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Behavior of Independent-Minded Particle Swarm Optimization

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Abstract

This study proposes an improved Independent-mined Particle Swarm Optimization (IIPSO) algorithm. Particles of IIPSO have independence, and it is decided stochastically that each dimension of each particle is affected by *gbest* or not. IIPSO is applied to various optimization problem. The effectiveness of IIPSO for multimodal functions is confirmed in terms of robustness, accuracy based on evaluation criteria and parameter dependence.

1. Introduction

Particle Swarm Optimization (PSO) [1] is an algorithm to simulate the movement of flocks of birds. Due to the simple concept, easy implementation and quick convergence, PSO has attracted much attention and is used to wide applications in different fields in recent years. Since each particle flies toward its personal best position *pbest* and the best position among the whole swarm *gbest*, all the particles of the standard PSO are fully-connected and always influence each other.

On the other hand, various topological neighborhoods for PSO have been considered [2]–[5]. In these papers, each particle shares its best position among neighboring particles on the network. It is an application of the network topology to the particle swarm, and investigations of the suitable network for PSO have attracted attention in these years [6], [7].

Our previous study has proposed a novel application of the complex network to PSO; an Independent-minded Particle Swarm Optimization (IPSO) [8]. The most important feature of IPSO is that it is decided stochastically that each particle depends on *gbest* or becomes independent from the swarm and moves depending only on *pbest*. In other words, the particles are not always connected each other, and they act with self-reliance. IPSO was applied to some benchmarks used widely in the literature, and it has been confirmed that IPSO is effective for multimodal problems with numerous local optima.

However, the information of all the dimensions of one benchmark are obtained by same equation. In fact, all the dimensions of the benchmark function have a same feature, and the shape of functions are symmetric in all the dimensions. Meanwhile, many real-world optimization problems are generally asymmetric and the feature of objective function is different in each dimension. In IPSO, the information of all the dimensions of each particle are simultaneously updated depending on *gbest* if the particle is decided to be connected to the swarm. Although IPSO has improved the performances on the benchmark problems, we consider that this updating feature is not effective for the real-world optimization problem.

In this study, we modify the updating rule of IPSO and propose an improved IPSO (IIPSO). The particles of IIPSO have independence as the conventional IPSO. The difference between IPSO and IIPSO is the number of dimensions of the particle affected by the swarm. In IIPSO, whether the particle is affected by the swarm is judged for every dimension. This effect helps to easy escape from the local optima and to apply PSO to the complicated optimization problem whose each dimension has different feature. We apply IIPSO to various benchmark optimization functions and investigate the behavior of IIPSO by carrying out the simulation using different probability of independence. IIPSO is compared with the standard PSO and the conventional IPSO. we confirm that IIPSO can effectively enhance the searching efficiency for the multimodal functions.

2. Improved Independent-minded Particle Swarm Optimizer (IIPSO)

In the algorithm of the PSO, multiple potential solutions called "particles" coexist. Each particle i ($i = 1, 2, \cdots, M$) has two information; position and velocity, represented by $\boldsymbol{X}_i = (x_{i1}, \cdots, x_{id}, \cdots, x_{iD})$ and $\boldsymbol{V}_i = (v_{i1}, \cdots, v_{id}, \cdots, v_{iD})$ ($d = 1, 2, \cdots, D$), respectively. At each time step, each particle flies toward its own past best position (*pbest*) and the best position among all particles (*gbest*). In other words, they always influence each other.

In this study, we propose the improved IPSO (IIPSO) for

Table 1: Four Test Functions.

Function name	Test Function	Initialization Space	Criterion
Sphere;	$f_1(\boldsymbol{X}) = \sum_{d=1}^{D} x_d^2,$	$x \in [-5.12, 5.12]^D,$	0.01
Rastrigin;	$f_2(\boldsymbol{X}) = \sum_{d=1}^{D} \left(x_d^2 - 10\cos(2\pi x_d) + 10\right),$	$x \in [-5.12, 5.12]^D,$	50
Ackley;	$f_3(\boldsymbol{X}) = \sum_{d=1}^{D-1} \Big(20 + e - 20e^{-0.2\sqrt{0.5(x_d^2+x_{d+1}^2)}} - e^{0.5(\cos(2\pi x_d)+\cos(2\pi x_{d+1}))}\Big),$	$x \in [-30, 30]^D,$	1.0
Stretched V;	$f_4(\boldsymbol{X}) = \sum_{d=1}^{D-1} (x_d^2 + x_{d+1}^2)^{0.25}\left(1 + \sin^2(50(x_d^2 + x_{d+1}^2)^{0.1})\right),$	$x \in [-10, 10]^D$	10

the application to the complicated optimization problem. The particles of both the conventional IPSO and IIPSO have independence, thus, it is decided stochastically whether they are connected to others at every step. In other words, they are not always affected by *gbest* and their *pbest* does not always affect the swarm. The difference between IPSO and IIPSO is the number of dimensions of the particle affected by the swarm. If the particle of IPSO is decided to be connected to the swarm, the particle information of all the dimensions are simultaneously updated depending on *gbest*. On the other hand, in IIPSO, whether the particle is affected by the swarm is judged for every dimension.

