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Search of Good Solutions of QAPs by Hopfield Neural Network with Chaos Noise to Matrix Elements

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Abstract

In this study, we propose a novel algorithm pouring the chaos noise to the matrix elements for solving the quadratic assignment problems (QAPs). We investigate the effect of chaos noise poured in the matrix elements with Hopfield Neural Network (NN). In addition, we investigate comparing chaos noise and random noise. By carrying out computer simulations for various problems, we confirm that the chaos noise has a good effect to avoid local minima and achieves to a good solution of the QAPs.

1. Introduction

Although it would be possible to solve combinatorial optimization problems with a huge number of elements if we have infinite long time, it does not make any sense for practical problems. In several approximation methods, the solutions are trapped into local minima and do not escape. In order to avoid this critical problem, technical methods to escape from local minima are required. Many researchers have proposed that the approximation method with the chaos noise [1][2]. The chaos noise is directly poured to the algorithm in these proposed methods.

In this study, we propose an algorithm that pouring the chaos noise to the matrix elements of quadratic assignment problems (QAPs). It supports to find good solutions and avoid local minima. In the past study, we have investigated the effect of chaos noise poured in the city placement of traveling salesman problems (TSPs) with 2-opt algorithm [3][4]. From results of the past study, we confirmed that the chaos noise supports to find the good solutions and avoid the local minima.

The QAP belongs to a class of NP-hard in the computational complexity. Solving for QAPs, it is reported that the good solution is achieved to used by neural network as other efficient method [5][6].

By carrying out computer simulations for various problems, we confirm that the chaos noise has a good effect to avoid local minima and achieves a good solution of the QAPs. In addition, we investigate the effect of chaos noise comparing to random noise.

2. Solve to QAP with Hopfield NN

The QAP of N-element is expressed by two $N\times N$ matrices, the distance matrix D and the flow matrix F. The objective function $C(p)$ is given as follows;

$$C(p) = \sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}F_{p(i)p(j)}, \tag{1}$$

where D_{ij} and F_{ij} is the (i, j)-th elements of D and F, $p(i)$ is the (i)-th element of vector p, N is the size of problem, respectively. A good solution becomes close to the minimum value of $C(p)$. One example of QAP is to find the arrangement of factories to make a cost the minimum. The cost is given by the distance between cities and the flow of products between factories. Because the QAP is very difficult, it is almost impossible to solve the optimal solutions in large problems.

For solving N-element QAP by Hopfield NN, $N \times N$ neurons are required. The energy function is following Eq. (2).

$$E = \sum_{i,m=1}^{N}\sum_{j,n=1}^{N} w_{im}, x_{jn} + \sum_{i,m=1}^{N} \theta_{im}, x_{im}. \tag{2}$$

The neurons are coupled each other with following weight between the (i,m)-th neuron and the (i,n)-th neuron and threshold the (i,m)-th neuron are described by:

$$w_{im;jn} = -2\left\{A(1-\delta_{mn})\delta_{ij} + B\delta_{mn}(1-\delta_{ij}) + \frac{D_{ij}F_{mn}}{q}\right\}, \tag{3}$$

$$\theta_{im} = A + B, \tag{4}$$

where A and B are positive constants and δ_{ij} is the Kronecker's delta. The states of $N \times N$ neurons are asynchronously updated as following Eq. (5).

$$x_{im}(t+1) = f\Big(\sum_{j,n=1}^{N} w_{im;jn} x_{im}(t) x_{jn}(t) - \theta_{im} + \beta z_{im}(t)\Big), \quad (5)$$

where f is sigmodal functions following Eq. (6).

$$f(x) = \frac{1}{1+\exp(-\frac{x}{\epsilon})}, \quad (6)$$

z_{im} is additional chaos noise, β limits the amplitude of noise. In this study, we use the noise poured to matrix elements of QAP.

3. Chaos Noise

In this study, we use the time series of the chaos generated by the logistic map as a noise. The logistic map is given as following equation.

$$x_n(t+1) = \alpha x_n(t)(1 - x_n(t)). \quad (7)$$

The chaotic sequence is normalized by

$$\hat{x}_n(t) = \frac{x_n(t) - \bar{x}}{\sigma_x}, \quad (8)$$

where $\bar{x}$ is the average of x_n and σ_x is the standard division of x_n. In this study, we use the bifurcation parameter $\alpha = 3.828$. The bifurcation parameter $\alpha = 3.828$ is the intermittency chaos near the three-periodic window obtained from the logistic map. It is reported that the intermittency chaos near the three-periodic window obtained from the logistic map gains good performance for combinatorial optimization problems [1]. The chaotic sequence is shown in Fig. 1.

The initial conditions are difference when the noise is poured to the Hopfield NN or matrix elements.

