2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Particle Swarm Optimization with Novel Concept of Complex Network

Haruna Matsushita†, Yoshifumi Nishio‡ and Toshimichi Saito†

†Dept. of Electrical and Electronics Engineering, Hosei University
3–7–2 Kajino-cho, Koganei, Tokyo 184–8584, Japan
‡Dept. of Electrical and Electronics Engineering, Tokushima University
2–1 Minami-Josanjima, Tokushima, 774–8506 Japan
Email: †{haruna.matsushita.pd, tsaito}@k.hosei.ac.jp, ‡nishio@ee.tokushima-u.ac.jp

Abstract—The paper proposes a novel concept of a complex network for the Particle Swarm Optimization (PSO); Independent-minded PSO (IPSO). Particles of the standard PSO always fly toward its own past best position (*pbest*) and the best position among the swarm (*gbest*). On the other hand, in the proposed IPSO, whether each particle and the swarm affected each other is stochastically decided according to a fixed parameter called "Cooperativeness". We confirm behavior of IPSO and its effectiveness by applying it to various benchmarks.

1. Introduction

Particle Swarm Optimization (PSO) [1] is an algorithm to simulate the movement of flocks of birds. Due to the simple concept, easy implementation and quick convergence, PSO has attracted much attention and is used to wide applications in different fields in recent years. Each particle position is updated according to its personal best position *pbest* and the best particle position among the whole swarm *gbest*. In other words, all the particles are fully-connected and always influence each other.

Meanwhile, in the real world, we are spending our life influencing each other in the human community, and it is important not to just depend on other people, but also to have own sense of independence.

On the other hand, various topological neighborhoods for PSO have been considered by researches [2]–[7]. In these search, each particle shares its best position among neighboring particles on the network. In other words, it is an application of the network topology to the particle swarm. Thus, investigations of the suitable network for PSO, especially using complex networks such as small-world network [8], have attracted attention in these years [9], [10].

In this study, we propose a novel application of the complex network to PSO; an Independent-minded Particle Swarm Optimization (IPSO). The most important feature of IPSO is that it is decided stochastically that each particle depends on *gbest* or becomes independent from the swarm and moves depending only on *pbest*. In other words, the particles are not always connected each other, and they act with self-reliance.

We apply the proposed IPSO to four benchmark optimization functions containing unimodal and multimodal functions, and investigate the behavior of IPSO by carrying out the simulation using different the probability of independence. IPSO is compared with various PSO, and from results, we confirm that IPSO can effectively enhance the searching efficiency for the multimodal functions. The results show that it is better that the particles value own information and are sometimes affected by the swarm, instead of being completely dependent on others.

2. Independent-minded Particle Swarm Optimizer (IPSO)

In the algorithm of the PSO, multiple potential solutions called "particles" coexist. At each time step, each particle flies toward its own past best position (*pbest*) and the best position among all particles (*gbest*). In other words, they always influence each other. In this study, we propose the novel concept of the complex network; the Independent-minded PSO (IPSO). The particles of IPSO have independence, thus, it is decided stochastically whether they are connected to others at every step. In other words, they are not always affected by *gbest* and their *pbest* does not always affect the swarm.

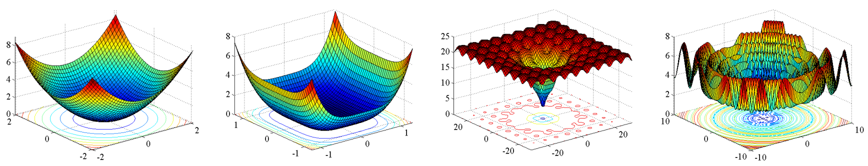
Each particle has two informations; position and velocity. The position vector of each particle i and its velocity vector are represented by $\boldsymbol{X}_i = (x_{i1}, \cdots, x_{id}, \cdots, x_{iD})$ and $\boldsymbol{V}_i = (v_{i1}, \cdots, v_{id}, \cdots, v_{iD})$, respectively, where $(d = 1, 2, \cdots, D)$, $(i = 1, 2, \cdots, M)$.

