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Particle Swarm Optimization Containing Plural Swarms Using Shared Velocity

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Abstract

Particle Swarm Optimization (PSO) is a popular optimization technique for solving objective functions and an algorithm to simulate the movement of flocks of birds toward foods. In this study, we propose an improved PSO algorithm; PSO containing shared velocity using plural swarms (PPSOV). Each particle of PPSOV belongs to one of plural swarms. Each particle belonging to each swarm shares a velocity with other particles in same swarm. We evaluate the performance of PPSOV by using four optimization functions and confirm that PPSOV obtain the best results among four PSOs.

1. Introduction

Particle Swarm Optimization (PSO) [1] is a popular optimization technique for solving objective functions and is algorithm to simulate the movement of flocks of birds toward foods. Due to its simple concept, easy implementation and quick convergence, PSO has attracted attentions and has been widely applied to different fields in recent years. Furthermore, PSO has demonstrated great performances for many optimization problems.

However, quick convergence often leads to a local optimum problem. It is important for multimodal functions with a lot of local optima to compromise between the quick convergence and being trapped in a local optimum. In order to escape from such local optima and to avoid the premature convergence, the search for a global optimum should be diverse. Many researchers have improved the performance of PSO by enhancing its ability with more diverse search [2][3]. In particular, some researchers have proposed the PSO method using plural swarms whose particles exchange their information among particles belong each swarm [4][5]. In real world, a bird becomes a flock that matched the speed and flies.
From this idea, we studied the improved PSO algorithm that using shared velocity (PSOV) [6]. The important feature of PSOV is that each particle of PSOV shares the same velocity (called shared velocity) that represented the average velocity of all the particles. Therefore, the velocity of each particle influence the other particle velocity, each other. The study showed that the shared velocity is effective for the optimizations using PSO.

In this study, we propose an improved PSO algorithm; PSO containing plural swarms using shared velocity (PPSOV). The important features of PPSOV are follows. PPSOV consists of plural swarms. The number of particles of each swarm is all the same.

A mean velocity called "shared velocity" is calculate for every swarm, and the each particle shares the shared velocity with all the particles in the swarm. Thus, each swarm has the different shared velocity from other swarm. At every generation, the shared velocity of each swarm is updated depending on the velocity of the particles belonging to each swarm.

We explain the algorithm of PPSOV in detail in Section 2. In Section 3, we perform basic numerical experiments by using four algorithm methods of PPSOV and other PSOs. From these results, we confirm that the mean values of PPSOV are the best among four PSOs, therefore, we can confirm that PPSOV algorithm is more effective than the standard PSO.

2. PPSOV

The standard PSO consists of M particles. Each particles has two information; position and velocity. The position vector of each particle i and its velocity vector are represented by $\boldsymbol{X}_i = (x_{i1}, x_{i2}, \dots, x_{iD})$ and $\boldsymbol{V}_i = (v_{i1}, v_{i2}, \dots, v_{iD})$, respectively, where $(i = 1, 2, \dots, M)$ and $x_{id} \in [x_{\min}, x_{\max}]$.

In this study, we propose the PSO containing plural swarms using shared velocity (PPSOV). PPSOV consists of not only uni-swarm but plural swarms. The number of swarms is K, and each swarm is denoted by S_k $(k = 1, 2, \cdots, K)$. Each particle of PPSOV is sorted into one of swarms, and the number of particles in one swarm is M/K. The particles belonging

