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Anti-Phase Synchronization of Switching Phenomena in Globally Coupled Chaotic Circuits without Parameter Mismatches

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Abstract

In this study, synchronizations of switching phenomena are investigated in globally coupled chaotic circuits without parameter mismatches. Especially, relationship between phenomena and the number of a chaotic circuits is investigated.

1. Introduction

Many kinds of complex phenomena can be observed on the large-scale coupled chaotic circuits. Investigations of these phenomena are very important works in order to declare nonlinear phenomena in the natural world. A electric circuit is suitable for models of large-scale coupled nonlinear systems because of the following reasons. Getting electric parts is easy and inexpensive, short experiment time, high repeatability of experiments and it's a real physical system. Therefore, there are many studies of large-scale coupled circuit systems. In these studies, synchronization phenomena are attracted attentions.

On the other hand, some chaotic circuits have coexisting attractors. In these circuits, switching phenomena of attractors can be observed. Normally, in the case of a synchronization state, the switching of attractors is also synchronized. And in the case of asynchronous states, the switching of attractors is also asynchronized. However, synchronization of self-switching phenomena on full-coupled chaotic oscillators is reported by [1]. Additionally, we could observe similar phenomena in an other system [2][3].

Let us describe below the points of a synchronization of switching phenomena:

- The system keeps asynchronous state.
- Each circuit switches in the almost same time.

In previous study, we investigated in case that the number of elements is two. Following results were obtained.

- In-phase and anti-phase synchronization of switching phenomena are observed.

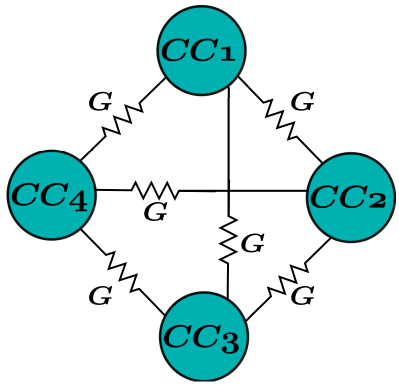


Figure 1: System model ($N = 4$)

- Anti-phase synchronization of switching phenomena can be observed whether a parameter has mismatches or not.

In this study, a relationship between the number of chaotic circuits and anti-phase synchronization of switching phenomena is investigated in globally coupled chaotic circuits without parameter mismatches.

2. System Model

In this study, globally coupled chaotic circuits shown in Fig. 1 is investigated. Elements of this system are coupled by resistors. The element is a chaotic circuit [4] as shown in Fig. 2. This circuit consists of three memory elements, one linear negative resistor and bi-directionally-coupled diodes. In this circuit, two attractors which are symmetrical about a origin are observed by adjusting the value of the negative resistor.

In order to derive the system equation, bi-directionally-coupled diodes are modeled as Fig. 3. Then, the v-i char-

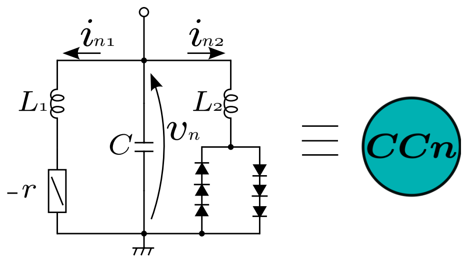
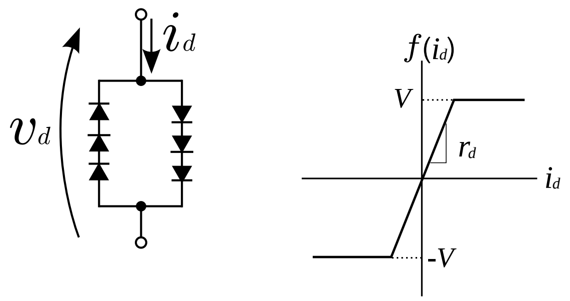


Figure 2: Circuit model



(a) Circuit schematic (b)v-i characteristics

Figure 3: Coupled diodes model

acteristic is described as follows.