(Step1) (Initialization) Let a generation step $t = 0$. Randomly initialize the particle position $\boldsymbol{X}_i$ ($x_{id} \in [x_{\min}, x_{\max}]$), initialize its velocity $\boldsymbol{V}_i$ to zero, and initialize $\boldsymbol{P}_i = (p_{i1}, p_{i2}, \cdots, p_{iD})$ with a copy of $\boldsymbol{X}_i$. Evaluate the objective function $f(\boldsymbol{X}_i)$ for each particle i and find $\boldsymbol{P}_g$ with the best function value among all the particles.

(Step2) Decide whether each particle i is connected to the others according to rand_i which is a random number ($\in (0, 1)$) for the particle i. *If* $\text{rand}_i \leq C$, *the particle* i *is connected to other particles. If not, the particle* i *is isolated from the swarm, then, it and others does not affect each other.* C is a constant cooperativeness coefficient which is the independence probability of the particles.

Table 1: Four Test Functions.

Function name	Test Function	Initialization Space	Criterion
Sphere func.;	$f_1(x) = \sum_{d=1}^{D} x_d^2,$	$x \in [-5.12, 5.12]^D,$	0.01
4^{th} De Jong's func.;	$f_2(x) = \sum_{d=1}^{D} d x_d^4,$	$x \in [-1.28, 1.28]^D,$	0.01
Ackley's func.;	$f_3(x) = \sum_{d=1}^{D-1} \left(20 + e - 20e^{-0.2\sqrt{0.5(x_d^2+x_{d+1}^2)}} - e^{0.5(\cos(2\pi x_d)+\cos(2\pi x_{d+1}))}\right),$	$x \in [-30, 30]^D,$	1.0
Stretched V sine wave func.;	$f_4(x) = \sum_{d=1}^{D-1} (x_d^2 + x_{d+1}^2)^{0.25} \left(1 + \sin^2(50(x_d^2 + x_{d+1}^2)^{0.1})\right),$	$x \in [-10, 10]^D$	10



(a) (b) (c) (d)

Figure 1: Test functions with two variables. First and second variables are on the x- and y-axis, respectively, and z-axis shows its function value. (a) Sphere function. (b) 4^{th} De Jong's function. (c) Ackley's function. (d) Stretched V sine wave function.

(Step3) Evaluate the fitness $f(\boldsymbol{X}_i)$ for each particle i. Update the personal best position (*pbest*) as $\boldsymbol{P}_i = \boldsymbol{X}_i$ if $f(\boldsymbol{X}_i) < f(\boldsymbol{P}_i)$.

(Step4) Let $\boldsymbol{P}_g$ represents the best position with the best *pbest* among particles being connected to others (*gbest*). Update *gbest* $\boldsymbol{P}_g = (p_{g1}, p_{g2}, \ldots, p_{gD})$ according to

$$g = \arg\min_i f(\boldsymbol{P}_i), \quad \text{rand}_i \leq C. \tag{1}$$

In other words, even if the $f(\boldsymbol{P}_i)$ is the minimum pbest among all the particles, gbest is not updated if i is not connected to others.

(Step5) Update $\boldsymbol{V}_i$ and $\boldsymbol{X}_i$ of each particle i according to

$$\boldsymbol{V}_i(t+1) = \begin{cases} w\boldsymbol{V}_i(t) + c_1\boldsymbol{r}_1(\boldsymbol{P}_i - \boldsymbol{X}_i(t)) + c_2\boldsymbol{r}_2(\boldsymbol{P}_g - \boldsymbol{X}_i(t)), & \text{rand}_i \leq C \\ w\boldsymbol{V}_i(t) + c_1\boldsymbol{r}_1(\boldsymbol{P}_i - \boldsymbol{X}_i(t)), & \text{rand}_i > C \end{cases} \tag{2}$$

$$\boldsymbol{X}_i(t+1) = \boldsymbol{X}_i(t) + \boldsymbol{V}_i(t+1), \tag{3}$$

where w is the inertia weight determining how much of the previous velocity of the particle is preserved. c_1 and c_2 are two positive acceleration coefficients, generally $c_1 = c_2$. $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are d-dimensional uniform random number vectors from $U(0, 1)$. These equations mean that whether each particle is affected by *gbest* is decided at random with the cooperativeness C. When $C = 0$, all the particles move depending only on own *pbest*, and when $C = 1$, the algorithm is completely the same as the standard PSO.

