2011 International Workshop on Nonlinear Circuits,
Communication and Signal Processing
NCSP'11, Tianjin, China, March 1-3, 2011

Synchronization Phenomena in Simultaneous Oscillators Coupled by an Inductor

Yoshifumi Nishio [(1)], Yang Yang [(2)] and Yoko Uwate [(3)]

Tokushima University
2-1 Minami-Josanjima, Tokushima 770-8506, Japan
Tel: +81-88-656-7470, Fax: +81-88-656-7471
Email: (1) nishio@ee.tokushima-u.ac.jp, (2) yyang_993@hotmail.com, (3) uwate@ee.tokushima-u.ac.jp

Abstract

In this study, synchronization phenomena observed from two inductively coupled simultaneous oscillators are reported. Since inductively coupled oscillators can exhibit both in-phase and anti-phase synchronizations, the circuit generate various synchronization patterns. Computer simulations confirm that 9 different synchronization patterns appear in the circuits.

1. Introduction

In 1954, Schaffner reported that an oscillator with two degrees of freedom could oscillate simultaneously at two different frequencies when the nonlinear characteristics are described by a fifth-power polynomial function [1]. Kuramitsu investigated the simultaneous oscillations for three or more degrees case theoretically and confirmed the generation of triple mode oscillation by circuit experiments [2]. Simultaneous oscillations are definitely one of the most common nonlinear phenomena observed in various higher-dimensional systems in the natural fields. However, after their pioneering works, as far as the authors know, there have not been many researches clarifying the basic mechanism of the simultaneous oscillations except [3][4].

On the other hand, studies on coupled oscillators become more and more popular not only in the field of physics but also in the engineering and the biological fields. For example, synchronization of complex networks is one of the hot topics in the engineering field, because the outcome could be used to develop better schemes attaining synchronization of communication networks [5]-[8]. In the biological field, neuronal oscillations are said to play a role in a variety of brain operations [9]-[11].

In our previous study, we reported some synchronization phenomena observed from two resistively coupled simultaneous oscillators [12]. However, because resistively coupled oscillators exhibit in-phase synchronization only, the variety of synchronization patterns were poor. In this study, we investigate synchronization phenomena observed from two inductively coupled simultaneous oscillators. Since inductively coupled oscillators can exhibit both in-phase and anti-phase synchronizations [13], we expect the generation of various synchronization patterns. Computer simulations confirm that 9 different synchronization patterns, including the state where all oscillations stops, appear in the circuits.

2. Circuit model

The circuit model is shown in Fig. 1.

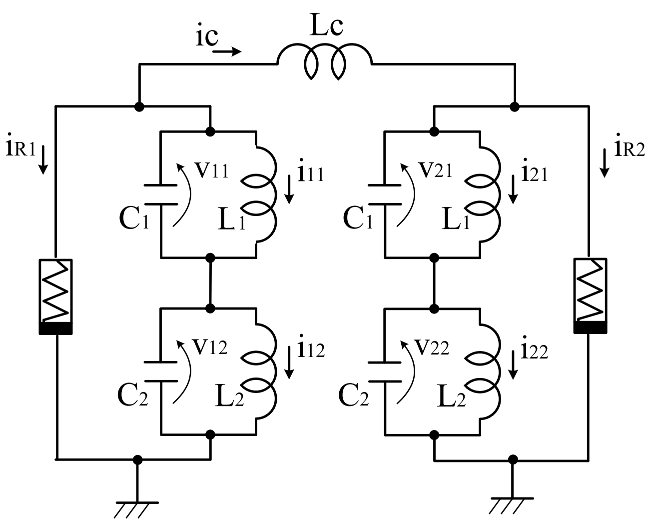

Figure 1: Circuit diagram.

In the circuit, two simultaneous oscillators are coupled by an inductor L_C and each simultaneous oscillator consists of a nonlinear negative resistor, whose $v - i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \qquad (g_1, g_3, g_5 > 0) \tag{1}$$

and two resonators with different natural frequencies $\omega_1 = 1/\sqrt{L_1 C_1}$ and $\omega_2 = 1/\sqrt{L_2 C_2}$.