$$v_d = \left(\left| i_d + \frac{V}{r_d} \right| - \left| i_d - \frac{V}{r_d} \right| \right). \quad (1)$$

The others elements are modeled as linear elements. Each circuit number is defined as $1 \leq n \leq N$. By using these models, the system equation is described as follows:

$$\begin{cases} L_1 \dfrac{di_{n1}}{dt} = v_n + r i_{n1}, \\ L_2 \dfrac{di_{n2}}{dt} = v_n - \dfrac{r_d}{2} \left(\left| i_{n2} + \dfrac{V}{r_d} \right| - \left| i_{n2} - \dfrac{V}{r_d} \right| \right), \\ C \dfrac{dv_n}{dt} = -(i_{n1} + i_{n2}) - G \left(N v_n - \displaystyle\sum_{k=1}^{N} v_k \right). \end{cases} \quad (2)$$

Changing parameters and variables as follows,

$$t = \sqrt{L_1 C}\tau, \ i_{n1} = V\sqrt{\frac{C}{L_1}} x_n, \ i_{n2} = V\sqrt{\frac{C}{L_1}} y_n,$$
$$v_n = V z_n, \ \text{"}\cdot\text{"} = \frac{d}{d\tau}, \ \alpha = r\sqrt{\frac{C}{L_1}}, \quad (3)$$
$$\beta = \frac{L_1}{L_2}, \ \gamma = \sqrt{\frac{C}{L_1}} r_d \ \text{and} \ \delta = G\sqrt{\frac{L_1}{C}}.$$

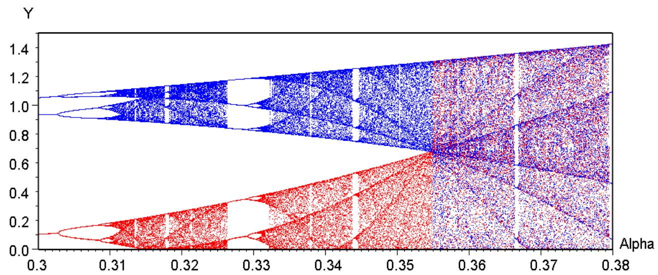


Figure 4: Bifurcation diagram of Fig. 2. Vertical axis: y. Horizontal axis: α. $\beta = 3.0$ and $\gamma = 470.0$. Red: Initial values are $x = 0.1$, $y = 0.1$ and $z = 0.1$. Blue: Initial values are $x = -0.1$, $y = 0.1$ and $z = -0.1$.

The normalized system equation is described as follows.

$$\begin{cases} \dot{x}_n = z_n + \alpha x_n, \\ \dot{y}_n = \beta \left\{ z_n - \dfrac{\gamma}{2} \left(\left| y_n + \dfrac{1}{\gamma} \right| - \left| y_n - \dfrac{1}{\gamma} \right| \right) \right\}, \\ \dot{z}_n = -x_n - y_n - \delta \left(N z_n - \displaystyle\sum_{k=1}^{N} z_k \right) \end{cases} \quad (4)$$

where N is corresponding the number of the circuits and n is corresponding the circuit number. Computer simulations are carried out using this equation.

3. Computer Simulations

First of all, we describe how to distinguish two attractors. Next, the computer simulation results are shown in the case of changing the number of elements.

3.1. Definition of two attractors

Figure 4 shows a bifurcation diagram by using two kinds of initial values. These points are plotted when the solution hits $z = 1$ and $\dot{y} < 0$. From this diagram, two attractors are distinguished by $y = 0.675$. Coexisting of two attractors are observed in $\alpha > 0.356$. Therefore, we defined the state R and B when the solution is $y \leq 0.675$ and $y > 0.675$ respectively. Figure 5 shows a simulation result of the chaotic circuit as shown in Fig. 2. Coexisting attractors are observed. This result is color-coded by the definition. In this paper, this definition is applied in all of the simulations.

3.2. Case of $N = 2$

Figure 6 shows a computer simulation results in the case of $N = 2$. In Fig. 6(a), horizontal axis shows the time. The vertical axes z_1 and z_2 are corresponding to voltages v_n of the circuit model as shown in Fig. 2. Fig. 6(b) shows the difference of two waveforms. The horizontal axis shows the time.