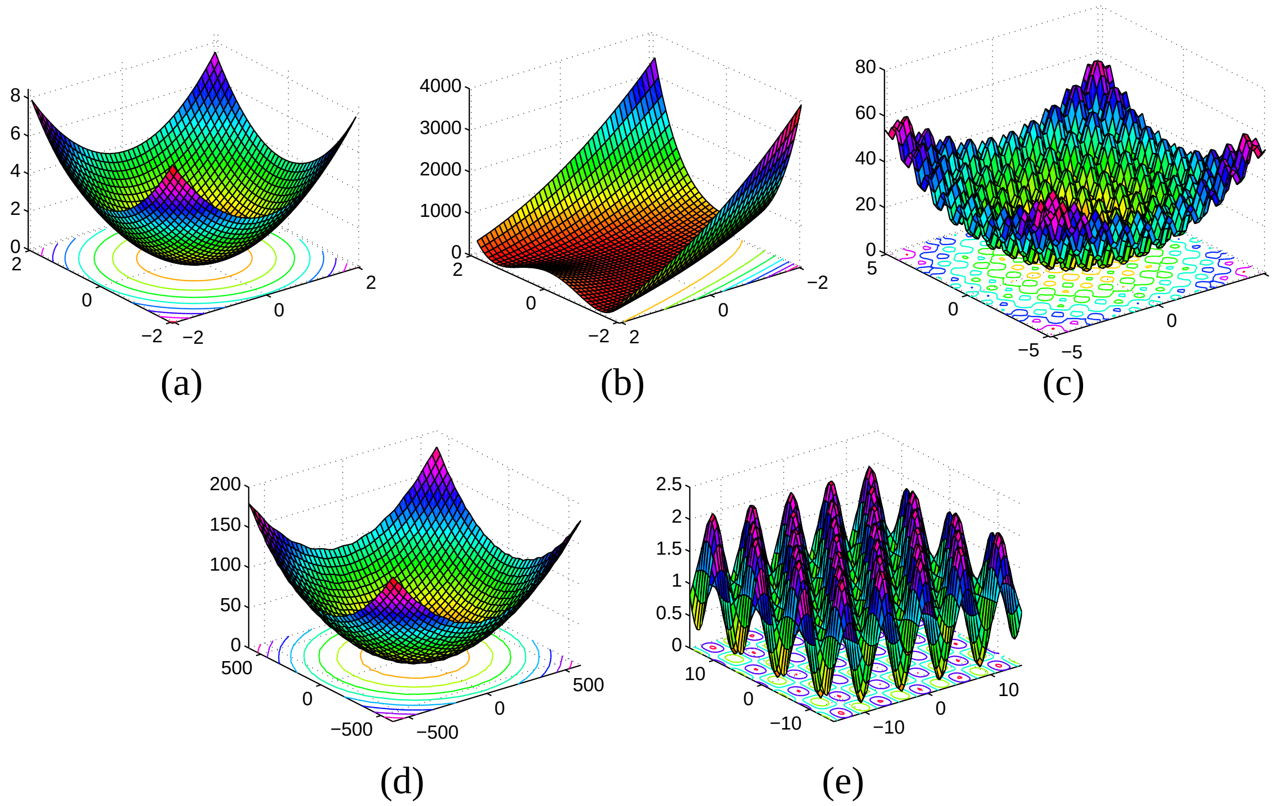

Figure 1: Landscape of four bench marks for $D = 2$. (a) Sphere function. (b) Rosenbrock's function. (c) Rastrigin's function. (d) Griewank's function with full definition range. (e) Inner area of Griewank's function.

to each swarm share same velocity (called "shared velocity") with other particles in the swarm.

[PPSOV1] (Initialization) Let a generation step $t = 0$. Randomly initialize the position $\boldsymbol{X}_i$ and its velocity $\boldsymbol{V}_i$ for all particles i and initialize $\boldsymbol{P}_i$ with a copy of $\boldsymbol{X}_i$.

[PPSOV2] Evaluate the current cost $f(\boldsymbol{X}_i)$. Update the personal best position *pbest* $\boldsymbol{P}_i = (p_{i1}, p_{i2}, ..., p_{iD})$ for each particle i and the global best position *gbest* $\boldsymbol{P}_g = (p_{g_1}, p_{g_2}, ..., p_{g_D})$ among all the particles.

[PPSOV3] Let $\boldsymbol{Ps}_k = (p_{s_k1}, p_{s_k2}, \cdots, p_{s_kD})$ represents the swarm best position with the best cost among the particles belonging to the swarm S_k so far (called *sbest*). Update $\boldsymbol{Ps}_k$ for each swarm S_k if needed.

$$s_k = \arg\min_i \{f(\boldsymbol{X}_i)\}, \qquad i \in S_k. \tag{1}$$

[PPSOV4] Updated $\boldsymbol{V}_i$ and $\boldsymbol{X}_i$ of each particle i depending on its *pbest*, its swarm best *sbest* and *gbest*;

$$\begin{aligned} v_{id}(t+1) =& w_k v_{id} + c_1 r_1 \{p_{id} - x_{id}(t)\} \\ &+ c_2 r_2 \{p_{s_k d} - x_{id}(t)\} + c_3 r_3 \{p_{gd} - x_{id}(t)\}, \end{aligned} \tag{2}$$

where r_1, r_2 and r_3 are three random variables distributed uniformly on $[0, 1]$, w is an inertia weight of all particles, and c_1, c_2 and c_3 are positive acceleration coefficients.

