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SPICE-Oriented Algorithm for Assessment of Stability for Periodic Solutions

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Abstract—The assessment of the stability for periodic solutions is very important for designing the circuit. There are many method for the assessment of the stability. In this article, we propose a SPICE-oriented method for the assessment of the stability, that is based on the Floquet theory. By using our method, we can assess the stability of the circuit easily. First, we obtain the periodic solutions of the circuit by using the SPICE. Next, we calculate the eigenvalues of a Jacobian matrix by solving variational circuits based on the Floquet theory. As an example, we assess the stability of the periodic solutions for one order resonance circuit including nonlinear capacitors.

1. Introduction

When we simulate the circuit, we often refer to the assessment of the stability. In this paper, we propose a SPICE-oriented algorithm to the assessment of the stability for periodic solutions which is based on the Floquet theory [1]. In the conventional method, we have to calculate the Jacobian matrix for the periodic solutions by solving the variational equations. In this study, we obtain the Jacobian matrix by the transient analysis of SPICE for variational circuits which is easily derived from the original circuit.

Section 2.1 shows how to use the sine-cosine circuits [2], which is based on the HB (harmonic balance) method. We use the sine-cosine circuit to obtain the value of the voltages which are required in order to solve variational circuits. Section 2.2 shows the solution curve-tracing circuit. It is based on the arc-length method [3][4]. Section 2.3 shows the Floquet theory. Section 3 shows an illustrative example and how to solve the variational circuits by using SPICE. Section 4 shows the results and confirms the effectiveness of the proposed method. Section 5 concludes this article.

2. Frequency analysis and assessment of stability

2.1. Sine-cosine circuit

We introduce the sine-cosine circuit corresponding to the determining equation of the HB method. If we set the voltage through a capacitor C

$$v_C = V_{CS} \sin\omega t + V_{CC} \cos\omega t, \qquad (1)$$

the current i_C is given by

$$i_C = C\frac{dv_C}{dt} = -\omega C V_{CC} \sin\omega t + \omega C V_{CS} \cos\omega t. \qquad (2)$$

Thus, the coefficients of $\sin\omega t$ and $\cos\omega t$ are described by

$$I_{CS} = -\omega C V_{CC}, \quad I_{CC} = \omega C V_{CS}. \qquad (3)$$

Namely, the capacitor is replaced by coupled voltage-controlled current sources in the sine-cosine transformation of the HB method. In the same way, let the current through an inductor L be

$$i_L = I_{LS} \sin\omega t + I_{LC} \cos\omega t. \qquad (4)$$

Then, the voltage v_L is given by

$$v_L = L\frac{di_L}{dt} = -\omega L I_{LC} \sin\omega t + \omega L I_{LS} \cos\omega t. \qquad (5)$$

Thus, the coefficients of $\sin\omega t$, $\cos\omega t$ are described by

$$V_{LS} = -\omega L I_{LC}, \quad V_{LC} = \omega L I_{LS}. \qquad (6)$$

Namely, the inductor is replaced by coupled current-controlled voltage sources in the sine-cosine transformation.

As an example, Fig. 1 shows an RLC ladder circuit and Fig. 2 shows the corresponding sine-cosine circuits.

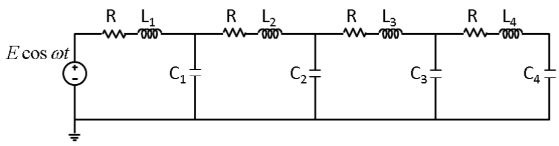

Figure 1: RLC ladder circuit

2.2. Solution curve-tracing circuit

Even we use our sine-cosine circuits, we can not obtain unstable periodic solutions, because we set the frequency as time in SPICE. In this section, we explain the STC (solution trace circuit) realizing the arc-length method.

First, we can express the ark length in $(n+1)$ dimensional euclidean space as Eq. (7)

$$ds = \sqrt{(dx_1)^2 + (dx_2)^2 + (dx_3)^2 + \ldots + (dx_{n+1})^2} \qquad (7)$$

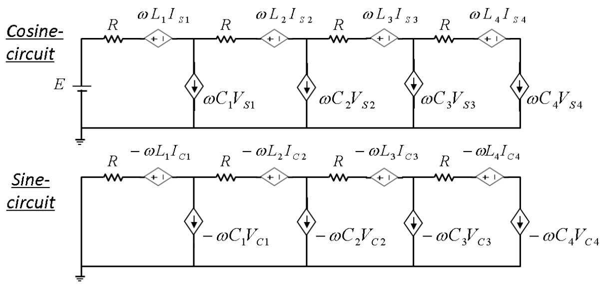

Figure 2: Sine-cosine circuit for Fig.1

In order to trace the solutions curve by SPICE, we replace the differentiation with respect to the arc-length s by the time t. We assume x_k as voltages in SPICE. From this, we can obtain Eq. (8).

