

## Hopfield Neural Network with Chaotic Pulse Wave Solving Traveling Salesman Problems

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### Abstract

In this study, in order to investigate the effect of chaotic oscillations of real biological signals on information processing ability, we investigate the performance of the Hopfield neural networks solving traveling salesman problems when the sampled data of chaotic pulse waves obtained from real biological experiments are poured into the neurons. Computer simulated results show that the performance of the chaotic pulse wave is better than the case of random noise.

### 1. Introduction

The pulse wave is a change of the volume caused by the blood current and is measured from the surface of various parts of the body. Biological signals including the pulse wave, the brain wave, and the heart beat wave are said to exhibit chaotic behaviors [1][2]. Some reports suggest that healthy biological systems possess chaotic features and periodic states indicate sick or fatal conditions.

On the other hand, in the optimization problems, many algorithms pouring chaotic oscillations to the Neural Networks (NN) have been proposed in order to avoid the local minimum problems [3]. We have also investigated various methods to exploit chaotic features to enhance the ability of the neural networks [4]-[9]. However, in the past studies, only mathematical abstract models, e.g. the logistic map and the cubic map, are considered as chaotic source.

In this study, in order to investigate the effect of chaotic oscillations of real biological signals, we use chaotic pulse waves obtained by real biological experiments and pour the sampled chaotic data to the Hopfield NN solving Traveling Salesman Problems (TSP). By computer simulations, we investigate the effect of the chaotic pulse wave. We also compare the results with the case of random noise. Further, this study may be a first step to clarify the relationship between the performance of the neural networks and the stress added to the subject.

### 2. Hopfield NN solving TSP

In this study, we choose a problem “bayg29” from TSPLIB [10]. The map points of “bayg29” is shown in Fig. 1. The optimum solution is known as 9.074e+03.

For solving  $N$ -element TSP by Hopfield NN,  $N \times N$  neurons are required and the following energy function is defined to fire  $(i, j)$ th neuron at the optimal position:

$$E = \sum_{i,m=1}^N \sum_{j,n=1}^N \omega_{im;jn} x_{im} x_{jn} + w_{ij} \sum_{i,m=1}^N \theta_{im} x_{im}. \quad (1)$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between  $(i, m)$ th neuron and  $(j, n)$ th neuron and the threshold of the  $(i, m)$ th neuron are described by:

$$\omega_{im;jn} = A\delta_{ij}(1 - \delta_{mn}) + B\delta_{mn}(1 - \delta_{ij}) - C - Dd_{ij}(\delta_{j,i+1} + \delta_{j,i-1}), \quad (2)$$

$$\theta_{im} = A + B, \quad (3)$$

where  $A$  and  $B$  are positive constants and  $\delta_{ij}$  is Kronecker's delta. The state of  $N \times N$  neurons are asynchronously updated due to the following difference equation:

$$x_i(t+1) = f \left( \sum_{j,n=1}^N \omega_{im;jn} x_{jm}(t) - \theta_{im}(t) + \beta z_{im}(t) \right) \quad (4)$$

where  $f$  is sigmoidal function defined as follows:

$$f(a) = \frac{1}{1 + e^{-a}}, \quad (5)$$

$z_{im}$  is additional noise, namely chaotic pulse wave in this study. Figure 2 shows a conceptual neuron model for this NN.

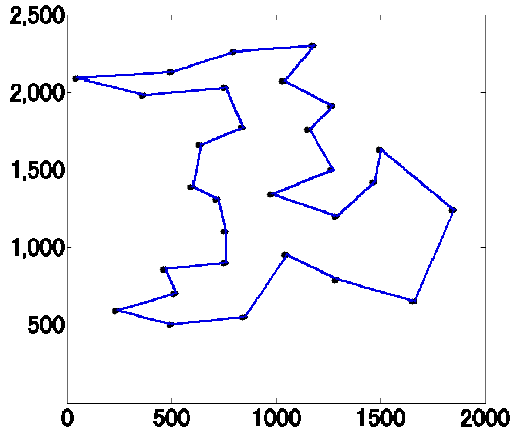


Figure 1: Example of TSP.

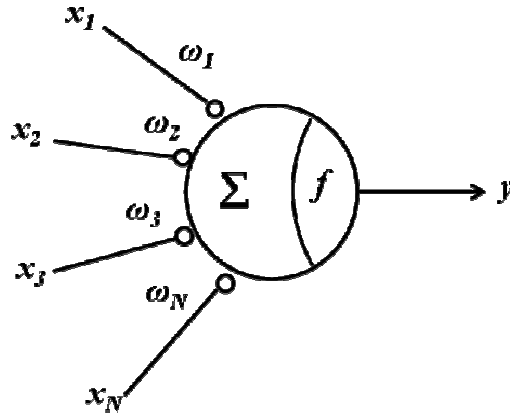


Figure 2: A neuron model.

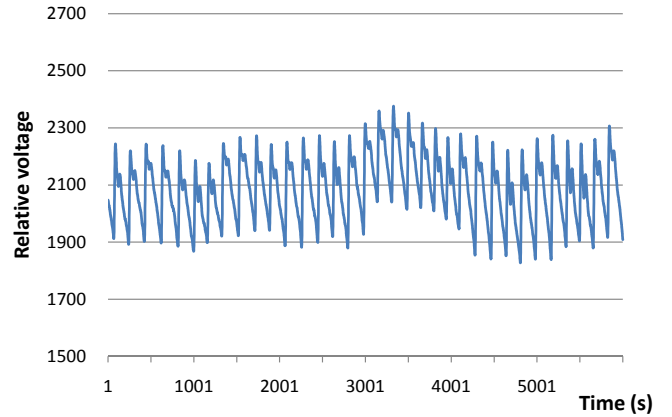


Figure 3: Example of pulse waves.