The equations governing the coupled oscillators are described by the following eighth-order differential equations including two nonlinear functions i_{R1} and i_{R2}.

$$\begin{cases} C_1 \dfrac{dv_{11}}{dt} = -i_{11} - i_{R1} - i_C \\ L_1 \dfrac{di_{11}}{dt} = v_{11} \\ C_2 \dfrac{dv_{12}}{dt} = -i_{12} - i_{R1} - i_C \\ L_2 \dfrac{di_{12}}{dt} = v_{12} \\ C_1 \dfrac{dv_{21}}{dt} = -i_{21} - i_{R2} + i_C \\ L_1 \dfrac{di_{21}}{dt} = v_{21} \\ C_2 \dfrac{dv_{22}}{dt} = -i_{22} - i_{R2} + i_C \\ L_2 \dfrac{di_{22}}{dt} = v_{22} \end{cases} \tag{2}$$

where i_C is the current through the coupling inductor and is given as

$$i_C = \frac{L_1(i_{11} - i_{21}) + L_2(i_{12} - i_{22})}{L_C} \tag{3}$$

and the currents through the nonlinear resistors i_{R1} and i_{R2} are given as

$$\begin{cases} i_{R1} = i_R(v_{11} + v_{12}) \\ i_{R2} = i_R(v_{21} + v_{22}) \end{cases} . \tag{4}$$

By using the following variables and parameters,

$$\begin{gathered} v_{mn} = \sqrt[4]{\frac{g_1}{5g_5}}\, x_{mn}, \quad i_{mn} = \sqrt[4]{\frac{g_1}{5g_5}} \sqrt{\frac{C_1}{L_1}}\, y_{mn}, \\ \alpha_C = \frac{C_1}{C_2}, \quad \alpha_L = \frac{L_1}{L_2}, \quad \gamma = \frac{L_1}{L_C}, \\ \varepsilon = g_1 \sqrt{\frac{L_1}{C_1}}, \quad \beta = \frac{3g_3}{g_1} \sqrt{\frac{g_1}{5g_5}}, \quad t = \sqrt{L_1 C_1}\, \tau, \end{gathered} \tag{5}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dfrac{dx_{11}}{d\tau} = -y_{11} - f(x_{11} + x_{12}) - y_C \\ \dfrac{dy_{11}}{d\tau} = x_{11} \\ \dfrac{dx_{12}}{d\tau} = \alpha_C\{-y_{12} - f(x_{11} + x_{12}) - y_C\} \\ \dfrac{dy_{12}}{d\tau} = x_{12} \\ \dfrac{dx_{21}}{d\tau} = -y_{21} - f(x_{21} + x_{22}) + y_C \\ \dfrac{dy_{21}}{d\tau} = x_{21} \\ \dfrac{dx_{22}}{d\tau} = \alpha_C\{-y_{22} - f(x_{21} + x_{22}) + y_C\} \\ \dfrac{dy_{22}}{d\tau} = x_{22} \end{cases} \tag{6}$$

where y_C corresponds to i_C and is given as

$$y_C = \gamma \left(y_{11} - y_{21} + \frac{y_{12} - y_{22}}{\alpha_L} \right) \tag{7}$$

and the nonlinear function $f(\cdot)$ which corresponds to the $v-i$ characteristics of the nonlinear resistors is given as

$$f(x) = \varepsilon \left(x - \frac{\beta}{3} x^3 + \frac{1}{5} x^5 \right) . \tag{8}$$

3. Synchronization Phenomena

Figure 2 shows computer simulated results. The coupled oscillators exhibit various synchronization patterns for different initial conditions. We can see from Fig. 2(b) that x_{11} and x_{21} are synchronized at in-phase while x_{12} and x_{22} are synchronized at anti-phase. We also found that the oscillation frequency of the pair x_{11} and x_{21} is different from that of the other pair x_{12} and x_{22}.

We tried to give different sets of initial conditions to the circuit in order to confirm the generation of all possible combinations of synchronization patterns. Figure 3 shows the observed 9 different synchronization patterns. The upper figures show the synchronization states between the upper resonators while the lower figures show the synchronization states between the lower resonators. So, for example, Fig. 3(a) shows the generation of the state where the upper resonators are in-phase and the lower resonators are also in-phase. While, for the case of Fig. 3(c), the uppers are in-phase but the lowers are anti-phase. In Figs. 3(e) and (f), the lower resonators stop their oscillations, while the uppers stop in Figs. 3(g) and (h).