(Step6) Let $t = t + 1$ and go back to (Step2).

3. Simulation

In order to evaluate the performance of IPSO, we use 4 benchmark optimization problems summarized in Table 1. f_1 and f_2 are unimodal functions, and f_3 and f_4 are multimodal functions with numerous local minima. The optimum solution $\boldsymbol{X}^*$ of all the functions are $[0, 0, \ldots, 0]$, and its optimum value $f(\boldsymbol{X}^*)$ is 0. All the functions have D variables, in this study, $D = 30$. The landscape maps of benchmark functions with two variables are shown in Fig. 1.

We compare IPSO with three PSOs; the standard PSO (PSO), PSO whose two acceleration coefficients are different (PSO2) and IPSO which use either *gbest* or *pbest* (IPSO2). Features of each algorithm are follows:

PSO: This is the standard PSO and is completely same as IPSO with $C = 1$. The velocities of all the particles are updated depending on its *pbest* and *gbest* at every generation.

PSO2: This is the standard PSO with different acceleration coefficients. All the particles are updated depending on its *pbest* and *gbest* at every generation, however the accelera-

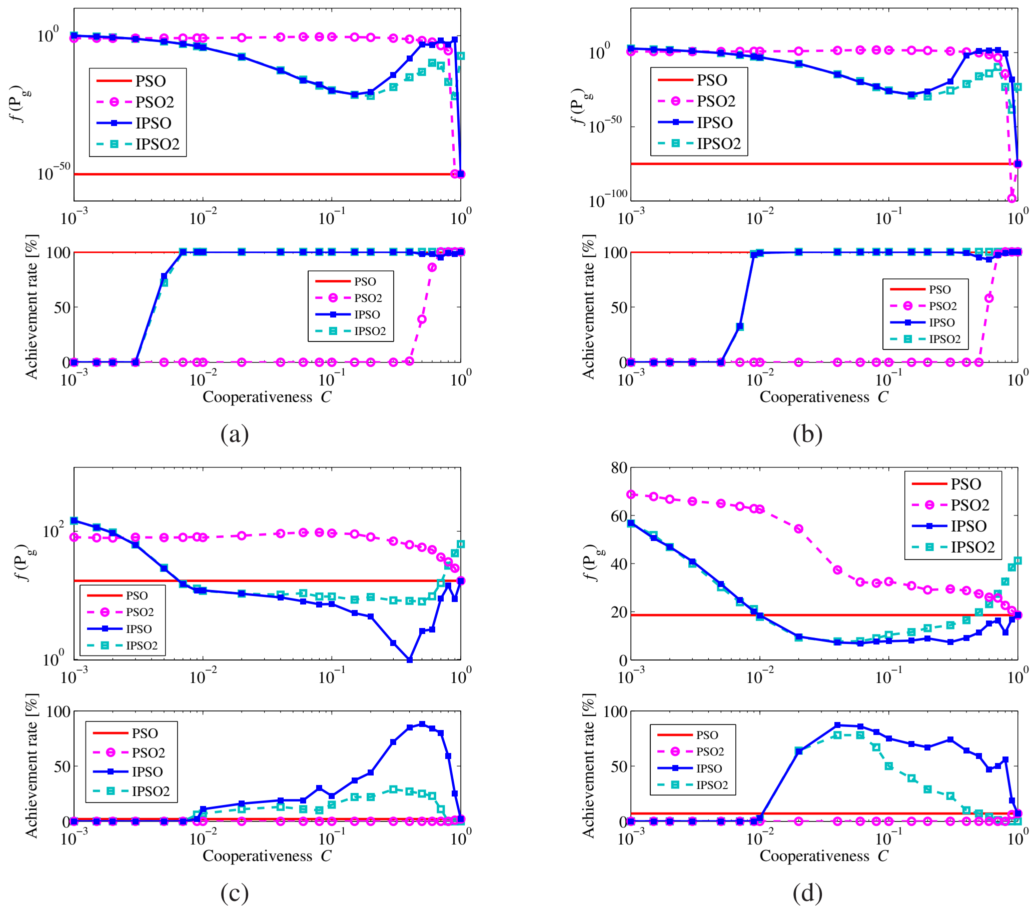


Figure 2: Simulation results of three PSOs using different C to four benchmarks. (a) Sphere function. (b) 4^{th} De Jong's function. (c) Ackley's function. (d) Stretched V sine wave function.

tion coefficient of *gbest* depends on the value of C, as

$$V_i(t+1) = wV_i(t) + c_1 r_1 (P_i - X_i(t)) + c_2 r_2 C (P_g - X_i(t)).$$

IPSO: This is the proposed PSO. It is decided according to the cooperativeness C whether each particle is connected to others. If the particle is connected to others, it is updated depending both on *pbest* and *gbest*, but otherwise, it depends only on own *pbest*, according as Eq. (2).

IPSO2: In IPSO2, it is decided according to the cooperativeness C whether each particle is connected to others, as the original IPSO. However, each particle is updated depending either on *pbest* or *gbest* as

$$V_i(t+1) = \begin{cases} wV_i(t) + c_2 r_2 (P_g - X_i(t)), & \text{rand}_i \le C \\ wV_i(t) + c_1 r_1 (P_i - X_i(t)), & \text{rand}_i > C. \end{cases}$$