(Step1) Let a generation step $t = 0$. Randomly initialize the particle position $\boldsymbol{X}_i$ ($x_{id} \in [x_{\min}, x_{\max}]$), initialize its velocity $\boldsymbol{V}_i$ to zero, and initialize $\boldsymbol{P}_i = (p_{i1}, p_{i2}, \cdots, p_{iD})$ with a copy of $\boldsymbol{X}_i$.

(Step2) Decide whether each particle i is connected to the others according to $\boldsymbol{r_3}_i = (r_{3i1}, r_{3i2}, \cdots, r_{3iD})$ which is a D-dimensional random vector ($\in (0,1)$) for i. *If any of each dimension of $\boldsymbol{r_3}_i$ satisfies $r_{3id} \leq C$, the information of i is transmitted to other particles. If not, the particle i is isolated from the swarm, then, it and others does not affect each other.* C is a constant cooperativeness coefficient which is the independence probability of the particles.

(Step3) Evaluate the fitness $f(\boldsymbol{X}_i)$ for each particle i. Update the personal best position (*pbest*) as $\boldsymbol{P}_i = \boldsymbol{X}_i$ if needed.

(Step4) Let $\boldsymbol{P}_g$ represents the best position with the best *pbest* among particles being connected to others, and it is denoted by *gbest*. Update *gbest* $\boldsymbol{P}_g = (p_{g1}, p_{g2}, \ldots, p_{gD})$ according to

$$g = \arg\min_i f(\boldsymbol{P}_i), \ \min\{r_{3id}, d\} \leq C. \tag{1}$$

In other words, even if $f(\boldsymbol{P}_i)$ is the minimum pbest among all the particles, gbest is not affected by it if the information of i is not transmitted to others according to (Step2).

(Step5) Update $\boldsymbol{V}_i$ and $\boldsymbol{X}_i$ of each particle i according to

$$v_{id}(t+1) = \begin{cases} wv_{id}(t) + c_1 r_1(p_{id} - x_{id}(t)) + c_2 r_2(p_{ig} - x_{id}(t)), & r_{3id} \leq C \\ wv_{id}(t) + c_1 r_1(p_{id} - x_{id}(t)), & r_{3id} > C \end{cases} \tag{2}$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1), \tag{3}$$

where w is the inertia weight determining how much of the previous velocity of the particle is preserved. c_1 and c_2 are two positive acceleration coefficients, generally $c_1 = c_2$. r_1 and r_2 are uniform random number values from $U(0,1)$. These equations mean that whether each particle is affected by *gbest* is decided at random with the cooperativeness C. When $C = 0$, all the dimensions of all the particles move depending only on own *pbest*, and when $C = 1$, the algorithm is completely the same as the standard PSO.

(Step6) Let $t = t + 1$ and go back to (Step2).

3. Computer Simulation

We apply IIPSO to 4 benchmark optimization problems summarized in Table 1. f_1 is unimodal functions, and f_2–f_4 are multimodal functions with numerous local minima. The optimum solution $\boldsymbol{X}^*$ of all the functions are $[0, 0, \ldots, 0]$, and its optimum value $f(\boldsymbol{X}^*)$ is 0. All the functions have D variables, in this study, $D = 30$.

IIPSO is compared with the standard PSO and IPSO. For all PSOs in all the simulations, the population size M is set to 36, and the parameters are set as $w = 0.7$ and $c_1 = c_2 = 1.6$. In order to investigate the behavior of IPSO and IIPSO, we carry out simulations using different cooperativeness C from 0 to 1.0. The maximum generation are set to 3000, and the results are evaluated in an achievement rate of the criterion attainment over 100 trials.

