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Synchronization Phenomena of Globally Coupled Logistic Maps with Time-Varying Parameters

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Abstract—Synchronization phenomena in globally coupled logistic maps whose parameters are forced into periodic varying are investigated when four and five maps are coupled. Various synchronization phenomena are observed by choosing a coupling intensity in both cases. The observed synchronization phenomena are fall into five general categories, which are asynchronous, self-switching phenomenon of synchronization, coexistence phenomena of synchronization states, synchronization of the total number of the coupled maps minus one map and synchronization of all the maps.

1. Introduction

Synchronization is one of the fundamental phenomena in nature, and one of typical nonlinear phenomena. Therefore, studies on synchronization phenomena of coupled systems are extensively carried out in various fields, physics [1], biology [2], engineering and so on. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena for future engineering applications. In a simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referred in Refs. [4] and [5]. The network of chaotic elements can be modeled by a system of coupled one-dimensional maps. Behavior generated in coupled system of chaotic one-dimensional map is investigated in Refs. [6]-[8]. In particular, Coupled Map Lattice (CML) and Globally Coupled Map (GCM) are well known as mathematical models in discrete-time system. The research into CML and GCM is important for not only modeling of nonlinear systems of multiple degree of freedom but also application to biological networks and engineering. In the past we have observed effects of parametric excitation of coupled van der Pol oscillators [9]. Previously, we investigate synchronization phenomena in three coupled logistic maps involving parametric force [10]. In this study, we propose now to investigate synchronization phenomena generated by a larger number of coupled maps with globally coupling. A typical scheme for global coupling is given by:

$$x_i(n+1) = (1-\varepsilon)f[x_i(n)] + \frac{\varepsilon}{N}\sum_{j=1}^{N} f[x_j(n)] \qquad i = 1, 2, \cdots, N, \tag{1}$$

where $\varepsilon \in [0, 1]$ is the coupling intensity. The globally coupled maps are a scheme such that an average number of all the maps affect each of the map. The one-dimensional map used in this study is a logistic map, since the map can be described by a simple discrete equation. Mathematically, the logistic map is written as

$$x(n+1) = \alpha x(n)(1 - x(n)). \tag{2}$$

In this study, we investigate synchronization phenomena in the globally coupled logistic maps whose parameters are forced into periodic varying when four or five maps are coupled. The paper is organized as follows. In the next section, we present the parametrically forced logistic map. Synchronization phenomena observed in the globally coupled maps are in Section 3. The last section is devoted to the conclusion.

2. Parametrically forced logistic map

A parametrically forced logistic map used in this study is described as:

$$x(n+1) = \alpha_f(n)x(n)(1 - x(n)), \tag{3}$$

and

$$\alpha_f(n) = \begin{cases} \alpha_1, & \text{for each even value of “}n\text{”} \\ \alpha_2, & \text{for each odd value of “}n\text{”} \\ & (n = 0, 1, 2, ...) \end{cases}, \tag{4}$$

where $\alpha_f(n)$ is a term of the parametric force and time-varying. In this system, two kinds of parameters, α_1 and α_2, are alternately replaced every update. Figure 1 shows an example of a return map of the parametrically forced logistic maps. For the original logistic map, two-periodic solution is observed for $\alpha = 3.0$, while, three-periodic solution is observed for $\alpha = 3.83$. These two solutions are periodic, whereas in the logistic map involving parametric force, a solution is chaotic as shown in Fig. 1 when the parameters α_1 and α_2 are set 3.0 and 3.83, respectively. Namely, chaotic solution can be observed in the combination of two parameters that generate two kinds of periodic solutions. In the following, the parameter values are fixed as $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$.

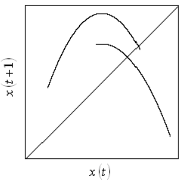

Figure 1: Return map of parametrically forced logistic map for $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$.

3. Synchronization

Synchronization phenomena generated in the coupled logistic map involving parametric force are investigated for one control parameter ε which is coupling intensity when four and five maps are coupled. In the following computer calculations, the logistic map parameters are fixed as $\alpha_1 = 3.0$ and $\alpha_2 = 3.8$.

3.1. Four maps case

In this subsection, we consider the case of $N = 4$, namely four parametrically forced logistic maps are coupled. Figure 2 shows maximum Lyapunov exponents, which calculates how complex trajectory of the solution is, for one-control parameter ε. Various synchronization phenomena are observed for ε. Examples of synchronization phenomena observed in the four coupled maps are shown in Figs. 3 and 4.