[PPSOV5] All the particles in each swarm S_k shares the shared velocity $\boldsymbol{V}_{s_k} = (v_{s_{k_1}}, v_{s_{k_2}}, ..., v_{s_{k_D}})$. $\boldsymbol{V}_{s_k}$ is defined as the average velocity of all the particles belonging to S_k;

$$v_{s_k d} = \frac{1}{M/K} \sum_{i=1}^{M/K} v_{id}, \qquad i \in S_k. \tag{3}$$

[PPSOV6] Update $\boldsymbol{X}_i$ depending on its $\boldsymbol{V}_s$ and $\boldsymbol{V}_i$;

$$x_{id}(t+1) = x_{id}(t) + c_v v_{s_k d}(t+1) + v_{id}(t+1), \tag{4}$$

where c_v is the shared velocity coefficients in each swarm S_k.

[PPSOV7] Let $t = t + 1$. Go back to [PPSOV2], and repeat until $t = T$.

3. Numerical Experiments

In order to confirm the performance of PPSOV algorithm, we carry out basic numerical experiments. The goal is to find the optimum (minimum) value of $f(x)$. Referring to [2], we use the following four benchmark functions as shown in Fig. 1.

Table 1: Comparison results PSO, PPSO, PSOV and PPSOV on test functions with $D = 60$.

f		PSO	PPSO	PSOV	PPSOV
f_1	Avg.	15.73	12.49	9.83	**6.43**
	Min.	3.69E-06	1.77	2.43E-16	4.69E-01
	Max.	32.77	55.43	19.66	12.47
f_2	Avg.	286.10	313.17	133.00	**120.62**
	Min.	154.16	206.45	53.71	74.32
	Max.	554.93	498.25	228.46	249.20
f_3	Avg.	275.52	361.62	267.81	**210.68**
	Min.	157.17	197.06	196.28	2.51
	Max.	380.32	686.85	396.83	420.59
f_4	Avg.	81.79	11.84	56.89	**11.68**
	Min.	22.99	3.71E-02	2.01E-06	0
	Max.	181.00	73.31	136.00	38.86

1. Sphere function:

$$f_1(x) = \sum_{d=1}^{D} x_d^2, \tag{5}$$

where $x \in [-2.048, 2.047]^D$ and the optimum solution x^* are all $[0, 0, \ldots, 0]$.

2. Rosenbrock's function:

$$f_2(x) = \sum_{d=1}^{D-1}(100(x_d^2 - x_{d+1})^2 + (1 - x_d)^2), \tag{6}$$

where $x \in [-2.048, 2.047]^D$ and the optimum solution x^* are all $[1, 1, \ldots, 1]$.

3. Rastrigin's function:

$$f_3(x) = \sum_{d=1}^{D}(x_d^2 - 5\cos(2\pi x_d) + 5), \tag{7}$$

where $x \in [-5.12, 5.12]^D$ and the optimum solution x^* are all $[0, 0, \ldots, 0]$.

4. Griewank's function:

$$f_4(x) = \sum_{d=1}^{D} \frac{x_d^2}{4000} + \prod_{d=1}^{D} \cos(\frac{x_d}{\sqrt{d}}) + 1, \tag{8}$$

where $x \in [-600, 600]^D$ and the optimum solution x^* are all $[0, 0, \ldots, 0]$.

The optimum function values $f(x^*)$ of all functions are 0. f_1 and f_2 are unimodal functions, and f_3 and f_4 are multimodal functions with numerous local minima. All the functions have D variables, in this study, D is set to 60. We compare the proposed PPSOV to three PSOs; the standard PSO, PPSO and PSOV. PPSO does not have the shared velocity but also have the plural swarms. Therefore, PSOV have the shared velocity but also have only the one swarm. The population size M is set to 60 in PSO and PSOV. PPSO and PPSOV have $K = 6$ swarms, and each swarm contains 10 particles, i.e. $M = 60$. The inertia weight is fixed as $w = 0.6$. For PSO and PSOV, the acceleration coefficients are set as $c_1 = c_3 = 1.8$ and $c_2 = 0$. For PPSO and PPSOV, the acceleration coefficients are set as $c_1 = 1.8$, $c_2 = 1.6$ and $c_3 = 0.2$. For PSOV and PPSOV, the shared velocity coefficients is set as $c_v = 0.3$ is used in the following simulation. We carry out the simulation 30 times for all the optimization functions with 2000 generations, namely $T = 2000$.

The performances with minimum, maximum and mean function values on four functions are listed in Table 1. We can see that the PPSOV can obtain the best mean values for all the test functions. Furthermore, for f_4, the minimum value of PPSOV is 0, therefore, only PPSOV was able to find the optimum function value. Figure 2 shows the mean *gbest* values of every generation over 30 runs for four test functions. From these results, we can see that the convergence of PPSO and PPSOV are quickly. Furthermore, the convergence of PSOV is not quickly. However, the values decline steadily. Therefore, we can investigated that the plural swarms prompt the quick search, and the shared velocity prompt the bust out from local optima.