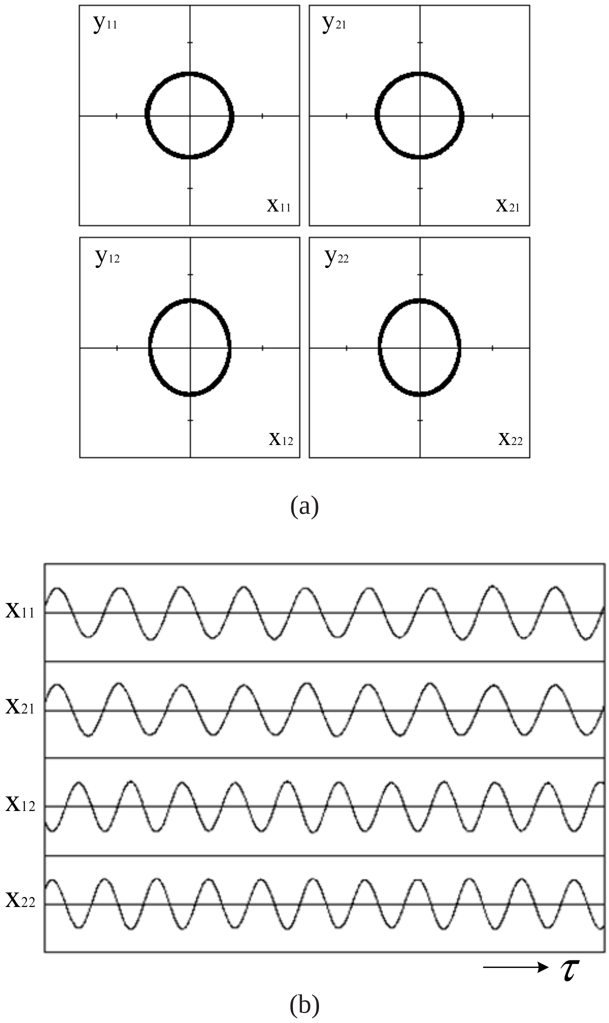

Figure 2: Computer simulated results. $\alpha_C = 1.0$, $\alpha_L = \sqrt{2}$, $\gamma = 0.01$, $\varepsilon = 0.1$ and $\beta = 3.14$. (a) Lissajous figures of four resonators. Horizontal x_{mn} and vertical y_{mn}. (b) Time waveforms of x_{mn}.

Finally, in the case of Fig. 3(i), all the resonators stop oscillating. Please note that the upper pair and the lower pair oscillate asynchronously in Figs. 3(a)-(d).

The result is encouraging because our next target is the investigation of the synchronization patterns when the number of resonators increases. We expect the generation of much larger number of synchronization patterns, if all possible combinations appear even for the case of larger number of resonators. However, we should notice the fact that the oscillation amplitudes of the resonators become smaller when the asynchronous oscillations appear. This may means that the simultaneous oscillations of many resonators are not easy to be synchronized. The detailed investigation of such cases will be reported elsewhere.

4. Conclusions

In this study, we have investigated the generation of synchronization phenomena observed from two inductively coupled simultaneous oscillators. By computer simulations we have confirmed that 9 different synchronization patterns appeared in the circuits.

Our future research topics include theoretical analysis and circuit experiments, and the investigation for the cases of more than two resonators in each oscillator and more than three coupled oscillators.

Acknowledgement

This work was partly supported by CASIO Science Promotion Foundation.

References

[1] J. Schaffner, "Simultaneous oscillations in oscillators," IRE Transactions on Circuit Theory, vol. 1, pp. 2–8, Jun. 1954.

[2] M. Kuramitsu and F. Takase, "Averaged potential analysis of multimode oscillators with hard operating conditions," IEICE Technical Report on NLP, vol. 81, no. 13, pp. 1–10, Sep. 1981.

[3] M. Matsuki and S. Mori, "Asynchronous simultaneous oscillations in negative resistance oscillatory circuit containing periodically operating analog switch," IEICE Technical Report on CAS, vol. 101, pp. 81–87, Jun. 1993.

[4] M. Matsuki and S. Mori, "Asynchronous excitation phenomena in negative resistance oscillatory circuit containing periodically operating analog switch," IEICE Technical Report on NLP, vol. 94, pp. 51–58, May 1994.

[5] C.W. Wu, *Synchronization in complex networks of nonlinear dynamical systems*, World Scientific, Oct. 2007.

[6] W. Wei, W. Zhou and T. Chen, "Cluster synchronization of linearly coupled complex networks under pinning control," IEEE Transactions on Circuits and Systems I, vol. 56, no. 4, pp. 829–839, Apr. 2009.

[7] J. Lu, G. Chen and M. Bernardo, "On some recent advances in synchronization and control of complex networks," Proceedings of ISCAS'10, pp. 3773–3776, May 2010.

[8] J. Lu, X. Yu and D. Hill, "A brief overview of some recent advances in pinning control of complex networks," Proceedings of NOLTA'10, pp. 398–401, Sep. 2010.

