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Ant Colony Optimization Changing the Rate of Dull Ants and its Application to QAP

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Abstract— **In our previous study, we have proposed an Ant Colony Optimization with Intelligent and Dull Ants (IDACO) which contains two kinds of ants. We have applied IDACO to various Traveling Salesman Problems (TSPs) and confirmed its effectiveness. This study proposes an Ant Colony Optimization Changing the Rate of Dull Ants (IDACO-CR) and its Application to Quadratic Assignment Problems (QAPs). In addition to the existence of the dull ants which cannot trail the pheromone, the rate of dull ants in IDACO-CR is changed flexibly and automatically in the simulation, depending on the problem. We investigate the behavior of IDACO-CR in detail and the effect of changing the rate of dull ants. Simulation results show that IDACO-CR gets out from the local optima by changing the rate of dull ants, and we confirm that IDACO-CR obtains the effective results in solving complex optimization problems.**

I. Introduction

ANT COLONY OPTIMIZATION (ACO) [1] is an evaluational optimization algorithm inspired by a pheromone effect of ants and is effective to solve difficult combinatorial optimization problems, such as graph coloring problem, routing in communications networks, Traveling Salesman Problem (TSP) and so on [2]-[4]. It has also been used to produce near-optimal solutions to the Quadratic Assignment Problem (QAP) [5]. QAP is a generalization of the TSP, and it is also an NP-hard combinatorial optimization problem. Given two matrices corresponding to a distance and a flow between activities, the task is to assign all activities to different locations with the minimum cost among all of the possible combinations. In ACO algorithm, multiple solutions called "ants" coexist, and the ants drop pheromone about an assignment of activity. Pheromone trails are updated depending on the behavior of the ants. The ants find a food source through paths having strong pheromone. By communicating with other ants according to the pheromone strength, the algorithm tries to find the optimal solution. However, ACO has a problem which is to fall into local solutions. Therefore, it is important to enhance algorithm performances by improving its flexibility.

Meanwhile, it has been reported that about 20 percent of the ants are unnecessary ants called "dull ant" in the real ant's world [6]. The dull ant keeps still around its colony whereas the other ants in the colony perform feeding behavior. In a computational experiment, the researchers performed the feeding behavior by using intelligent ants, which can exactly trail the pheromone, and dull ants which cannot trail the pheromone. In the results, the ants group including the dull ants was able to obtain more foods than the group containing only the intelligent ants. It means that the coexistence of the dull ant improves the effectiveness of the feeding behavior.

In our previous study [7], we have proposed a new type of ACO algorithm, which is called Ant Colony Optimization with Intelligent and Dull ants (IDACO). The important feature of IDACO is that two kinds of ants coexist. The one is an *intelligent ant* and the another is a *dull ant*. The intelligent ant can trail the pheromone exactly, however the dull ant cannot trail the pheromone exactly. Because their features are essentially similar to the real ant's world, we can say that IDACO algorithm is nearer to the real ant colony than the conventional ACO algorithm. We have applied the IDACO to various TSPs and have confirmed an interesting result that IDACO including the dull ants obtained better results than the standard ACO consisted of only the intelligent ants. However, since the difficulty of getting out from the local optima varies from problem to problem, we have to consider changing the rate of dull ants flexibly depending on the problem.

In this study, we propose an improved IDACO; ACO Changing the Rate of Dull Ants (IDACO-CR). In addition to the existence of the dull ants, the rate of dull ants in IDACO-CR flexibly changes in the optimization. In other words, the rate of dull ants is automatically determined depending on the problem. Furthermore, we propose an application of IDACO-CR to QAP. By applying the proposed method to various QAPs, we confirm the behavior of IDACO-CR, its efficiency and an importance of the dull ants.