First, the Lyapunov exponent is positive and all the maps behave chaotic when ε is zero, namely all the maps are not coupled. When ε is small, the maps are not synchronized. Increasing ε, a self-switching phenomenon of synchronization on four coupled maps is observed as shown in Fig. 3 when ε is set around 0.045. The phenomenon is that three among the four maps are synchronized and the combination of the synchronized pair changes with time. Figure 3 shows time series of differences of $x(n)$ between two maps. Areas where the amplitudes of the time series are small correspond to in-phase synchronization in the figure. In Fig. 3, firstly, map 1, map 3 and map 4 are synchronized. However, after a while, the synchronous state breaks up and map 1, map 2 and map 4 are synchronized. As seen above, the synchronous states switch with time. More increasing ε, the Lyapunov exponent becomes around zero or negative. In some parts of ε between 0.05 and 0.08, there exist some

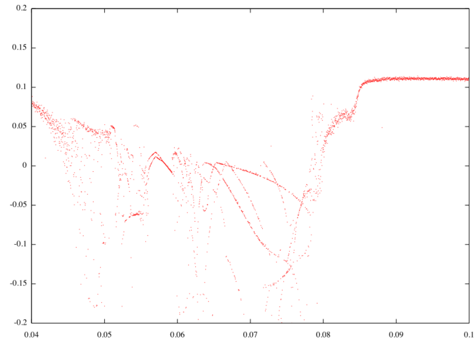

Figure 2: Lyapunov exponents in globally coupled parametrically forced logistic maps for $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$. Horizontal axis: ε. Vertical axis: λ.

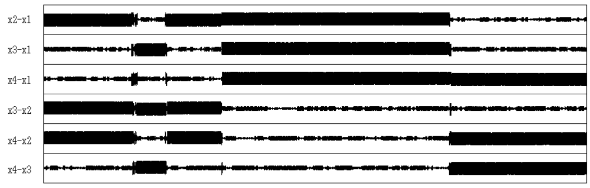

Figure 3: Time series of differences between two maps when a self-switching phenomenon of the synchronization are observed. $\varepsilon = 0.045$, $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$.

lines of Lyapunov exponent. It means that the synchronization state is multi stable. Coexistence phenomena of various synchronization states are observed for the values of ε as shown in Figs. 4(a-1) and (a-2). Two pairs of two chaotic maps are synchronized in Fig 4(a-1). While, three among the four chaotic maps are synchronized in Fig 4(a-2). These two synchronization state are observed for the same parameters, although initial values are different. The synchronization states depend on the initial values. In the region of the multi stable state, other coexistence phenomena of synchronization states are confirmed as;

- coexistence of three kinds of synchronization states that are synchronization of two among the four chaotic maps, synchronization of two pairs of two periodic maps and synchronization of three among the four periodic maps,
- coexistence of two kinds of synchronization states that are synchronization of two pairs of two periodic maps and two pairs which are synchronization of two periodic maps and quasi-synchronization of two maps,
- coexistence of two kinds of synchronization states that are synchronization of two among the four chaotic maps and synchronization of two pairs of two chaotic maps.