$$\sum_{i=1}^{p}\left(\frac{dv_i}{dt}\right)^2+\left(\frac{dv_\omega}{dt}\right)^2=1 \tag{8}$$

In this paper, where v_i ($i = 1, 2, \ldots, p$) are the coefficient of voltages in Eqs. (1) and (4) and v_ω corresponds to ω. They are realized by using differentiators (simply realized by capacitors with $1(F)$ in SPICE). The circuit in Fig. 3 realizes to satisfy the arc-length method Eq. (8). In this

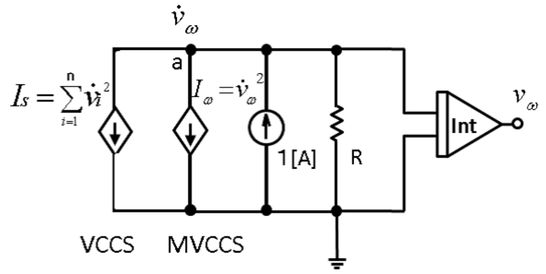

Figure 3: STC (Solution curve-tracing circuit)

circuit, the voltages corresponding to the coefficients are inputted after differentiated and squared via the voltage-controlled current source (VCCS). If we set the voltage of node a as $\dot{v}_\omega$, $I_\omega = \dot{v}_\omega^2$ can be obtained by multiplier and voltage-controlled current source VCCS (MVCCS). Then, the node voltage $\dot{v}_\omega$ is integrated to obtain v_ω. Note that R in Fig. 3 is a very large resistance used only to avoid the $L-J$ cut-set.

2.3. Stability of periodic solutions

We suppose that there is a circuit equation as

$$f(\dot{x},\ x,\ y,\ \omega t) = 0, \tag{9}$$

and make the variational equation for the regular period solution of $\hat{x}$. First, we assume the small change quantity as $(\Delta x, \Delta y)$ as

$$\begin{cases} x = \hat{x} + \Delta x \\ y = \hat{y} + \Delta y \end{cases} \tag{10}$$

and substitute Eq. (10) to Eq. (9). We obtain the equation as

$$f(\dot{\hat{x}},\ \hat{x},\ \hat{y},\ \omega t) + \left[\frac{\partial f}{\partial \dot{x}}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\right]\Big|_{x=\hat{x},y=\hat{y}} \begin{bmatrix} \dot{\Delta x} \\ \Delta x \\ \Delta y \end{bmatrix} = 0. \tag{11}$$

In Eq. (11), the first term is regular period solution and second term is variational equation. We change the second term as

$$\dot{\Delta x} = A(t)\Delta x. \tag{12}$$

In Eq. (12), $A(t)$ is the periodic function. We apply the Floquet theory for this periodic function. We write the Jacobian matrix of the periodic solution as $\Phi(t)$. From this, the solution after one period from initial value of $\Delta x(0)$ is given as follows;

$$\Delta x(T) = \Phi(T)\Delta x(0). \tag{13}$$

Hence, when the eigenvalues $(\lambda_1, \lambda_2, \ldots, \lambda_n)$ of $\Phi(T)$ satisfy $|\lambda_k| < 1$ ($k = 1, 2, \ldots, n$), the regular periodic solution $\hat{x}$ is stable.

3. Illustrative example

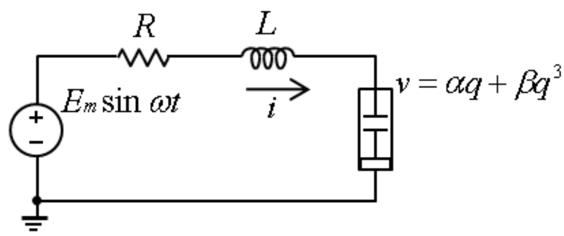

Figure 4: Example model

Figure 4 shows a circuit for an illustrative example. The nonlinear characteristics can be solved by using the SPICE model in Fig. 5.