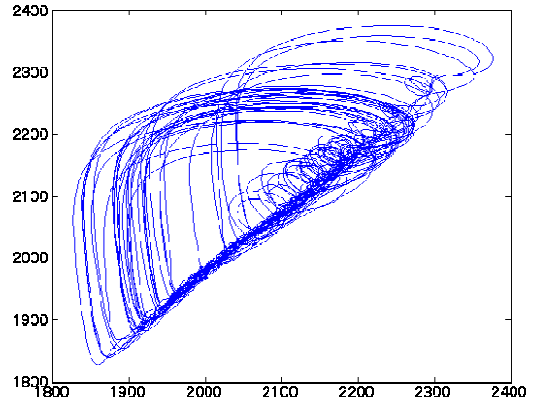


Figure 4: Example of reconstructed attractors of pulse wave.

### 3. Chaotic pulse wave

We obtained pulse waves from real biological experiments. The subjects were 4 healthy male student (21 to 22 years old). We gave them the body and mental stress and measured their pulse waves. The pulse waves measured at 8pm are assumed to be their reference data. After the four hours, the pulse waves are measured five times every two hours; 0am, 2am, 4am, 6am, and 8am. The sampling frequency of the data is 200 Hz and the measurement time length is 5 minutes. It is not allowed for the subjects to sleep during the night, because we keep to give the stress to the subjects.

An example of the time waveforms of the pulse wave and the corresponding reconstructed attractor are shown in Fig. 3 and Fig. 4, respectively. The attractor is reconstructed based on the Takens' theorem [11] with the following parameters: embedding dimension = 4, super-sphere size = 0.08, data points = 60000 and calculation points = 36000.

By Sano-Sawada method [12][13], we calculate the Lyapunov exponents  $\lambda_i$  as

$$\lambda_i = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^{N-1} \ln |e'_i(t)| \quad (6)$$

where  $e'$  is the Lyapunov spectrum. At least, if the value of  $\lambda_1$  is positive, the orbital instability exists that is the feature of chaos. The Lyapunov exponent of the attractor in Fig. 4 is calculated as 4.5032 and hence we can say the pulse wave in Fig. 3 is chaotic. Similar can be said for other subjects' pulse waves.

Further, we calculated the Lyapunov exponent with sliding the time interval for calculations. The Lyapunov exponent of the reference data changes like Fig. 5.

The Lyapunov exponents of all 4 subjects are summarized in Fig. 6.

In order to pour this chaotic pulse wave to the Hopfield NN, we normalize the wave and use only one of every 15 samples. Further, in this study, we fix the amplitude parameter  $\beta = 0.7$ .



Figure 5: Time evolution of Lyapunov exponents with sliding time interval.

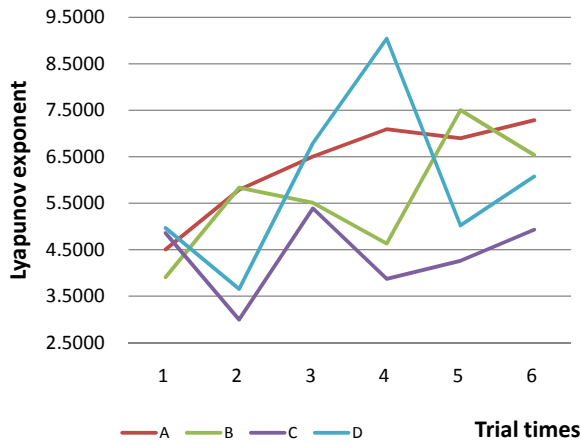


Figure 6: Change of Lyapunov exponents of 4 subjects with stress.

#### 4. Simulation results

We carry out computer simulations for 10 times of 100 iterations and recorded the minimum and the average values of the tour length. The simulated results of 4 subjects (A, B, C, D) are summarized in Fig. 7. We compared the results with the result of random noise case. We use the error rate as follows;

$$\text{Error rate}[\%] = \frac{(\text{obtained solution}) - (\text{optimal solution})}{(\text{optimal solution})} \quad (7)$$

The results are summarized in Table 1. From this table, we can confirm that the chaotic pulse wave gains better performance than the random noise.

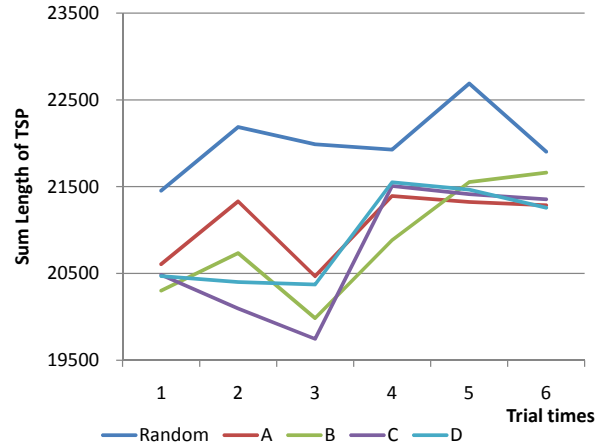


Figure 7: Simulated results 1.

Table 1: Simulated results 2.

	Average	Error rate
Noise	2.2025e+04	143%
A	2.1068e+04	132%
B	2.0853e+04	130%
C	2.0767e+04	129%
D	2.0920e+04	131%

#### 5. Conclusions

We have investigated the effect of the chaotic pulse wave poured in the Hopfield NN solving TSP. By carrying out computer simulations for the problem, we have confirmed that the chaotic pulse wave had an effect to avoid local minimum problems and achieved a performance to find good solutions of TSP.

In this study, we could not obtain an enough results to conclude on the relationship between the performance of the neural networks and the stress added to the subject. That is our important future research topic.

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