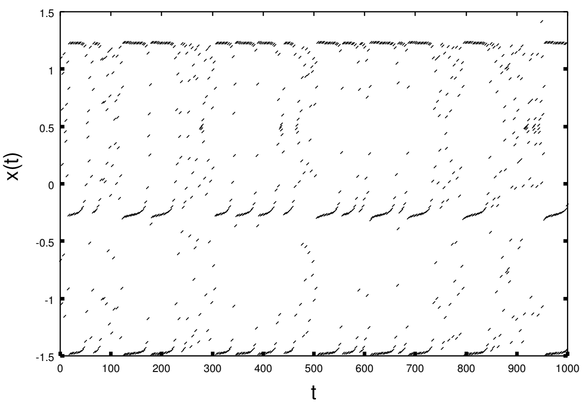

Figure 1: The time series of logistic map ($\alpha = 3.828$).

4. Simulation Results

4.1. Simulation 1

In this section, the simulation results of Hopfield NN with two kinds method of the chaos noise to matrix elements are summarized in Tables 1 and 2. Here, the problems used from QAPLIB [7] are “Nug12” and “Nug15”. The optimal solution of “Nug12” is known “578” and “Nug15” is known “1150”. The number of iterations is from 1000 to 10000 times. The results are shown by the values of total cost. The parameter of Hopfield NN is $A = 0.9$, $B = 0.9$, $\beta = 0.7$, $q = 140$ and $\epsilon = 0.02$.

In these tables, the conventional method is Hopfield NN with chaos noise only for state Eq. (5). The propsed method 1 is the chaos noise poured to matrix elements once before executing Hopfield NN. The propsed method 2 is the chaos noise poured to matrix elements when the neurons of Hopfield NN are updated. The states of the neurons are updated many times. Then, the chaos noise is poured to the original matrix elements at every time. In the proposed methods 1 and 2, the chaos noise is poured to distance matrix and flow matrix.

Table 1: The result for Nug12.

Iteration	Conventional method	Proposed method 1	Proposed method 2
1000	643.8	706.4	638.4
5000	631.6	684.6	624.6
10000	629.6	680.0	620.6

Table 2: The result for Nug15.

Iteration	Conventional method	Proposed method 1	Proposed method 2
1000	1273.8	1321.4	1281.2
5000	1240.8	1304.8	1255.0
10000	1240.8	1304.8	1241.8

From Table 1, we can confirm that the proposed method 2 is better performance than the conventional method and the proposed method 1. This means that the chaos noise is effective to avoid the local minima. From Table 2, we can confirm that the result of the proposed method 2 is similar to the conventional method.

4.2. Simulation 2

In this section, the simulation results of comparing the proposed method 2 to random noise for two QAPs are summarized in Tables 3 - 6. Here, we use two problems "Nug12" and "Nug15" from QAPLIB [7]. The result of "Nug12" is summarized in Tables 3 and 4 and "Nug15" is summarized in Tables 5 and 6. The simulation conditions are the same as "Simulation 1".

In these tables, Chaos noise 1 is Hopfield NN with chaos noise poured to two matrices, 2 is Hopfield NN with chaos noise poured to distance matrix, 3 is Hopfield NN with chaos noise poured to flow matrix, Random noise 1 is Hopfield NN with random noise poured to two matrices, 2 is Hopfield NN with random noise poured to distance matrix, 3 is Hopfield NN with random noise poured to flow matrix.

Table 3: Solving for Nug12 with chaos noise.

Iteration	Chaos noise 1	Chaos noise 2	Chaos noise 3
1000	638.4	623.6	627.0
5000	624.6	617.4	618.4
10000	620.6	614.8	616.2

Table 4: Solving for Nug12 with random noise.

Iteration	Random noise 1	Random noise 2	Random noise 3
1000	764.0	752.4	747.8
5000	754.2	751.2	729.0
10000	737.2	759.2	730.2

Table 5: Solving for Nug15 with chaos noise.

Iteration	Chaos noise 1	Chaos noise 2	Chaos noise 3
1000	1281.2	1281.2	1273.8
5000	1255.0	1255.0	1240.8
10000	1241.8	1241.8	1229.4

From these tables, we can confirm that the conventional method exhibits better performance than the random noise poured to matrices. However, we can confirm that the chaos noise is better performance than the random noise and the conventional method.

The simulation results of typical examples of the error with iteration are shown in Figures 2 - 6. The errors show the values between the average or the minimum values and the optimal solution calculated by the following equations.

Table 6: Solving for Nug15 with random noise.

Iteration	Random noise 1	Random noise 2	Random noise 3
1000	1322.0	1324.6	1324.6
5000	1288.0	1284.0	1284.0
10000	1274.6	1274.2	1274.6

$$Error_{ave} = \frac{Ave - Opt}{Opt} \times 100, \tag{9}$$

$$Error_{min} = \frac{Best - Opt}{Opt} \times 100, \tag{10}$$

where Opt shows the optimal solution of "Nug15" from QAPLIB [7]. A known optimal total cost of "Nug15" is 1150. The horizontal axis shows the iteration of Hopfield NN. The vertical axis shows the error. The result of the conventional method is Fig. 2, the chaos noise to two matrices is Fig. 3, the chaos noise to flow matrix is Fig. 4, the random noise to two matrices is 5, the random noise to flow matrix is Fig. 6.