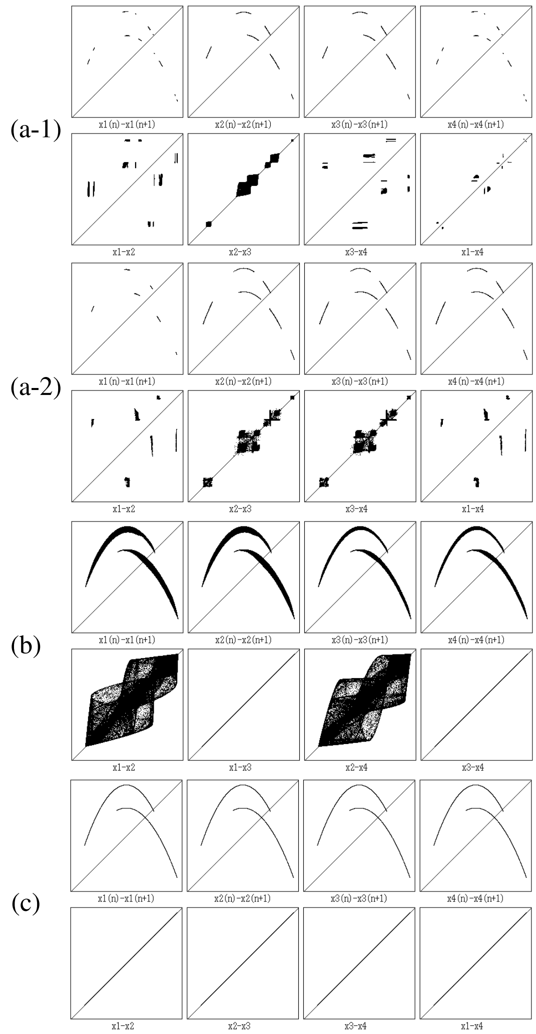

Figure 4: Synchronization of four maps. $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$. (a) $\varepsilon = 0.050$. (b) $\varepsilon = 0.160$. (c) $\varepsilon = 0.200$.

Next, more increasing ε, the Lyapunov exponent becomes positive again. Synchronization of three among the four chaotic maps is observed when $\varepsilon = 0.160$ as shown in Fig. 4(b). Finally, when ε is equal and over 0.200, all the maps are synchronized as shown in Fig. 4(c).

3.2. Five maps case

In this subsection, we consider the case of $N = 5$. Figure 5 shows maximum Lyapunov exponents for one-control parameter ε. Various synchronization phenomena are observed for ε. Examples of synchronization phenomena observed in the five coupled maps are shown in Figs. 6 and 7.

When ε is small, the maps are not synchronized. Increasing ε, a self-switching phenomenon of synchronization on five coupled maps is observed as shown in Fig. 6 when ε is set around 0.045. The phenomenon is that three among the five maps are synchronized and another two maps are synchronized and the combination of the synchronized pair changes with time. Figure 6 shows time series of differences of $x(n)$ between two maps. More increasing ε, the Lyapunov exponent becomes around zero or negative. In some parts of ε between 0.05 and 0.095, there exist some lines of Lyapunov exponent. Coexistence phenomena of various synchronization states are observed in the parts of ε as shown in Figs. 7(a-1) and (a-2). Three among the five periodic maps are synchronized in Fig 7(a-1). While, Three among the five periodic maps are synchronized and another two periodic maps are synchronized in Fig 7(a-2). In the region of the multi stable state, other coexistence phenomena of synchronization states are confirmed. It confirms the existence of some kinds of coexistence phenomena which are constructed as combinations of following synchronization states;

- quasi-synchronization of three among the five maps and quasi-synchronization of two other maps,
- quasi-synchronization of three among the five maps and synchronization of two other maps,
- synchronization of three among the five maps and synchronization of two other maps,
- two pairs of quasi-synchronization of two maps,
- two pairs of synchronization of two maps,
- synchronization of two maps and quasi-synchronization of two maps,
- two pairs of the synchronization of two maps and quasi-synchronization of one of the two pairs and another map,
- two pairs of the synchronization of two maps and quasi-synchronization of the two pairs,
- quasi-synchronization of three among the five maps.

Next, more increasing ε, the Lyapunov exponent becomes positive again. Synchronization of four among the five chaotic maps is observed when $\varepsilon = 0.190$ as shown in Fig. 7(b). Finally, when ε is equal and over 0.210, all the maps are synchronized as shown in Fig. 7(c).

4. Conclusion

In this study, we have investigated synchronization phenomena in globally coupled logistic maps whose parameters are forced into periodic varying when four and five maps are coupled. Various synchronization phenomena can be observed by choosing a coupling intensity in both the four coupled maps case and the five coupled maps case. The observed synchronization phenomena fall into five general categories, which are asynchronous, self-switching phenomenon of synchronization, coexistence phenomena

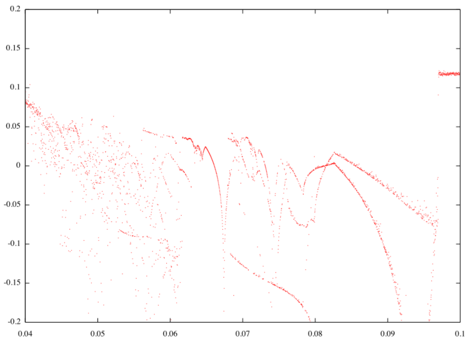

Figure 5: Lyapunov exponents in globally coupled parametrically forced logistic maps for $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$. Horizontal axis: ε. Vertical axis: λ.

