anti}$ toward f_{in-in}, because $\Phi_{(5,6)(5,7)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization. $f_{(4,6)}$ also starts to change from $f_{in-anti}$ toward f_{in-in}, because a phase-inversion wave of adjacent horizontal direction arrives at 6th column line at the same time.
3	$\Phi_{(5,5)(6,5)}$ and $\Phi_{(5,4)(6,4)}$ start to from the in-phase synchronization toward the anti-phase synchronization, because $f_{(6,5)}$ and $f_{(6,4)}$ start to change from $f_{in-anti}$ toward f_{in-in}.	$\Phi_{(5,5)(5,6)}$ and $\Phi_{(4,5)(4,6)}$ start to from the in-phase synchronization toward the anti-phase synchronization, because $f_{(5,6)}$ and $f_{(4,6)}$ start to change from $f_{in-anti}$ toward f_{in-in}.
4	A horizontal phase-inversion wave and a vertical phase-inversion wave arrives at OSC(5,5). $f_{(5,5)}$ starts to change from $f_{in-anti}$ toward $f_{anti-anti}$. $f_{(4,5)}$ and $f_{(5,4)}$ start to change from $f_{in-anti}$ toward $f_{anti-anti}$, because $\Phi_{(5,4)(6,4)}$ and $\Phi_{(4,5)(4,6)}$ start to change from the in-phase synchronization toward the anti-phase synchronization.	
5	$\Phi_{(4,5)(5,5)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization, because $f_{(5,5)}$ starts to change from $f_{in-anti}$ toward $f_{anti-anti}$.	$\Phi_{(5,4)(5,5)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization, because $f_{(5,5)}$ starts to change from $f_{in-anti}$ toward $f_{anti-anti}$.
The phase-inversion waves penetrate each other by above mechanism.		

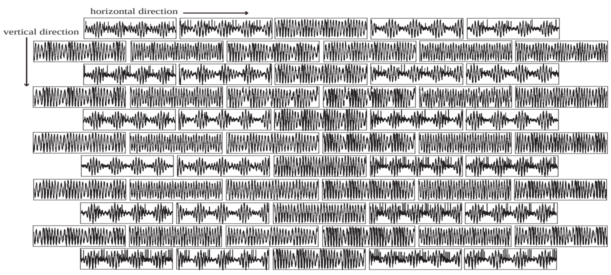

Figure 6: Actual circuit experimental result of N=6. ($R_1 \simeq R_2 \simeq 16$[kΩ],$R_3 \simeq 1$[kΩ], $C \simeq 4.7$[nF],$L \simeq 100$[mH] and $L_0 \simeq 50$[μ H]).

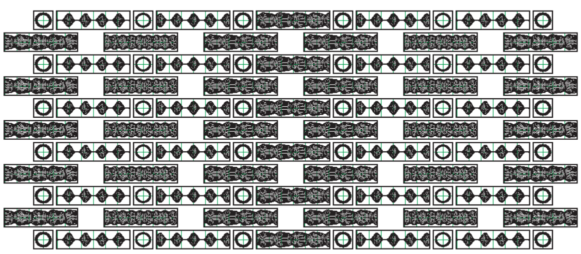

Figure 7: Computer simulation result of N=6. (α= 0.05 and ε=0.15.)

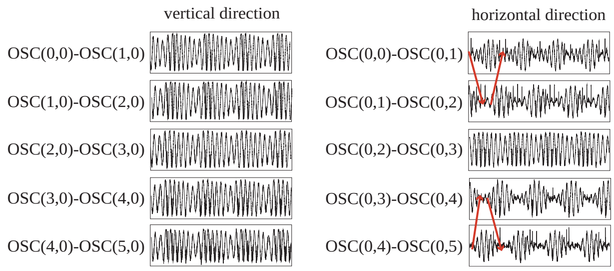

Figure 8: Extraction of the Fig.6 in vertical direction and horizontal direction.

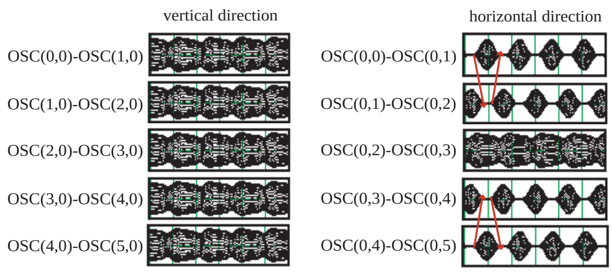

Figure 9: Extraction of the Fig.7 in vertical direction and horizontal direction.

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