We can express the circuit equations as follows;

$$\begin{cases} e(t) = Ri + L\dfrac{di}{dt} + \alpha q + \beta q^3 \\ \dfrac{dq}{dt} = i \end{cases} \tag{14}$$

If we write the variables as periodic solutions with small variations:

$$\begin{cases} i = i_0 + \Delta i \\ q = q_0 + \Delta q, \end{cases} \tag{15}$$

we obtain the following variational equations.

$$\begin{cases} e(t) = R\Delta i + L\dfrac{d\Delta i}{dt} + (\alpha + \beta 3{q_0}^2)\Delta q \\ \dfrac{d\Delta q}{dt} = \Delta i \end{cases} \tag{16}$$

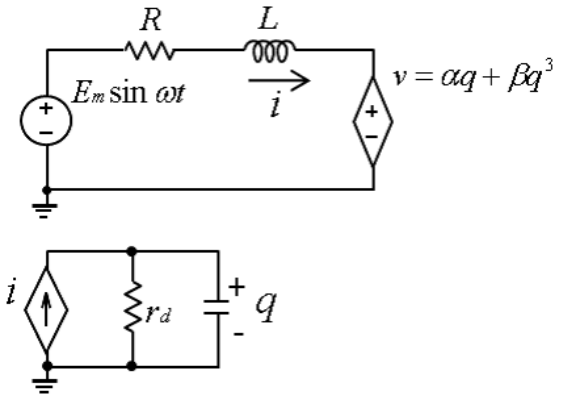

Figure 5: SPICE model for Fig.4

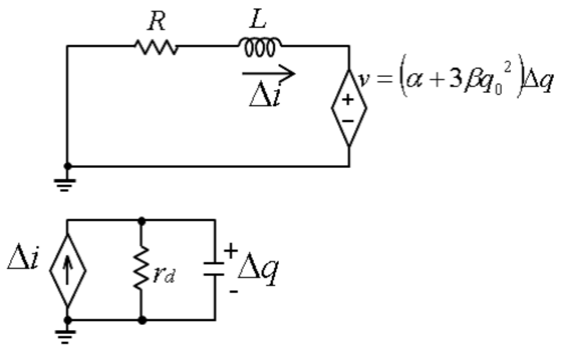

Figure 6: Variational circuit for Fig. 5

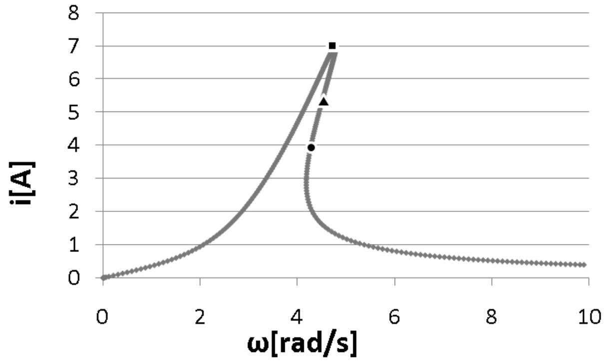

Figure 7: Frequency response of i

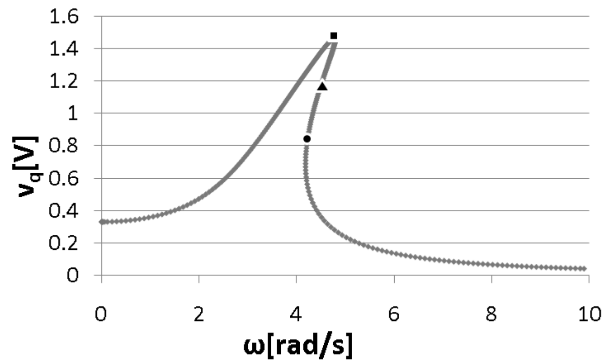

Figure 8: Frequency response of q

where we neglect higher-order small terms.

From these equations, we can make the variational circuit for Fig. 5. In Fig. 6, q_0 is the steady solutions given as

$$q_0 = Q_c \cos \omega t + Q_s \sin \omega t. \quad (17)$$

We analyze this circuit and calculate the values of two variables after one period from two different initial conditions; $(\Delta i_0, \Delta q_0) = (1, 0)$ or $(\Delta i_0, \Delta q_0) = (0, 1)$. We obtain 4 variational values for $\Phi(T)$.

$$\Phi = \begin{bmatrix} \Delta i_{(\Delta i_0, \Delta q_0)=(1,0)} & \Delta q_{(\Delta i_0, \Delta q_0)=(1,0)} \\ \Delta i_{(\Delta i_0, \Delta q_0)=(0,1)} & \Delta q_{(\Delta i_0, \Delta q_0)=(0,1)} \end{bmatrix}$$

We calculate the eigenvalues of $\Phi(T)$ by using MATLAB and assess the stability of the periodic solutions.

4. Simulation results

Figures 7 and 8 show the simulation results of the frequency response of i and q, respectively, which are obtained by the sine-cosine circuits and the STC with SPICE.

We set the parameters as follows; $E_m = 0.35[V]$, $\alpha = 1.0$, $\beta = 0.8$, $R = 0.05[\Omega]$, $L = 0.1[H]$. In this section, we compare and check the our results with the results which obtained by transient analysis of Fig. 4. We analyze the case $\omega = 4.78[rad/s]$, $\omega = 4.51[rad/s]$ and $\omega = 4.26[rad/s]$.

