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Relationship between Synchronization and Parameter in Two-Template CNN with Periodic Boundary Conditions

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Abstract

In our previous study, we have proposed Two-Template CNN. This system was proposed in order to investigate a new class of coupled oscillatory systems. The system was investigated in some conditions of templates or boundaries. As a result, some interesting phenomena are observed. In this study, we investigate a relationship between parameters of templates and oscillatory phenomena in Two-Template CNN with periodic boundary conditions.

1. Introduction

Cellular Neural Networks (CNN) [1],[2] is one kinds of mutually neural networks. The main characteristic is the local connection and the parallel signal processing. There have been many studies on CNN and many kinds of modified CNNs have been proposed. One of them is Two-layer CNN. Two-layer CNN [3], [4] can generate many interesting phenomena. For instance, self-organizing pattern, active wave propagation, clustering and so on are observed.

In our previous studies [5], [6], we have proposed Two-template CNN. The architecture of Two-Template CNN is very simpler than Two-layer CNN. In the case of comparing a conventional CNN, control signal lines of template values becomes two times only. The architecture and the number of coupling lines of one cell are same as a conventional CNN. Additionally, this system can generate similar phenomena to Two-layer CNN and some interesting phenomena can be obtained. In these studied, fixed boundary conditions cases are investigated mainly. Since many kinds of complex phenomena are observed. Additionally, some simple oscillatory phenomena are observed only in the case of periodic boundary conditions. However, it is important to be able to investigate oscillatory phenomena in the case of periodic boundary conditions. Basically, this condition of our system is corresponding to the case of non-oscillation in conventional CNN. Therefore, to confirm oscillatory phenomena is very interesting.

In this study, relationship between template parameters and oscillatory phenomena are investigated in the case of periodic boundary condition. Additionally, observed oscillatory phenomena are investigated in details.

2. Two-Template CNN

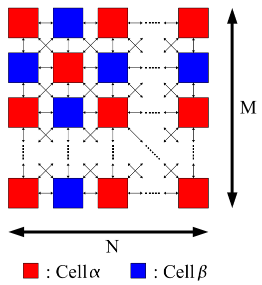

Figure 1: Structure of Two-Template CNN.

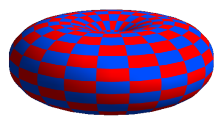

Figure 2: A periodic boundary conditions.

Figure 1 shows an architecture of Two-Template CNN. We assume that the system has a two-dimensional M by N array architecture. Each cell in the array is denoted as $c(i, j)$, where (i, j) is the position of the cell, $1 \leq i \leq M$ and $1 \leq j \leq N$. The coupling radius is assumed to be one. Cells having one template are called as cell α and the other are called as cell β. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

1: The case that $i + j$ is an even number.

$$\begin{aligned}\frac{dx_{ij}}{dt} =& -x_{ij} + I_\alpha \\ &+ \sum_{c(k,l)} A_\alpha(i,j;k,l)y_{kl} \\ &+ \sum_{c(k,l)} B_\alpha(i,j;k,l)u_{kl}\end{aligned} \tag{1}$$

2: The case that $i+j$ is an odd number.

$$\begin{aligned}\frac{dx_{ij}}{dt} =& -x_{ij} + I_\beta \\ &+ \sum_{c(k,l)} A_\beta(i,j;k,l) y_{kl} \\ &+ \sum_{c(k,l)} B_\beta(i,j;k,l) u_{kl}\end{aligned} \tag{2}$$

$A_{\{\alpha\beta\}}(i,j;k,l)y_{kl}$, $B_{\{\alpha\beta\}}(i,j;k,l)u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively. The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}), \tag{3}$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \tag{4}$$

This system is more complex than a conventional CNN. Although this system has a unique characteristic. Namely, a pair of cell α and cell β are needed for a simple oscillation. Additionally, one cell α connects with four neighbor cells β and one cell β also connects with four neighbor cells α. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators. Hence, we consider that this system is a new class of coupled oscillatory systems.

3. Relationship between oscillatory phenomena and parameters

In our previous study, some kinds of boundary conditions have been investigated. As a result, some kinds of interesting phenomena was observed. Additionally, it was assumed that boundary conditions affect oscillatory phenomena. In fact, there are complete stability theorem [7] of conventional CNN shown as follows.

Any $M \times N$ space-invariant CNN of arbitrary neighborhood size with constant inputs and constant threshold is completely stable if the following three hypotheses are satisfied.

1. The A template is symmetric

$$A(i,j;k,l) = A(k,l;i,j) \tag{5}$$

2. The nonlinear function $y_{ij} = f(x_{ij})$ is differentiable, bounded, and

$$f'(x_{ij}) > 0, \quad \text{for all} - \infty < x_{ij} < \infty \tag{6}$$

3. All equilibrium points are isolated.

In this study, Two-Template CNN is set the condition which is corresponding to the condition which satisfy these hypotheses. Therefore, the investigation of oscillatory phenomena is very interesting. In order to carry out computer simulations, following configurations are applied. The initial state values set as random values. A boundary condition is set as a periodic boundary condition as shown in Fig. 2.

