Propagation Mechanism of Phase-Inversion Wave in In-and-Anti-Phase Synchronization on 2D Lattice Oscillator

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Abstract—We analyze synchronization phenomena on coupled oscillators systems as a ladder and a lattice. On the systems, we observed phase-inversion waves, which are phenomena of change phase states between two adjacent oscillators from in-phase synchronization to anti-phase synchronization or from anti-phase synchronization to in-phase synchronization in steady state. Some characteristics of phase-inversion waves are propagations, penetrations, reflections, and disappearances. In this paper, we discover the phase-inversion waves in in-and-anti-phase synchronization. We clarify regions which the phase-inversion wave can be observed in in-and-anti-phase synchronization, and clarify a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization on the lattice system.

1. Introduction

A lot of synchronization phenomena can be observed in nature world. For example, there are biological clocks, schools of sardines, the synchronization of fireflies, and so on. Recently, synchronization phenomena are researched in various fields[1]-[2].

In our previous study, we observed synchronization phenomena on coupled oscillators system. This system is made by using van der Pol oscillators which are coupled by inductor as a lattice[3]. We predicted the time-series data by using this system including nine oscillators[4]. We observed phase-inversion waves on this system including over 25 oscillators. We analyzed a mechanism of disappearance between two phase-inversion waves. Further, we analyzed a mechanism of reflection when two phase-inversion waves arrive at a corner at same time. However, these phase-inversion waves are observed in double-in-phase synchronization which all oscillators synchronize to in-phase for a vertical direction and a horizontal direction. In other hand, on ladder system, the phase-inversion waves are observed in in-and-anti-phase synchronization[5]. In-and-anti-phase synchronization is in-phase and anti-phase synchronizations exist alternately.

In this study, we observe the phase-inversion waves in in-and-anti-phase synchronization. We clarify regions which the phase-inversion wave can be observed in in-and-anti-phase synchronization when N equals 9, and clarify a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization using instantaneous frequency of each oscillator and phase differences between adjacent oscillators on the lattice system.

2. Circuit model

The van der Pol oscillators are coupled by inductors $L_0$ as a ladder (see Fig. 1). The number of column and row of this system are assumed as “$N + 1$” respectively. We name each oscillator $OSC(k,l)$. A voltage of each oscillator is named $v(k,l)$, and a current of inductor of each oscillator is named $i(k,l)$ (see Fig. 1). The circuit equations of this circuit model are normalized by Eq. (1), and the normalized circuit equations are shown as Eqs. (2)–(6).

\[ i(k,l) = \sqrt{\frac{L_0}{C_0}} \cdot \frac{dx(k,l)}{dt}, \quad v(k,l) = \sqrt{\frac{g_0}{C_0}} y(k,l), \]
\[ t = \sqrt{\frac{L_0}{C_0} \tau}, \quad \frac{dx}{dt} = “\cdot”, \quad \alpha = \frac{b}{L_0}, \quad \epsilon = g_1 \sqrt{\frac{L_0}{C_0}}. \]

[Corner–top] (left: $(a,b)=(0,1)$, right: $(a,b)=(N,N-1)$)

\[ \frac{dy_{N,a}}{dt} = y_{(0,a)}, \]
\[ \frac{dy_{(0,a)}}{dt} = -x_{(0,a)} + \alpha(x_{(0,b)} + x_{(1,a)}) - 2x_{(0,a)} + \epsilon(y_{(0,a)} - \frac{1}{3}y_{(0,a)}^3). \]

[Corner–bottom] (left: $(a,b)=(0,1)$, right: $(a,b)=(N,N-1)$)

\[ \frac{dy_{N,a}}{dt} = y_{(N,a)}, \]
\[ \frac{dy_{(0,a)}}{dt} = -x_{(N,a)} + \alpha(x_{(N-1,a)} + x_{(N,b)}) - 2x_{(N,a)} + \epsilon(y_{(N,a)} - \frac{1}{3}y_{(N,a)}^3). \]