This paper is organized as follows. In the Section II, the QAP is introduced. The basic method of ACO for QAP is introduced in the Section III. We explain about the proposed algorithm of IDACO-CR for QAP in detail in the Section IV. In the Section V, we apply IDACO-CR to five QAPs whose number of locations and facilities are different. Furthermore, we investigate the behavior of IDACO-CR in detail. We, in addition to IDACO-CR, consider the conventional IDACO for QAP whose algorithm does not change the rate of the dull ants in the simulation. Performances are evaluated both visually and quantitatively in comparison with the conventional ACO and the conventional IDACO. Finally, the Section VI concludes this paper.

II. Quadratic Assignment Problem

The QAP of order n is a problem which looking for the best allocation of n activities in n locations. The QAP has been recognized as a model of many different real situations;

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arrangement of departments in the hospitals, minimization of the total wire length in electronic circuits and so on. Mathematically the problem is defined by two matrices of the dimension $n \times n$;

$\boldsymbol{D}$ =$[l_{ih}]$ = matrix of the distances (between location i and location h).

$\boldsymbol{F}$ = $[f_{jk}]$ = matrix of the flows (between activity j and activity k).

In order to solve the QAP, we find a permutation Π which corresponds to the minimum value of the total assignment cost L in Eq. (1).

$$L = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} f_{\pi(i)\pi(j)}, \tag{1}$$

where l_{ij} and f_{ij} are the (i, j)-th elements of $\boldsymbol{D}$ and $\boldsymbol{F}$, respectively, $\pi(i)$ is the i-th element of the vector Π, and n is the size of the problem. From the two matrices $\boldsymbol{D}$ and $\boldsymbol{F}$, one sums the elements of each line to obtain two vectors $\mathcal{D}$ and $\mathcal{F}$ called *distance potentials* and *flow potentials*, respectively. Furthermore, from the two vectors $\mathcal{D}$ and $\mathcal{F}$, one may obtain a matrix $\boldsymbol{A}$ called the *coupling matrix*. The element a_{ij} of $\boldsymbol{A}$ is given by the product $\mathcal{D}_i \cdot \mathcal{F}_j$. We show an example of calculate by using Nugment's problem of order 5. Two matrices $\boldsymbol{D}$ and $\boldsymbol{F}$ is shown as

$$\boldsymbol{D} = \begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 1 & 0 & 2 & 1 & 2 \\ 1 & 2 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 & 1 \\ 3 & 2 & 2 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{F} = \begin{bmatrix} 0 & 5 & 2 & 4 & 1 \\ 5 & 0 & 3 & 0 & 2 \\ 2 & 3 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 5 \\ 1 & 2 & 0 & 5 & 0 \end{bmatrix}.$$

We obtain the vectors of the potentials $\mathcal{D}$ and $\mathcal{F}$;

$$\mathcal{D} = \begin{bmatrix} 7 \\ 6 \\ 6 \\ 5 \\ 8 \end{bmatrix}, \mathcal{F} = \begin{bmatrix} 12 \\ 10 \\ 5 \\ 9 \\ 8 \end{bmatrix}.$$

$\boldsymbol{A}$ is given from the two vectors $\mathcal{D}$ and $\mathcal{F}$. In Nugent's example, this corresponds to the following values;

$$\boldsymbol{A} = \begin{bmatrix} 84 & 70 & 35 & 63 & 56 \\ 72 & 60 & 30 & 54 & 48 \\ 72 & 60 & 30 & 54 & 48 \\ 60 & 50 & 25 & 45 & 40 \\ 96 & 80 & 40 & 72 & 64 \end{bmatrix}.$$

$\boldsymbol{A}$ shows potential cost of assignment. Thus, a_{ij} nearer 0 is more desirable.

III. Ant Colony Optimization for QAP

We explain the conventional ACO algorithm for QAP in detail. A flowchart of ACO algorithm for QAP is shown in Fig. 1. The number of ants is denoted by M. Total assignment cost L is same as Eq. (1).