The template is set as follows.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -u & v & -u \\ v & w & v \\ -u & v & -u \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} u & -v & u \\ -v & -w & -v \\ u & -v & u \end{pmatrix},$$
$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0. \tag{7}$$

We consider that coupling coefficients and oscillation factors are corresponding to u and v, respectively. These templates are symmetric each other. This setting satisfy the hypothesis 1. The number of cells is fixed as 8×8. In the case of lower than 8×8, only one kind of synchronization phenomena is observed and in the case of over 8×8, various kinds of oscillatory phenomena are observed. Hence, 8×8 which is lowest number in various kinds of oscillatory phenomena is selected.

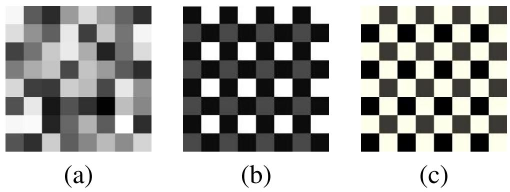

Figure 3: Simulation result in the case of periodic boundary condition. The number of cells is 8×8. $u = 2$, $v = 5$ and $w = 1$. (a) Initial state. (b) and (c) Snapshot of a stable state.

Figure 3 shows a computer simulation result in the case of $u = 2$, $v = 5$ and $w = 1$. Figure 3 (a) shows an initial state. Figures 3 (b) and (c) show snapshots of a stable state. The oscillatory phenomena are observed. These oscillations are divided into four groups is shown in Fig. 4. Cells of each groups are synchronized. In order to investigate the phenomena in detail, some computer simulations are carried out.

Figure 5 shows state variables of cells. Vertical axes show state variables. Horizontal axes show time. The first row shows state variables of group α_1. In the same way, the second, third and fourth rows show state variables of groups β_1, β_2 and α_2, respectively. Each row includes sixteen waveforms. This result is corresponding to Fig. 3. Synchronization phenomena of each group are observed. Additionally, groups β_1 and β_2 are also synchronized. Moreover, a difference of groups α_1 and α_2 is an equilibrium point only. Namely, groups α_1 and α_2 are also synchronized. In this simulation, the other result is obtained. The difference of two results is that α_2 is an inverse value of α_1. On some parameter sets observed oscillatory phenomena in this study, same

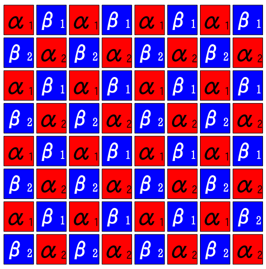

Figure 4: Classification of cells for the investigation of oscillatory phenomena.

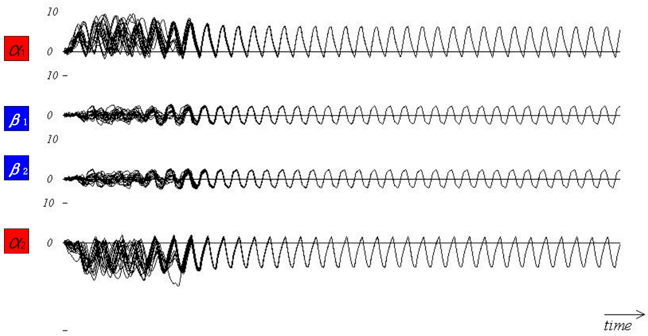

Figure 5: Waveforms of state values. $u = 1.0$, $w = 1.0$ and $v = 5.0$.

results are obtained.

First of all, in order to investigate the oscillation region in detail,some computer simulation are carried out.

Figure 6 shows the relationships between oscillatory phenomena and parameters u and v. Horizontal axis shows v. Vertical axis shows u. “Osc” means that the oscillatory phenomena are observed. “N” means that the oscillatory phenomena are not observed. Intervals of u and v are set as 1 in Fig. 6. Line-symmetric oscillatory regions are confirmed. Figures 7 and 8 show waveforms of state variables of cells. In Figure 7, parameters are set as $u = 2$, $v = 7$ and $w = 1$. In Figure 8, parameters are set as $u = 3$, $v = 5$ and $w = 1$. In an oscillatory region of first quadrant of Fig. 6. Following relatioinships are confirmed by comparing with Fig. 5, 7, 8 and 9.

- By increasing parameter u, the amplitude is increased and the frequency is decreased.
- By increasing parameter u, distances between horizontal axes and equilibrium points of groups α_1 and α_2 are incrased.
- By increasing parameter w, the frequency is decreased and the amplitude is decreased.

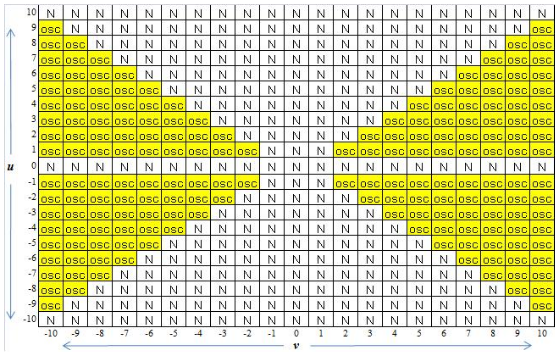

Figure 6: Relationship between oscillatory phenomena and parameters u and v. “Osc” means an oscillation. “N” means no oscillation. $-10 \leq v \leq 10$ and $-10 \leq u \leq 10$. The interval is set as 1.0.