For all PSOs, the population size M is set to 36, and the parameters are set as $w = 0.7$ and $c_1 = c_2 = 1.6$. In order to investigate the behavior of PSO2, IPSO and IPSO2, we carry out simulations using different cooperativeness C from 0 to 1.0. The maximum generation are set to 3000 for all the benchmarks, and the results are evaluated in an achievement rate of the criterion attainment over 100 trials.

3.1. Experimental Results

Figure 2 shows respective mean results and their achievement rate over 100 runs in different cooperativeness C. Note that the standard PSO used $C = 1.0$ for all the simulations. The best mean result among different C, and its value of C and achievement rate [%] are listed in Table 2.

As shown in Figs. 2(a) and (b), in IPSO for the unimodal functions, the case that all the particles are connected each other at every generations was the most effective, namely $C = 1.0$. In other words, the standard PSO was the most suitable to unimodal functions. Then, let us consider the importances of *gbest* and *pbest* for the unimodal functions from the results of PSO2 and IPSO2. In the results of PSO2, the larger acceleration rate of *gbest* was able to obtain better result, especially, it obtained effective results when the acceleration rate of *gbest* was about from 90 to 100 percent of that of *pbest* ($C = 0.9$–1.0). Moreover, for IPSO2, the performance was effective when the particles moved depending on *gbest* in almost generations and on *pbest* in about every 10 generations. From these results, we can say that quick communication to the swarm is more im-

Table 2: Comparison results of four PSOs on 4 test functions.

D	f		PSO	PSO2	IPSO	IPSO2
30	f_1	Best avg.	4.73e-51	4.73e-51	4.73e-51	1.12e-22
		C	-	1.0	1.0	0.9
		Achievement	100%	100%	100%	100%
	f_2	Best avg.	9.08e-76	5.07e-99	9.08e-76	3.02e-39
		C	-	0.9	1.0	0.9
		Achievement	100%	100%	100%	100%
	f_3	Best avg.	16.87	16.87	0.98	8.01
		C	-	1.0	0.5	0.5
		Achievement	2%	2%	85%	25%
	f_4	Best avg.	18.6	18.68	6.92	7.69
		C	-	1.0	0.06	0.04
		Achievement	7%	7%	86%	78%

portant for the unimodal functions than a particle diversity. Meanwhile, for f_1 and f_2, there were insignificant differences between the best mean results of four PSOs, as shown in Table 2. All the PSOs obtained 100% achievement rate.