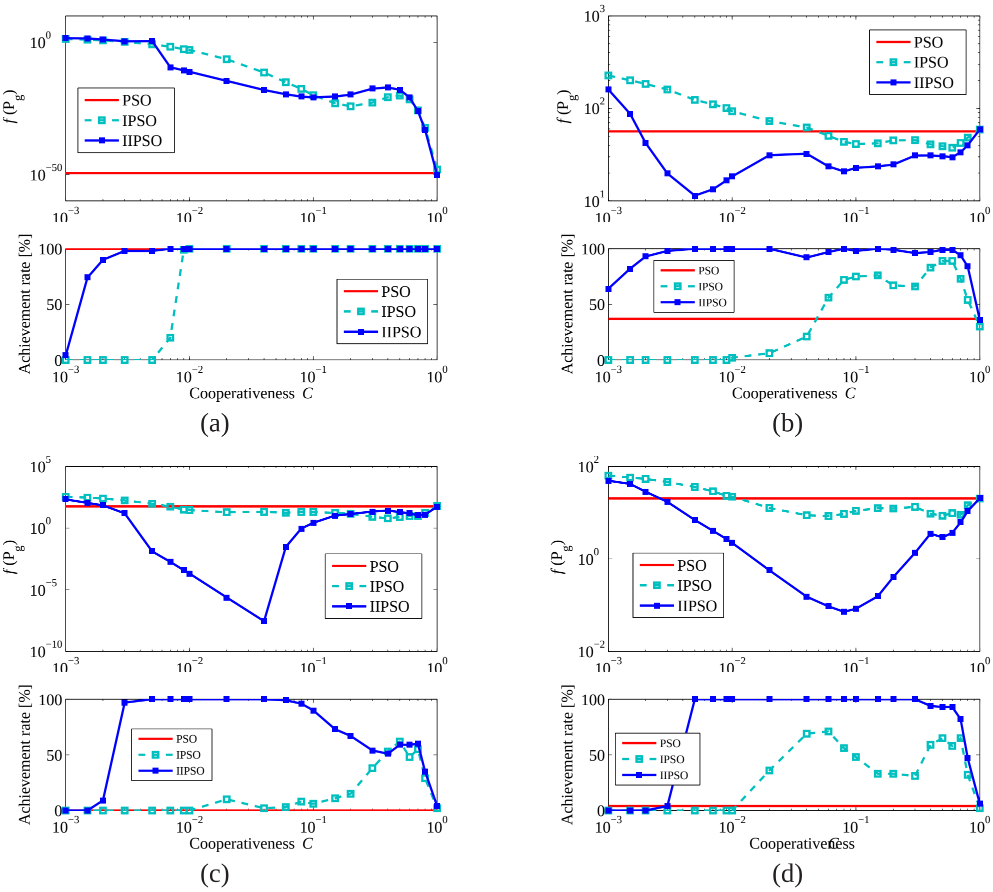

Figure 1: Simulation results of three PSOs using different C to four benchmarks. (a) Sphere function. (b) Rastrigin's function. (c) Ackley's function. (d) Stretched V sine wave function.

Table 2: Comparison results of 3 PSOs on 4 test functions.

f		PSO	IPSO	IIPSO
f_1	Best avg.	3.37e-50	3.37e-50	3.37e-50
	C	-	1.0	1.0
	Achievement	100%	100%	100%
f_2	Best avg.	56.45	37.56	11.41
	C	-	0.6	0.005
	Achievement	37%	89%	100%
f_3	Best avg.	57.34	6.13	2.85e-08
	C	-	0.4	0.04
	Achievement	0%	53%	100%
f_4	Best avg.	20.32	8.44	7.20e-02
	C	-	0.06	0.08
	Achievement	4%	71%	100%

Figure 1 shows respective mean results and their achievement rate over 100 runs in different cooperativeness C. Note that the standard PSO used $C = 1.0$ for all the simulations. The best mean result among all C, its value of C and achievement rate [%] are listed in Table 2. For the unimodal function f_1, the fully-connected PSOs with $C = 1.0$, namely the standard PSO, obtained the best result. In fact, the results became better as the value of C increased. This is because quick communication is more important for the simple problem than the diversity.

On the other hand, for the multimodal functions as f_2–f_3, IPSO and IIPSO obtained more effective results than when it was fully-connected ($C = 1.0$). In addition, IIPSO significantly improved performances of not only the standard PSO but also IPSO although IPSO improved the performance of the standard PSO. These results mean that the particle diversity is more important for the multimodal functions than the quick communication, and the particles of IIPSO is more diverse than IPSO. At the same time, IIPSO obtained perfect achievement rate for all the multimodal problems although not only the standard PSO but also IPSO was not able to the 100% achievement rate for f_2–f_4. It is the significant improved performances from the standard PSO and IPSO. Furthermore, IIPSO kept 100% for long range of C. It means that

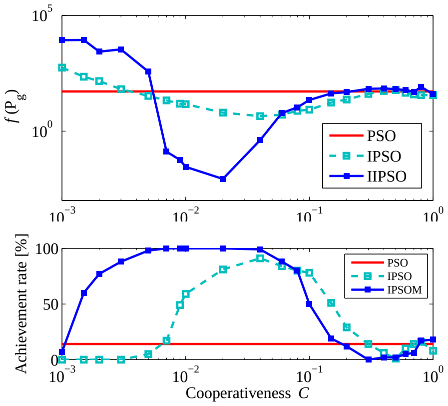

Figure 2: Simulation results of combined problem.