From these results, we can say that PPSOV is the effective algorithm which has the advantage of the plural swarms and shared velocity.

4. Conclusions

In this study, we have proposed the new PSO algorithm, PSO containing plural swarms using shared velocity (PPSOV). PPSOV consists of not uni-swarm but plural swarms. Each particle of PPSOV belongs to one of plural swarms. The particles belonging to each swarm share the ve-

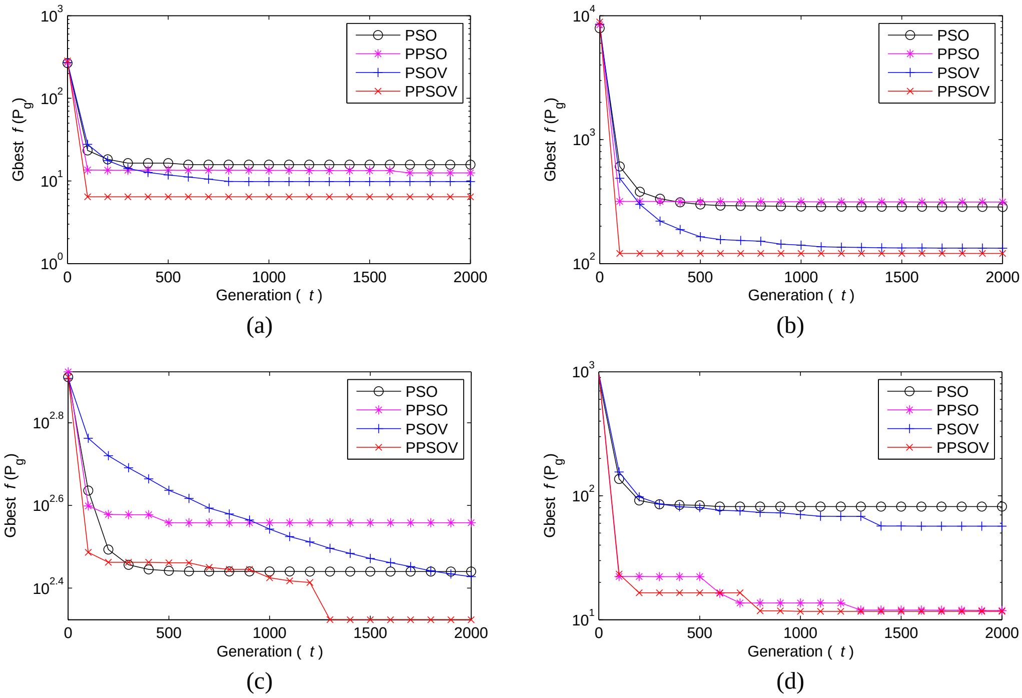

Figure 2: Mean *gbest* value of every generation for 60-dimensional four functions. (a) Sphere function. (b) Rosenbrock's function. (c) Rastrigin's function. (d) Griewank's function.

locity (called "shared velocity") with other particles in the swarm. We have investigated behavior of PPSOV with the simulation and have confirmed the efficiency of PPSOV.

References

[1] J. Kennedy and R. Eberhart, "Particle swarm optimization", in *Proc. IEEE Int. Conf. Neural Netw.*, vol. 4, pp. 1942–1948, 1995.

[2] J. Kennedy and R. Medes, "Population structure and particle swarm performance" in *Proc. Congr. Evol. Comput.*, pp. 1671-1676, 2002.

[3] A. Ratnaweera, S. K. Halgamuge, and H. C. Watson, "Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients" in *IEEE Trans. Evol. Comput.*, vol. 8, no. 3, pp. 240-255, 2004.

[4] G. Yen and M. Daneshyari, "Diversity-based Information Exchange among Multiple Swarm in Particle Swarm Optimization" in *IEEE Congress on Evolutionary Computation.*, pp. 6150-6157, 2006.

[5] M. Iwamatsu, "Multi-Species Particle Swarm Optimizer for Multimodal Function Optimization" in *IEICE Trans. Inf. & Syst.*, vol. E89-D, no. 3, 2006.

[6] M. Sugimoto, H. Matsushita and Y. Nishio, "Particle Swarm Optimization Containing Shared Velocity" in *IEEE Workshop on Nonlinear Circuit Networks*, 2010.