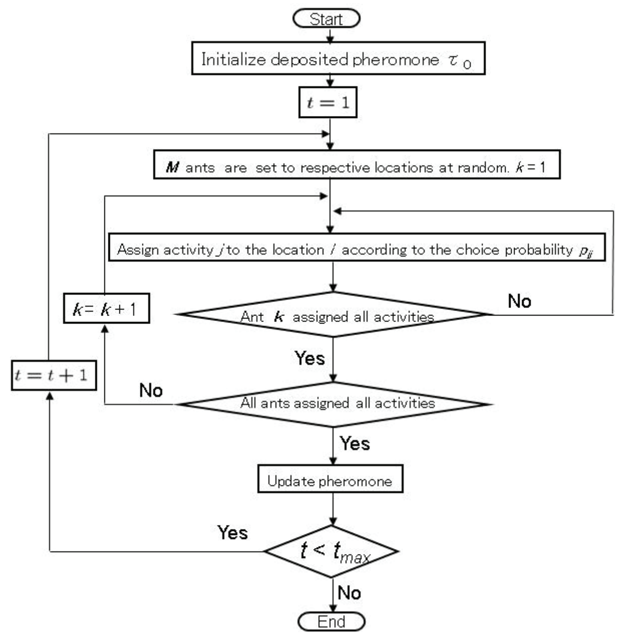

Fig. 1. Flowchart of the conventional ACO algorithm for QAP

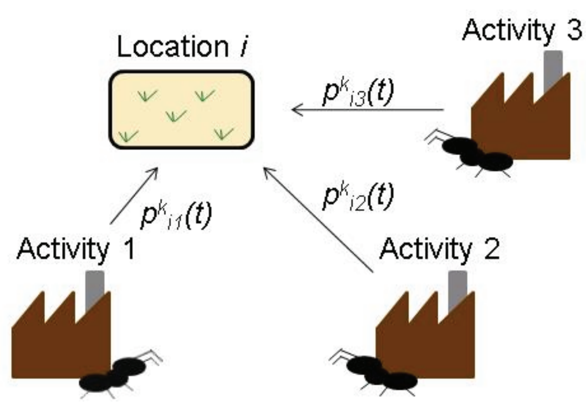

Fig. 2. Choice probability $p_{ij}(t)$ of ants. The assigning activity is chosen by probability $p_{ij}(t)$

[ACO1](Initialization): Let an iteration number $t = 0$. $\tau_{ij}(t)$ is an amount of pheromone trail on a coupling (i, j) to assign an activity j to the location i, and $\tau_{ij}(0) = \tau_0$.

[ACO2](Assign activity): The assigning activity is chosen by according to the probability $p_{ij}(t)$ as shown in Fig. 2. The probability that k-th ant ($k = 1, \cdots, M$) assigns activity j to the location i is decided by

$$p_{k_{ij}}(t) = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_k}[\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta}}, \tag{2}$$