[Center] $(0 < k < N, 0 < l < N)$

\[ \frac{dy_{k,l}}{dt} = y_{(k,l)}, \]
\[ \frac{dy_{(k,l)}}{dt} = -x_{(k,l)} + \alpha(x_{(k-1,l)} + x_{(k+1,l)} + x_{(k,l+1)} + x_{(k,l-1)} - 4x_{(k,l)}) + \epsilon(y_{(k,l)} - \frac{1}{3}y_{(k,l)}^3). \]
In our circuit, an oscillator, which is not an oscillator on the edge, has four adjacent oscillators. When phase states between the oscillator and two of four oscillators are anti-phase synchronization, phase states between the oscillator and other two oscillators are in-phase synchronization. Oscillators on the edges stay in anti-phase synchronization in in-and-anti-phase synchronization. These phase states are called “in-and-anti-phase synchronization.” The phase-inversion waves in in-and-anti-phase synchronization are classified into two patterns. Pattern A can be observed if N is an odd number. Odd number’s phase-inversion waves propagate in vertical direction and horizontal direction respectively. Pattern B can be observed if N is an even number. Even number’s phase-inversion waves propagate in vertical direction and horizontal direction respectively. Simulation results of pattern A and B show in Figs. 2 and 3 respectively. Figure 4 shows regions which the phase-inversion wave can be observed in in-and-anti-phase synchronization when N equals 9. The coupling parameter $\alpha$ and nonlinearity $\varepsilon$ are changed from 0.050 to 1.0, every 0.050. The phase-inversion wave in in-and-anti-phase synchronization can be observed in region(i) (see Figs. 2 and 4). The complex phenomena on in-and-anti-phase synchronization can be observed in region(ii) (see Figs. 4 and 5). We can observe some characteristics of phase-inversion waves in in-and-anti-phase synchronization. These characteristics are a propagation, a penetration, a reflection at an edge, and a reflection between two phase-inversion waves (see Figs. 2 and 3). These characteristics are shown in Table 1.

4. Mechanism

We analyze a mechanism of propagation of a phase-inversion wave. The mechanism is made clear by using instantaneous frequency of each oscillator and phase differences between adjacent oscillators. Figure 6 shows the signs of the initial values of the voltages and currents of each oscillator. The coupling parameter $\alpha$ is fixed as 0.05, and nonlinearity $\varepsilon$ is fixed as 0.15. An equation of the instantaneous frequency of OSC($k, l$) is obtained as follows (see Eq. (7)). The instantaneous frequency is named $f_{k,l}(a)$ where “a” expresses the number of times of the peak value of the voltage. Time of a-th peak value of the voltage of OSC($k, l$) is assumed as $\tau_{k,l}(a)$ (see Fig. 7). Similarly, $\tau_{l,k}(a)$ and $\tau_{l,k+1}(a)$ are decided.

$$f_{k,l}(a) = \frac{1}{\tau_{k,l}(a) - \tau_{k,l}(a - 1)}.$$  

Figure 7: Pattern A - Phase-inversion waves on 9x9 oscillators ($\alpha=0.05$ and $\varepsilon=0.15$).
Phenomena
When a phase-inversion wave arrives at an edge, the phase-inversion wave reflects and propagates to where they came from. Sometimes this phenomenon is happened with penetration.

Reflections at an edge
When a phase-inversion wave arrives at an edge, the phase-inversion wave reflects and propagates to where they came from. Sometime this phenomenon is happened with penetration.

Reflections between two phase-inversion waves
When two phase-inversion waves coming from the opposite directions arrive to two adjacent oscillator at same time, the phase-inversion waves reflect and propagate to where they came from.

4.1. Propagation mechanism
We can observe a phenomenon that a phase-inversion wave to a vertical direction in each column propagate in in-and-anti-phase synchronization.

Table 1: Characteristics of the phase-inversion waves on in-and-anti-phase synchronization.

<table>
<thead>
<tr>
<th>Names of characteristics</th>
<th>Phenomena</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagations</td>
<td>The phase-inversion waves propagate for vertical direction or horizontal direction. The vertical phase-inversion waves independently move from the horizontal phase-inversion waves.</td>
</tr>
<tr>
<td>Penetrations</td>
<td>Two phase-inversion waves arrive at an oscillator from vertical direction and horizontal direction, and each phase-inversion wave penetrates each other.</td>
</tr>
<tr>
<td>Reflections at an edge</td>
<td>When a phase-inversion wave arrives at an edge, the phase-inversion wave reflects and propagates to where they came from. Sometime this phenomenon is happened with penetration.</td>
</tr>
<tr>
<td>Reflections between two phase-inversion waves</td>
<td>When two phase-inversion waves coming from the opposite directions arrive to two adjacent oscillator at same time, the phase-inversion waves reflect and propagate to where they came from.</td>
</tr>
</tbody>
</table>

Figure 5: An example of complex phenomena in region(ii) (\(a=0.05\) and \(\varepsilon=0.85\)).

Figure 6: Sign of initial value of each oscillator of in-and-anti-phase synchronization.

Figure 7: The detection method of frequencies and the phase differences.

4.2. Comparison between a propagation in double in-phase synchronization and a propagation in in-and-anti-phase synchronization.

We discovered the phase-inversion waves in in-and-anti-phase synchronization. We fixed the \(N\) as 19. Propagation mechanism is shown in Table. 2 (see Fig. 8). In Fig. 8(a), the vertical axis is instantaneous frequency, and horizontal axis is time. In Fig. 8(b), the vertical axis is the phase difference, and the horizontal axis is time.

5. Conclusion
We discovered the phase-inversion waves in in-and-anti-phase synchronization. We clarified regions which the phase-

Figure 8: Transitions of phase difference and frequencies by propagation of a phase-inversion wave on in-and-anti-phase synchronization and anti-phase synchronization.
A phase inversion wave propagates by this mechanism. A reflection at an edge, and a reflection between two phase-inversion waves.

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References