Table 1: Relationship between parameters and oscillation frequencies or amplitude

Quadrant	By increasing v	By increasing u
First	Freq. High	Freq. Low, Amp. High
Second	Freq. Low	Freq. Low, Amp. High
Third	Freq. Low	Freq. High, Amp. Low
Fourth	Freq. High	Freq. High, Amp. Low

Quadrant	By increasing w
First	Freq. Low
Second	Freq. Low
Third	Freq. High
FourthHigh	Freq. High

In the other quadrant, there are similar relationships. These relatioinships are shown in Table. 1 and 2. Like this, relationships are line-symmetries. In almost oscillatory regions of Fig. 6, oscillations of cells are divided four groups as shown in Fig. 5, 7, 8 and 9. However, in some areas which are near non-oscillatory regions, oscillations of cells are divided five or more groups as shown in Fig 10.

4. Conclusions

In this study, we have investigated relationship between oscillatory phenomena and parameters in Two-Template CNN with periodic boundary conditions. Additionally, we have investigated the oscillation region. Furthermore, we could observed coexistences of some kinds of stable states in the case of nearly convergents of the template parameters u and v.

As future works, we will investigate these phenomena in detail.

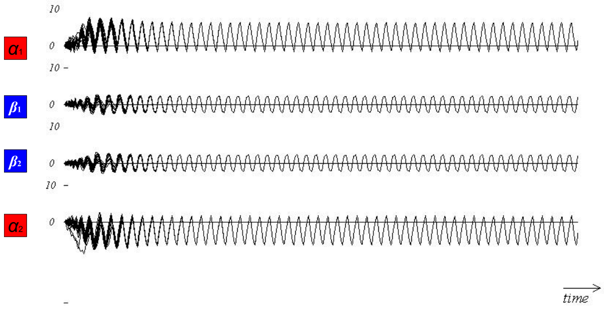

Figure 7: Waveforms of state values. $u = 2.0$, $w = 1.0$ and $v = 7.0$.

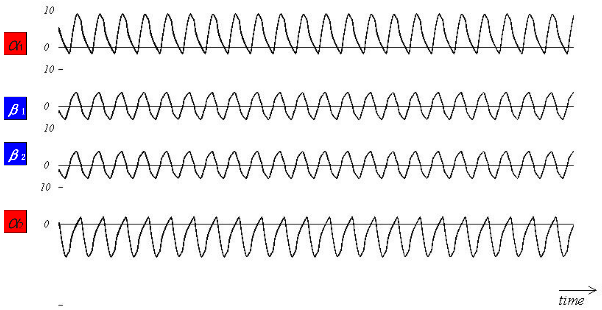

Figure 8: Waveforms of state values. $u = 3.0$, $w = 1.0$ and $v = 5.0$.

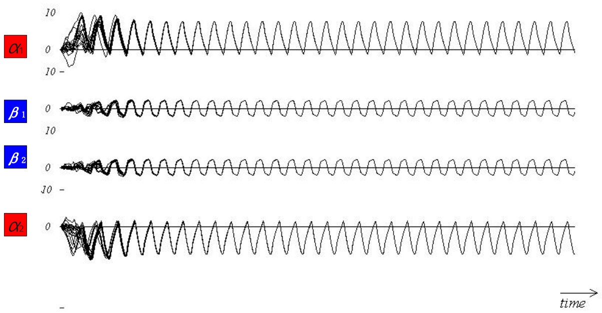

Figure 9: Waveforms of state values. $u = 2.0$, $w = 2.0$ and $v = 5.0$.

Table 2: Relationship between parameter u and distances between equilibrium points and horizontal axes.

Quadrant	By increasing u
First	Distances of group α_1 and α_2 are increased.
Second	Distances of group α_1 and α_2 are increased.
Third	Distances of group β_1 and β_2 are increased.
Fourth	Distances of group β_1 and β_2 are increased.

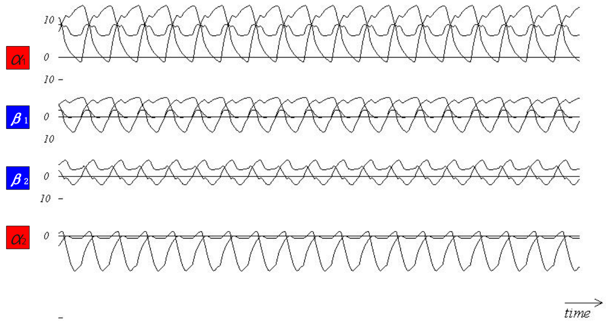

Figure 10: Waveforms of state values. $u = 3.0$, $w = 1.0$ and $v = 4.0$ in the case of the initial values set as random values.

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