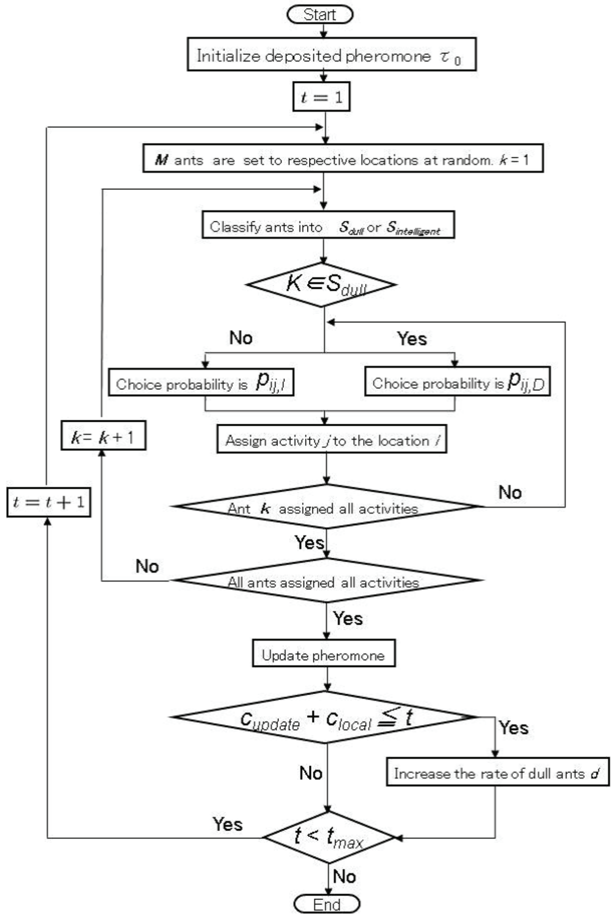

Fig. 3. Flowchart of IDACO-CR algorithm for QAP

where η_{ij} is defined as the inverse of the coupling matrix element a_{ij}. The adjustable parameters α and β control the weight of pheromone intensity and of the coupling matrix element, respectively. Therefore, a searching ability goes up and down by changing α and β. N^k is a set of activities such that k-th ant has not yet assigned any activity in the set. The ants repeat assigning activity until all the activities are assigned to the locations.

[ACO3](Pheromone update): After all the ants have completed assignment, the total cost $L_k(t)$ is computed, and the amount of the pheromone $\tau_{ij}(t)$ is updated. The amount of the pheromone $\Delta\tau_{ij}^k$ deposited by k-th ant on the coupling (i, j) is decided as

$$\Delta\tau_{ij}^k(t) = \begin{cases} 10/L^k, & \text{if } (i,j) \in T^k(t) \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $T_k(t)$ is obtained permutation by k-th ant and $L_k(t)$ is its total cost. $\rho \in [0, 1]$ is the pheromone trail decay coefficient. Update $\tau_{ij}(t)$ of each coupling (i, j) depending on its $\Delta\tau_{ij}^k$;

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \sum_{k=1}^{M} \Delta\tau_{ij}^k(t). \tag{4}$$

[ACO4] Let $t = t + 1$. Go back to [ACO2] and repeat until $t = t_{\max}$.

IV. ACO Changing the Rate of dull ants (IDACO-CR) for QAP

We propose ACO Changing the Rate of Dull Ants (IDACO-CR) and explain its optimization algorithm for QAP in detail. A flowchart of the proposed IDACO-CR algorithm for QAP is shown in Fig. 3. Important features of IDACO-CR for QAP are follows;

(1) *Two kinds of ants coexist; intelligent ants which can exactly trail pheromone, and dull ants which cannot trail the pheromone.*
(2) *The rate of dull ants are flexibly changed if the system is trapped into the local optima.*

The number of ants is denoted by M. Total assignment cost L is same as Eq. (1).

[IDACO-CR1](Initialization): Let the iteration number $t = 0$. $\tau_{ij}(0) = \tau_0$.

[IDACO-CR2](Classification): $(1-d) \times M$ ants and $d \times M$ ants are classified into a set of the intelligent ants $S_{\text{intelligent}}$ and of the dull ants S_{dull}, respectively. d is the rate of dull

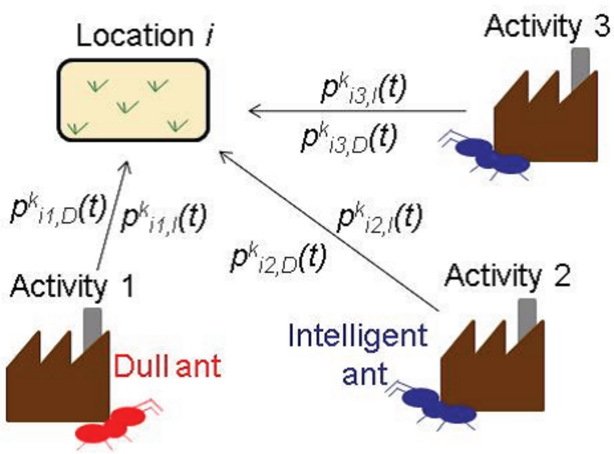

Fig. 4. Choice probability $p_{ij}(t)$ of Intelligent and Dull ants. The assigning activity of intelligent ant is chosen by the probability $p_{ij,I}(t)$. The assigning activity of dull ant is chosen by the probability $p_{ij,D}(t)$ which does not include the amount of pheromone.

ants in all the ants.