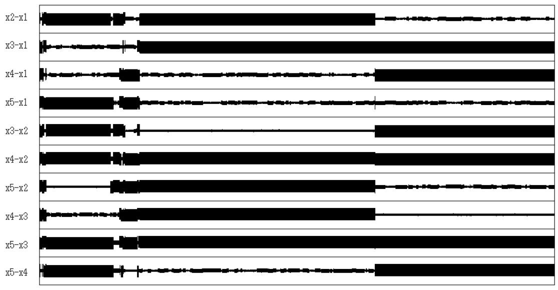

Figure 6: Time series of differences between two maps when a self-switching phenomenon of the synchronization are observed. $\varepsilon = 0.045$, $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$.

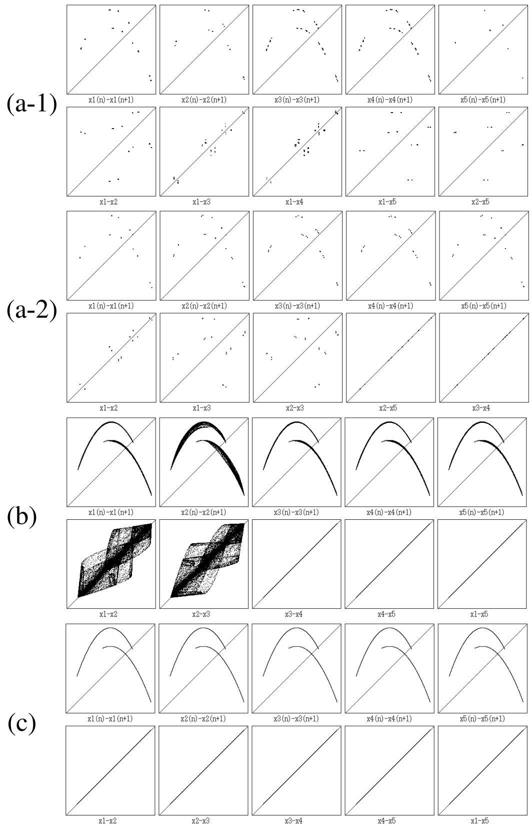

Figure 7: Synchronization of five maps. $\alpha_1 = 3.0$ and $\alpha_2 = 3.83$. (a) $\varepsilon = 0.050$. (b) $\varepsilon = 0.190$. (c) $\varepsilon = 0.210$.

of synchronization states, synchronization of the total number of the coupled maps minus one map and synchronization of all the maps.

References

[1] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, “Synchronization and Graph Topology,” Int. J. Bifurcation and Chaos, vol.15, no. 11, pp.3423-3433, Nov. 2005.

[2] J. Cosp, J. Madrenas, E. Alarcon, E. Vidal and G. Villar, “Synchronization of Nonlinear Electronic Oscillators for Neural Computation,” IEEE Trans. Neural Networks, vol. 15, no. 5, pp. 1315-1327, Sep. 2004.

[3] C. Hayashi, Nonlinear Oscillations in Physical Systems, Chap. 11, McGraw-Hill, New York 1964.

[4] C. Hayashi, M. Abe, K. Oshima and H. Kawakami, “The Method of Mapping as Applied to the Solution for Certain Types of Nonlinear Differential Equations,” Proc. of the Ninth International Conference on Nonlinear Oscillations, pp. 1-8, 1981.

[5] M. Inoue, “A Method of Analysis for the Bifurcation of the Almost Periodic Oscillation and the Generation of Chaos in a Parametric Excitation Circuit,” Trans. of IEICE, vol. J68-A, no. 7, pp. 621-626, 1985.

[6] K. Kaneko, “Spatiotemporal Intermittency in Coupled Map Lattice,” Progress of Theoretical Physics, vol. 74, no. 5, pp. 1033-1044, Nov. 1985.

[7] K. Kaneko, “Pattern Dynamics in Spatiotemporal Chaos,” Physica D, vol. 34, nos. 1-2, pp. 1-41, Jan. 1989.

[8] K. Kaneko, “Clustering, Coding, Switching, Hierarchical Ordering, and Control in a Network of Chaotic Elements,” Physica D, vol. 41, no. 2, pp. 137-172, Mar. 1990.

[9] H. Kumeno and Y. Nishio, “Synchronization Phenomena in Coupled Parametrically Excited van der Pol Oscillators,” Proc. of International Symposium on Nonlinear Theory and its Applications (NOLTA’ 08), pp. 128-131, Sep. 2008

[10] H. Kumeno and Y. Nishio, “Globally Coupled Parametrically Forced Logistic Maps,” Proc. of Asia Pacific Conference on Postgraduate Research in Microelectronics and Electronics (PrimeAsia’09), pp. 181-184, Nov. 2009.