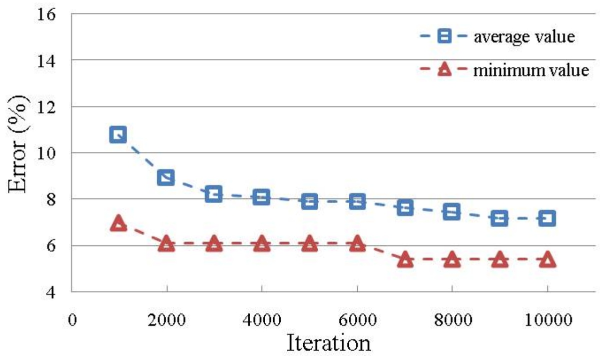

Figure 2: Conventional method.

From these figures, we can confirm that the Hopfield NN with chaos noise exhibits better performance than the conventional method and Hopfield NN with random noise. In addition, we can also confirm that the Hopfield NN with chaos noise to matrix elements is able to avoid the local minima comparing the conventional method and the Hopfield NN with chaos noise.

5. Conclusion

We have investigated the effect of chaos noise poured to the matrix elements with Hopfield NN for the QAP. By carrying out computer simulations, we have confirmed that the chaos

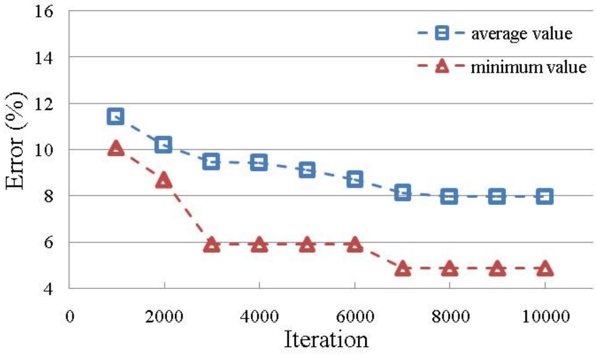

Figure 3: Chaos noise to two matrices.

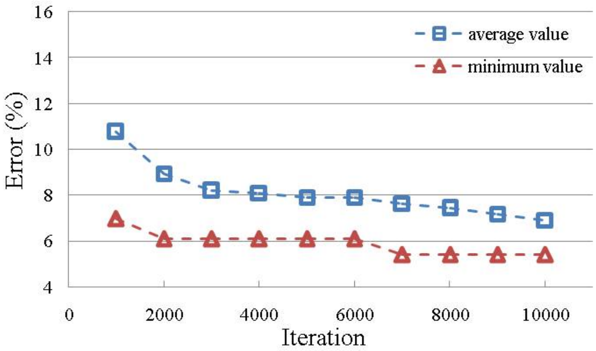

Figure 4: Chaos noise to flow matrix.

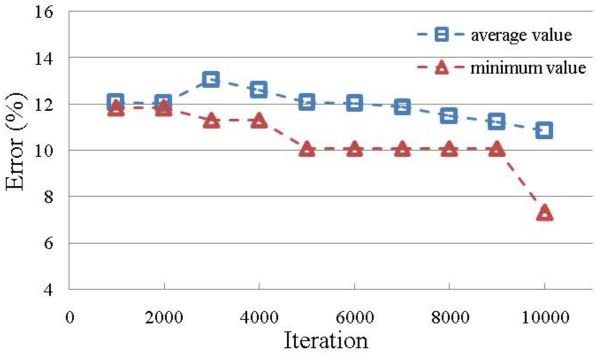

Figure 5: Random noise to two matrices.

noise had a good effect to avoid local minimum problems and achieved a good performance to find good solutions of the QAP.

As a future subject, we will investigate the effect to pour different noises to the matrix elements and changing each parameter.

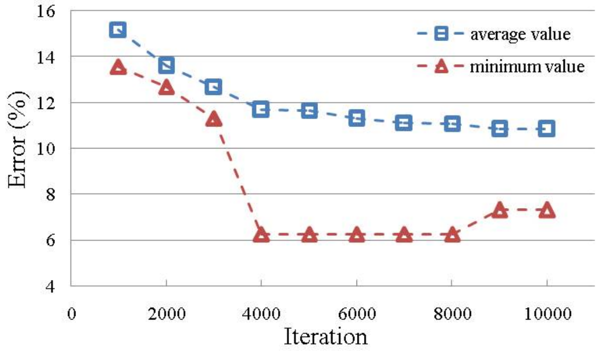

Figure 6: Random noise to flow matrix.

Acknowledgment

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