[IDACO-CR3](Assign activity): For the intelligent ants and the dull ants, the assigning activity is chosen by the probability $p_{ij,I}(t)$ and $p_{ij,D}(t)$ as shown in Fig. 4. IDACO-CR uses the two kinds of choice probabilities. The probability, that k-th ant ($k = 1, \cdots, M$) assigns activity j to the location i, is decided by

$$p_{k_{ij},D}(t) = \frac{[\eta_{ij}]^{\beta_D}}{\Sigma_{l \in N_k}[\eta_{il}]^{\beta_D}}, \quad \text{if } k \in S_{\text{dull}} \tag{5}$$

$$p_{k_{ij},I}(t) = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta_I}}{\Sigma_{l \in N_k}[\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta_I}}, k \in S_{\text{intelligent}}, \tag{6}$$

where η_{ij} is defined as the inverse of the coupling matrix element a_{ij}. The adjustable parameters β_I and β_D control the weight of the coupling matrix element of the intelligent ants and dull ants, respectively. Therefore, a searching ability goes up and down by changing α, β_I and β_D. The equation of intelligent ants is same as that of the conventional ACO. Since the dull ants cannot trail the pheromone, the equation of the dull ants does not include the amount of pheromone trail $\tau_{ij}(t)$. Therefore, the intelligent ants judge the next assigning activity by the pheromone and the coupling matrix element, however, the dull ants judge the next assigning activity by only the coupling matrix element. The ants repeat the assignment until all the activities are assigned to locations.

[IDACO-CR4](Pheromone update): After all the ants have completed assignment, the total cost $L_k(t)$ are computed and the amount of the pheromone $\tau_{ij}(t)$ are updated. Note that dull ants can deposit the pheromone, although they cannot trail the pheromone. The amount of the pheromone $\Delta\tau^k_{ij}$ deposited by k-th ant on the coupling (i, j) is same as Eq. (3). The update $\tau_{ij}(t)$ of each coupling (i, j) depending on its $\Delta\tau^k_{ij}$ is same as Eq. (4).

[IDACO-CR5](Change the rate of dull ants): The rate of dull ants d is changed if the system is trapped into the local optima. Whether IDACO-CR is trapped into the local solution or not, is judged by following equation;

$$c_{update} + c_{local} \leq t, \tag{7}$$

where c_{local} is the fixed parameter corresponding to the maximum steps in case IDACO uses the same parameter d, c_{update} is the iteration number which finds the best solution. The rate of dull ants d is changed according to

$$d = \begin{cases} d + d_{up} \text{ and } c_{update} = t, & \text{if Eq. (7) is satisfied.} \\ d_0, & \text{otherwise,} \end{cases} \tag{8}$$

where the adjustable parameter d_{up} controls the addition of the rate of dull ants. Thus, the rate of dull ants d is flexibly varied from d_0 to 1.0 and $c_{updated}$ is reset to t, if the best solution is not updated within a fixed time c_{local}. If the best solution is updated, d is reset to the initial value d_0.

[IDACO-CR6] Let $t = t+1$. Go back to [IDACO-CR2] and repeat until $t = t_{\max}$.

V. Simulation Results

In order to investigate a feasibility and an effectiveness of the proposed IDACO-CR, we apply the proposed IDACO-CR to five QAPs. In addition, to confirm the effectiveness of changing the rate of dull ants, we consider the conventional IDACO for QAP whose algorithm does not change the rate of dull ants in the simulation. The rate of dull ants d of the conventional IDACO is set to 20%, 50% and 100%. The conventional IDACO with d = 100% contains only the dull ants, and the conventional ACO consisted of only the intelligent ants. The QAPs are conducted on *Nug12*, *Scr12* (composed of 12 locations and activities), *Nug20*, *Had20* and *lipa20a* (composed of 20 locations and activities).