Table 3: Comparison results of 3 PSOs on combined problem.

f		PSO	IPSO	IIPSO
f_5	Best avg.	51.91	4.43	8.57e-03
	C	-	0.04	0.02
	Achievement	14%	91%	100%

the parameter-dependence of IIPSO is weaker than IPSO, that is, IIPSO is easier to set the parameters than IPSO.

Next, we consider a combined optimization benchmark function. In the standard benchmark function, the feature of all the dimensions is same because the same equation is used for all the dimensions, even if it is a high dimension problem. To avoid this issue, we consider a following function which is a combination of the plural benchmarks.

$$
\begin{aligned}
f_5(\boldsymbol{X}) = & \textstyle\sum_{d=1}^{d_1} x_d^2 + \sum_{d=d_1+1}^{d_2} |x_d| + \sum_{d=d_2+1}^{d_3} dx_d^4 \\
& + \textstyle\sum_{d=d_3+1}^{d_4} \left(x_d^2 - 10\cos(2\pi x_d) + 10\right) \\
& + \textstyle\sum_{d=d_4+1}^{d_5-1} \Big(20 + e - 20e^{-0.2\sqrt{0.5(x_d^2+x_{d+1}^2)}} \\
& \qquad - e^{0.5(\cos(2\pi x_d)+\cos(2\pi x_{d+1}))}\Big) \\
& + \textstyle\sum_{d=d_5+1}^{d_6-1} (x_d^2 + x_{d+1}^2)^{0.25}\left(1+\sin^2(50(x_d^2+x_{d+1}^2)^{0.1})\right).
\end{aligned} \tag{4}
$$

Eq. (4) consists of 6 benchmarks, and different benchmark is used for each dimension. In particular, Sphere function, 3rd De Jong's function, 4th De Jong's function, Rastrigin's function, Ackley's function and Stretched V sine wave function are used for 1^{st}–d_1^{th}, $(d_1+1)^{\mathrm{th}}$–d_2^{th}, $(d_2+1)^{\mathrm{th}}$–d_3^{th}, $(d_3+1)^{\mathrm{th}}$–d_4^{th}, $(d_4+1)^{\mathrm{th}}$–d_5^{th} and $(d_5+1)^{\mathrm{th}}$–d_6^{th} dimensions, respectively. The optimum solution $\boldsymbol{X}^*$ and its optimum value $f(\boldsymbol{X}^*)$ are $[0, 0, \cdots, 0]$ and 0. In this simulation, we set each dimension as $d_1 = 4$, $d_2 = 8$, $d_3 = 12$, $d_4 = 16$, $d_5 = 20$ and $d_6 = 24$, and the criterion for f_5 is set as 10.

The mean results and achievement rate of 3 PSOs over 100 runs by changing the cooperativeness C are shown in Fig. 2. The best mean result among all C are listed in Table 3. When the particles were little affected by *gbest*, IPSO and IIPSO obtained more effective results than when it was fully-connected ($C = 1.0$). This result means that we can obtain better performance that the particles value own information and are sometimes affected by others, instead of connecting to all the particles. Furthermore, it is clear that IIPSO significantly improved the performances from IPSO. In fact, IIPSO obtained the perfect achievement rate at many values of C although IPSO has never obtained it even if C was changed.

From these results, we can say that IIPSO easy escape from the local optima and provides significantly improved performance in terms of robustness, accuracy based on evaluation criteria and parameter dependence. Therefore, IIPSO is effective algorithm for the multimodal functions.

4. Conclusions

This study has proposed the improved Independent-mined Particle Swarm Optimization (IIPSO) algorithm. The particles of IIPSO have independence, and it is decided stochastically that each particle depends on *gbest* or not. Furthermore, whether the particle is affected by the swarm is judged for every dimension. We have applied IIPSO to various optimization problem. From the results, we have confirmed that IIPSO easy escape from the local optima, and for the multimodal functions, IIPSO provides significantly improved performance in terms of robustness, accuracy based on evaluation criteria and parameter dependence.

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