In contrast, the changes in results depending on C for the multimodal functions were distinctly different from that for the unimodal functions. When the particles were little affected by *gbest*, the proposed IPSO obtained more effective results than when it was fully-connected ($C = 1.0$). As the results of IPSO2, IPSO with about $C = 0.5$ was effective for Ackley function f_3, and $C = 0.02$–0.8 was suitable for Stretched V sine wave function f_4. These results mean that the particle diversity is more important for the multimodal functions than the quick communication. At the same time, the proposed IPSO obtained high achievement rate as 85 and 86% for f_3 and f_4, respectively. It is the significant improved performances from the standard PSO, and the standard PSO has never obtained the same results although its parameters were changed as PSO2.

From these results, we can say that instead of connecting to all the particles, we can obtain better performance that the particles value own information and are sometimes affected by others. It is the interesting result, and the same is equally true of the human relationship in the real world.

4. Conclusions

In this study, we have proposed the Independent-minded Particle Swarm Optimization (IPSO). The particles of IPSO act with its sense of independence. Whether each particle is affected by the swarm and itself also affects the swarm is decided stochastically according to the fixed parameter "Cooperativeness".

We have applied IPSO to various benchmark functions containing unimodal and multimodal functions. From results, we have confirmed that the fully-connected IPSO, namely the standard PSO, was suitable for the unimodal functions because it can quickly transmit each particle information to the whole swarm. On the other hand, for the multimodal functions, IPSO, whose particles were little affected by the swarm, significantly improved the fully-connected PSO. This is because PSO with $C < 1.0$ brings diversity to the swarm and the particles can easily get out from the local optima. From these results, we can say that instead of connecting to all the particles, it is better that the particles value own information and are sometimes affected by others. In addition, we have confirmed a possibility of an appropriate value of C depending on the kinds of optimization problems. Our future problem is to investigate the association between the performance of IPSO and its parameters such as the cooperativeness C and the population size M.

References

[1] J. Kennedy and R. C. Eberhart, "Particle swarm optimization," in *Proc. of IEEE. Int. Conf. on Neural Netw.*, pp. 1942–1948, 1995.

[2] J. Kennedy and R. Mendes, "Population structure and particle swarm performance," in *Proc. of Cong. on Evolut. Comput.*, pp. 1671–1676, 2002.

[3] R. Mendes, J. Kennedy and J. Neves, "The Fully Informed Particle Swarm: Simpler, Maybe Better," in *IEEE Trans. Evolut. Comput.*, vol. 8, no.3, pp. 204–210, June 2004.

[4] J. Lane, A. Engelbrecht and J. Gain, "Particle Swarm Optimization with Spatially Meaningful Neighbors," in *Proc. of IEEE Swarm Intelligence Symposium*, pp. 1–8, 2008.

[5] S. B. Akat and V. Gazi, "Particle Swarm Optimization with Dynamic Neighborhood Topology: Three Neighborhood Strategies and Preliminary Results," in *Proc. of IEEE Swarm Intelligence Symposium*, pp. 1–8, 2008.

[6] H. Matsushita and Y. Nishio, "Network-Structured Particle Swarm Optimizer Considering Neighborhood Relationships," in *Proc. of IEEE. Int. Joint Conf. on Neural Netw.*, pp. 2038–2044, 2009.

[7] H. Matsushita and Y. Nishio, "Network-Structured Particle Swarm Optimizer with Various Topology and its Behaviors," in *Lecture Notes in Computer Science*, vol. 5629, pp. 163–171, 2009.

[8] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," Nature 393, pp.440–442, 1998.

[9] J. Kennedy, "Small worlds and mega-minds: effects of neighborhood topology on particle swarm performance," in *Proc. of Cong. on Evolut. Comput.*, pp. 1931–1938, 1999.

[10] H. Matsushita and Y. Nishio, "Network-Structured Particle Swarm Optimizer with Small-World Topology," in *Proc. of Int. Symposium on Nonlinear Theory and its Applications*, pp. 372–375, 2009.