In the experiments, the total number of ants M in IDACO-CR, the conventional IDACO and the conventional ACO are the same as the number of locations. IDACO-CR and the conventional IDACO include $d \times M$ dull ants and $(1-d) \times M$ intelligent ants in each simulation. Although the rate of dull ants d in the conventional IDACO is fixed, the rate of dull ants in IDACO-CR is changed according to Eq. (8) to get out from the local solution if IDACO-CR is trapped into the local solution. We repeat the simulation 20 times. The parameters of IDACO-CR, the conventional IDACO and the conventional ACO were set to the follows;

$$\begin{array}{l} \tau_0 = 10, \rho = 0.9, \ \alpha = 1, \ \beta = \beta_I = \beta_D = 1, \\ t_{max} = 15000, c_{local} = 2000, d_0 = 0, d_{up} = 0.2, \end{array} \tag{9}$$

where the evaporation rate ρ, the weight of pheromone α, the weight of coupling matrix element β, β_I and β_D, the search limit t_{max} and c_{local} are fixed value. The initial value of the rate of dull ants d_0 of IDACO-CR is set to 0. However, the

TABLE I
RESULTS OF THE CONVENTIONAL ACO, IDACO AND IDACO-CR FOR NUG12, SCR12, NUG20, HAD20 AND LIPA20A

		Nug12	Scr12	Nug20	Had20	lipa20a
The conventional ACO		4.76%	7.95%	11.84%	3.83%	4.02%
IDACO-CR		**4.57%**	**7.83%**	**11.66%**	**3.62%**	**3.96%**
IDACO	d = 20%	5.26%	9.47%	12.47%	3.9%	3.99%
	d = 50%	5.88%	8.16%	12.25%	3.91%	3.99%
	d = 100%	6.18%	9.01%	12.47%	3.9%	4.02%
Improved rate of IDACO-CR from ACO		4%	1.5%	1.5%	5.5%	1.5%

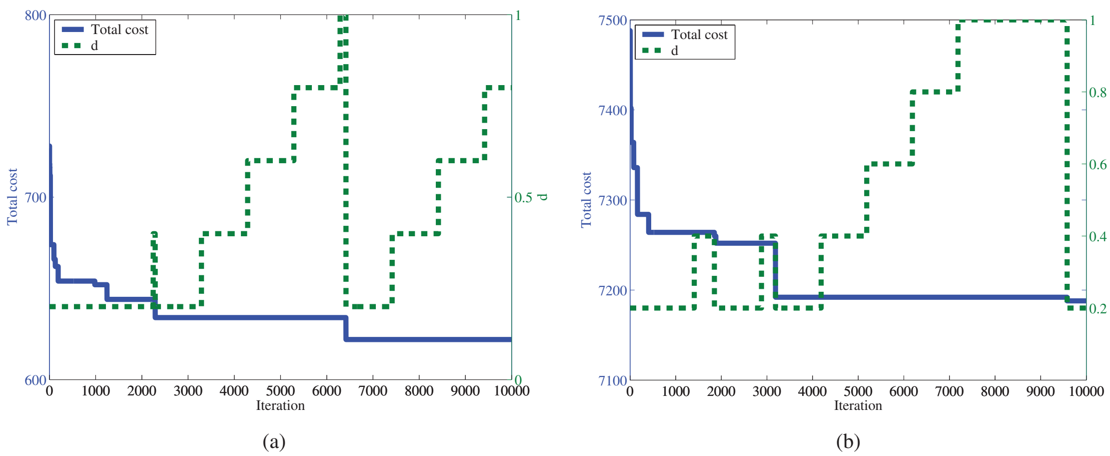

Fig. 5. Relationship between the performance and the change of the dull ants rate d in the simulation of (a) Nug12, (b) Had20.