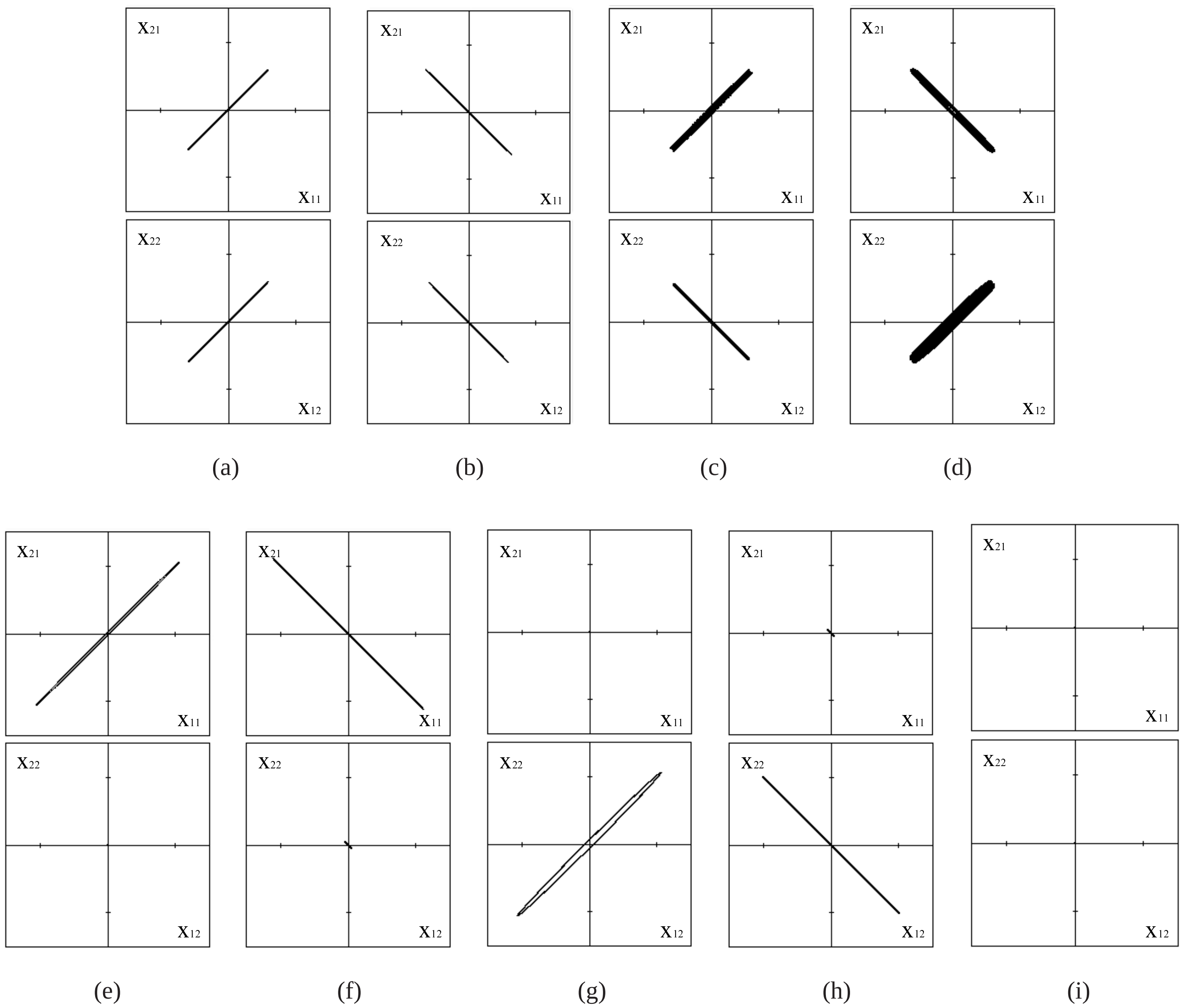

Figure 3: Different synchronization patterns. $\alpha_C = 1.0$, $\alpha_L = \sqrt{2}$, $\gamma = 0.01$, $\varepsilon = 0.1$ and $\beta = 3.14$. Upper figures: horizontal x_{11} and vertical y_{21}. Lower figures: horizontal x_{12} and vertical y_{22}.

[9] R.T. Canolty, E. Edwards, S.S. Dalal, M. Soltani, S.S. Nagarajan, H.E. Kirsch, M.S. Berger, N.M. Barbaro and R.T. Knight, "High gamma power is phase-locked to theta oscillations in human neocortex," Science, vol. 313, no. 5793, pp. 1626–1628, Sep. 2006.

[10] P. Lakatos, G. Karmos, A.D. Mehta, I. Ulbert and C.E. Schroeder, "Entrainment of neuronal oscillations as a mechanism of attentional selection," Science, vol. 320, no. 5872, pp. 110–113, Apr. 2008.

[11] G. Yu and J.-J. Slotine, "Visual grouping by neural oscillator networks," IEEE Transactions on Neural Networks, vol. 20, no. 12, pp. 1871–1884, Dec. 2009.

[12] C. Higashi and Y. Nishio, "Investigation of synchronization phenomena in coupled simultaneous oscillators," Proceedings of NCSP'10, pp. 113–116, Mar. 2010.

[13] T. Suezaki and S. Mori, "Mutual synchronization of two oscillators," Transactions of IECE, vol. 48, no. 9, pp. 1551–1557, Sep. 1965.