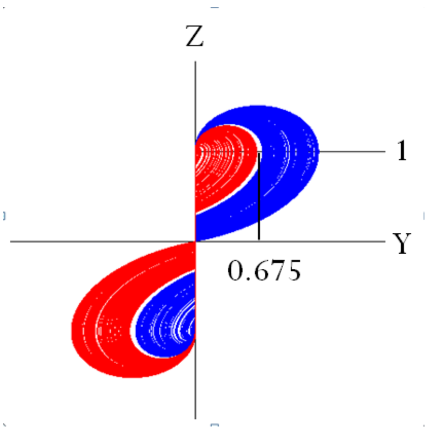


Figure 5: Coexisting attractors of a chaotic circuit as shown in Fig. 2. $\alpha = 0.35$, $\beta = 3.0$, $\gamma = 470.0$

The vertical axis shows as follows. A black line shows z_1 - z_2. A green line shows the difference of two coexisting attractors. Namely, when both z_1 and z_2 are red or blue, the line becomes 0. When z_1 is blue and z_2 is red, the line becomes 1. When z_1 is red and z_2 is blue, the line becomes -1. On the basis of above points, let us explain the phenomena in Fig. 6. The black line of Fig. 6(b) shows that an asynchronous state is kept at all time. The color of the waveforms as shown in Fig. 6 shows that two attractors are synchronized with in-phase. Thus, this phenomena is called as "in-phase synchronization of switching phenomena". Note that the circuits have parameter mismatches. Namely, this phenomena cannot be observed in the case that parameter mismatches become zero.

Figure 7 shows the other computer simulation result in the case of $N = 2$. In this case, an asynchronous state is also kept at all time. A difference from Fig. 6 is a relationship between colors of two waveforms. In this case, this phenomena is called as "anti-phase synchronization of switching phenomena". This phenomena can be observed weather parameter has mismatches or not.

From these results, we consider that to observe "in-phase synchronization of switching phenomena" is difficult in the case of increasing the number of elements because increasing the number of elements means increasing a range of parameter mismatches. However, we consider that "anti-phase synchronization of switching phenomena" may be observed in the case of increasing the number of elements and the result is very interesting.

3.3. Case of $N = 3$

Figure 8 shows a computer simulation results in the case of $N = 3$. In the center of Fig. 8, switching phenomena are observed. Additionally, it is confirmed that a proportion of

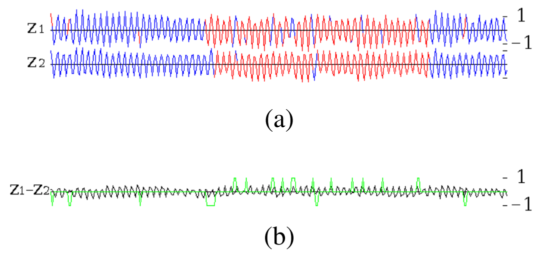


Figure 6: In-phase synchronization of switching phenomena. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 15.0$, $\gamma = 470.0$ and $\delta = 0.33$.

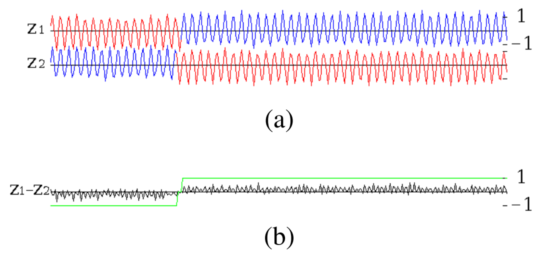


Figure 7: Anti-phase synchronization of switching phenomena case of $N = 2$. $\alpha_n = 0.40$, $\beta = 3.0$, $\gamma = 470.0$ and $\delta = 0.21$.