rate of dull ants d increases every d_{up} when it is judged that the system falls into the local solution. In order to compare the obtained result with the optimal solution, we use the error rate as follow;

$$\text{Error rate} = \frac{(\text{obtained solution}) - (\text{optimal solution})}{(\text{optimal solution})}, \quad (10)$$

where this equation shows how close to the optimal solution the conventional ACO obtains the total cost. Thus, the error rate nearer 0 is more desirable. Furthermore, in order to investigate how well the solution of IDACO-CR are improved from the solution of the conventional ACO, we use following improved rate;

$$\text{Improved rate} = \frac{(\text{AVG of ACO}) - (\text{AVG of IDACO-CR})}{(\text{AVG of ACO})}. \quad (11)$$

where AVG is the average result in 20 simulations. The simulation results of the conventional ACO, the conventional IDACO and the proposed IDACO-CR are shown in Table I. In this table, the average values nearer 0 are more desirable. We can confirm that the proposed IDACO-CR obtained better results than the conventional ACO and the conventional IDACO for all cases. However, we should note that the conventional IDACO was worse than the conventional ACO in the cases of Nug12, Scr12, Nug20 and Had20. This result means that the dull ants have a bad effect on the colony regardless of the number of the dull ants. However, IDACO-CR changing the rate of dull ants in the simulation was able to obtain the best results for all the problems because it flexibly changed the rate of dull ants depending on the problem. We can confirm that changing the rate of dull ants in the simulation is more effective than using the fixed rate of dull ants.

Furthermore, we investigate the behavior of IDACO-CR. Relation of the iteration t to the rate of dull ants d of IDACO-CR for *Nug12* and *Had20* in 1 simulation is shown in Fig. 5. We can see that IDACO-CR got out from the local optima when the iteration time was t = 1900, t = 3100 and t =

TABLE II
RESULTS OF CHANGING THE d_{up} OF IDACO-CR FOR NUG12, SCR12, NUG20, HAD20 AND LIPA20A

IDACO-CR	Nug12	Scr12	Nug20	Had20	lipa20a
d_{up} = 0.2	4.57%	7.83%	**11.66%**	**3.62%**	**3.96%**
d_{up} = 0.1	**4.13%**	**7.19%**	11.93%	3.75%	**3.96%**
d_{up} = 0.4	5%	8.49%	12.01%	3.86%	**3.96%**

9500 for *Had20*. At $t = 1000$ and $t = 2500$, IDACO-CR with $d = 0.2$ fell into local optima, however, the total cost improved after the rate of dull ants was changed from $d = 0.2$ to $d = 0.4$. This figure means that increasing the rate of dull ants d has an effect to get out from the local optima. We can say that the dull ants help getting out from the local optima because the behavior of the dull ants is different from that of the intelligent ants. Furthermore, in order to investigate the effect of varying the rate of dull ants d_{up}, we carried out the simulations by changing d_{up}. The simulation results of IDACO-CRs are shown in Table II. We can confirm that IDACO-CR with $d_{up} = 0.1$ obtained the best results for *Nug12* and *Scr12*. However, by increasing the order of QAP, IDACO-CR with d_{up} obtained the best results for *Nug20* and *Had20*. We consider that we need to set high d_{up} because it is easy for the QAP of large order to fall into local optima.

VI. CONCLUSION

This study has proposed novel ACO which contains the intelligent and dull ants and varies the rate of dull ants. We have confirmed that IDACO-CR changing the rate of dull ants in the simulation obtained better results than the conventional ACO consisted of only the intelligent ants and the conventional IDACO which does not change the rate of dull ants. We have investigated behavior of IDACO-CR and confirmed that changing the rate of dull ants flexibly has the effect to get out from the local optima because the behavior of the dull ants is different from that of the intelligent ants. From these results, we can say that we have to change the rate of dull ants depending on the problem and IDACO-CR is the effective algorithm in solving QAPs.

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