For the validation, we simulate the original circuit. The simulation results of transient analysis (Fig. 9, Fig. 10 and Fig. 11), indicate that $\omega = 4.78[rad/s]$ is stable, and that $\omega = 4.51[rad/s]$ and $\omega = 4.26[rad/s]$ are unstable.

Next, we show the results which obtained by our method. First, we show the solutions for the case of $\omega = 4.78[rad/s]$.

$$\Phi = \begin{bmatrix} 0.599 & 0.012 \\ -13.897 & 0.593 \end{bmatrix}$$

Second, we show the solutions for the case of $\omega = 4.51[rad/s]$.

$$\Phi = \begin{bmatrix} 0.0447 & 0.149 \\ -2.831 & 1.707 \end{bmatrix}$$

Lastly, we show the solutions for the case of $\omega = 4.26[rad/s]$.

$$\Phi = \begin{bmatrix} 0.4001 & 0.1209 \\ 0.7348 & 1.412 \end{bmatrix}$$

Table 1 shows the calculated eigenvalues of Φ for the two ω.

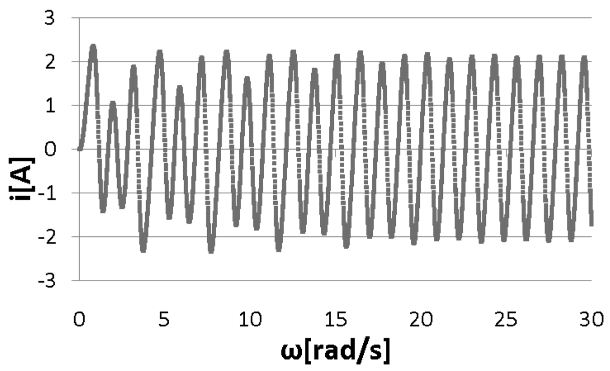

Figure 9: Transient analysis of $\omega = 4.78[rad/s]$

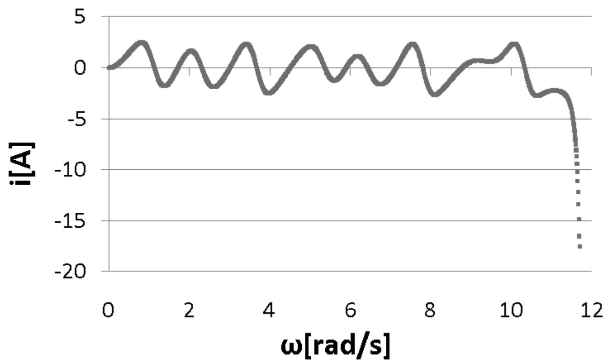

Figure 10: Transient analysis of $\omega = 4.51[rad/s]$

We can see that $\omega = 4.78[rad/s]$ is stable, because all of eigenvalues satisfy $|\lambda| < 1$. However, for the other patterns, the solutions are unstable, because one of the two eigenvalues does not satisfy $|\lambda| < 1$.

Namely, we can say that our method gives the same results as the results obtained by transient analysis with simpler SPICE-oriented algorithm.

5. Conclusion

We proposed a SPICE-oriented algorithm to assess the stability of periodic solutions for nonlinear circuits. We obtained periodic solutions by using SPICE and we assessed the stability based on the Floquet theory. In detail, we analyzed the second order resonance circuit with nonlinear capacitors for three different conditions of ω which gives both stable and unstable solutions. Our results agree well with the previously obtained results. We would like to improve the proposed method more effectively for the analysis of larger scale circuit.

Table 1: Eigenvalues of Φ

ω $[rad/s]$	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$
$\omega = 4.78$	0.5963+0.4093i	0.5963-0.4093i
$\omega = 4.51$	0.358	1.3936
$\omega = 4.26$	0.319	1.4928

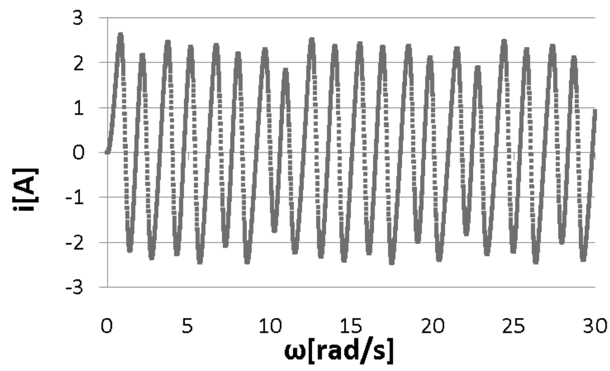

Figure 11: Transient analysis of $\omega = 4.26[rad/s]$

Acknowledgment

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