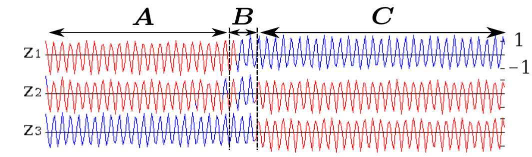
two attractors becomes $B : R = 1 : 2$ or $B : R = 1 : 2$ constantly. Furthermore, a switching time is not so short as shown in part B of Fig. 8. This switching time is observed in all the switching phenomena.

3.4. Case of $N = 4$

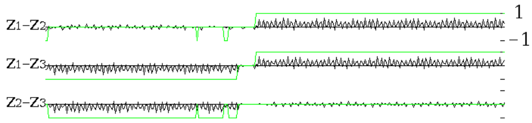
Figure 9 shows a computer simulation results in the case of $N = 4$. Switching phenomena are also observed. A proportion of two attractors becomes $B : R = 2 : 2$ constantly. Furthermore, as mentioned above, a switching time can be also observed. Whereas, there is another different point. The switching time is longer than the case of $N = 3$. The system needs much time to determine the attractor of each circuit.

3.5. Cases of $N = 5$ to 8

The cases of $N = 5$ to 8 were also investigated. Figure 10 shows one of computer simulation results. From results of investigations of $N = 2$ to 8, it is clear that the switching time is longer and longer as increasing the number of N. Additionally, a proportion of two attractors becomes $B : R = 1 : 1$ in the case of N is equal to even number. And a proportion of two attractors becomes $B : R = N/2 : N/2 + 1$ or $B : R = N/2 + 1 : N/2$ in the case of N is equal to odd number.



(a)



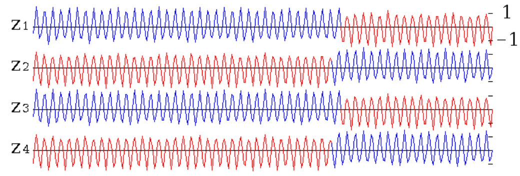
(b)

Figure 8: Anti-phase synchronization of switching phenomena case of $N = 3$. $\alpha = 0.40$, $\beta = 3.0$, $\gamma = 470.0$ and $\delta = 0.151$.

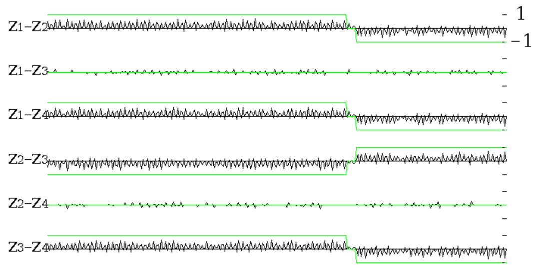
4. Conclusions

In this study, a relationship between the number of chaotic circuits and anti-phase synchronization of switching phenomena have been investigated in globally coupled chaotic circuits without parameter mismatches.

Consequently, we can be obtained following results. At least, anti-phase synchronization of switching phenomena can be observed in the case of $N = 2$ to 8. The switching time is longer and longer as increase the number of elements. There is a rule of proportion of two attractors.



(a)



(b)

Figure 9: Anti-phase synchronization of switching phenomena in the case of $N = 4$. $\alpha = 0.40$, $\beta = 3.0$, $\gamma = 470.0$ and $\delta = 0.095$.

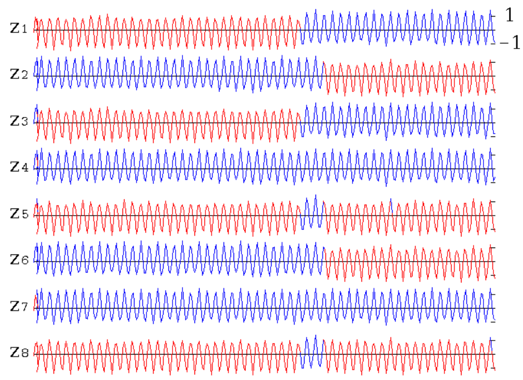


Figure 10: Anti-phase synchronization of switching phenomena in the case of $N = 8$. $\alpha = 0.40$, $\beta = 3.0$, $\gamma = 470.0$ and $\delta = 0.043$.

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