

# Space-Varying Cellular Neural Networks Designed by Hopfield Neural Network

Yasuhiro Ueda, Masakazu Kawahara, Takashi Inoue, Yoko Uwate and Yoshifumi Nishio

**Abstract**— In this study, we propose a space-varying cellular neural network (CNN) designed by Hopfield neural network (Hopfield NN). CNN is classified into two types of system like space-invariant system and space-varying system. Space-invariant means that all cells have identical template. On the other hand, space-varying means that all cells do not have identical template according to the state values of the cell and neighbor cells and so on.

The proposed CNN is the space-varying system and it is designed by using an associative memory ability of Hopfield NN. In general, the design of space-varying systems is not easy. However, we can set one of prepared existing templates on each cell of CNN according to the retrieved pattern by Hopfield NN to which some typical local image structures are embedded. Namely, we need only some existing templates and their associated patterns. Some simulation results show the basic properties of the proposed CNN and its effectiveness.

## I. INTRODUCTION

CELLULAR Neural Networks (CNN) [1] were introduced by Chua and Yang in 1988. The idea of CNN was inspired from the architecture of the cellular automata and the neural networks. A different point from the conventional neural networks is that CNN has local connectivity property. Since the structure of CNN resembles the structure of animals' retina, CNN can be used for various image processing applications [2]-[5]. CNN is composed of the basic circuit units called cells. The cell contains linear and nonlinear circuit elements which are typically linear capacitors, linear resistors, linear and nonlinear controlled sources.

Wiring weights of the cells are established by parameters called the template. The performance of the CNN is decided by the template. If the templates of all the cells in the CNN are identical, the system is called space-invariant, while if the templates of all the cells in the CNN are not identical, the system is called space-varying. Typically, the space-invariant system is mainly used in the studies of the CNN because the implementation cost is low. However, the performance of the space-invariant system may be limited because the essential processing dynamics are identical for every position. On the other hand, the performance of space-varying system can be better although the design of the template and implementation are much more difficult. Therefore, the research of the space-varying system is an interesting subject for developing a new direction of CNN. In the research of the space-varying system, the design method of CNN is important problem and

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it influences the performance of CNN. Sumitomo and Ushida proposed a design method using Fuzzy Inference [6]. They could design the space-varying CNN that takes advantage of Fuzzy inference and have confirmed the effectiveness of the space-varying CNN.

In this study, we propose a space-varying CNN designed by using an ability of the associative memory of Hopfield neural network (Hopfield NN) [7]. In the design method, firstly, we prepare some two-dimensional patterns with the size of  $5 \times 5$  and some existing templates are associated with these patterns as one to one. Secondly, the two-dimensional patterns are embedded to the Hopfield NN with the same size. Thirdly, we input each pixel and its two-neighborhood pixels (namely  $5 \times 5$  pixels) of an input image into Hopfield NN. Hopfield NN retrieves one of the embedded patterns which is the most similar to the pattern of each pixel and its two-neighborhood pixels. Fourthly, according to the retrieved pattern, its associated template is set on the cell of CNN. Namely, all the templates of the space-varying CNN are decided. Finally, the input image is processed by using the space-varying CNN.

In Sec. 2, we review the standard CNN. In Sec. 3, we review the standard Hopfield NN. In Sec. 4, we describe the structure of the space-varying CNN designed by Hopfield NN. In Sec. 5, simulation results of some image processing tasks are shown. Section 6 concludes the article.

## II. CELLULAR NEURAL NETWORKS [1]

CNN has  $M$  by  $N$  processing unit circuits called cells. Cells are arranged in a two-dimensional lattice pattern to  $M$  lines by  $N$  rows. We represent a cell  $C(i, j)$  using a variable  $i$  which denotes vertical position and a variable  $j$  which denotes horizontal position. Each cell is connected to its neighborhood cells according to a template. Usually, the same template is used for all the cells except for boundary cells. CNN has the features of time continuity, spatial discreteness, nonlinearity and parallel processing capability.

The state equation and the output equation of the cell  $C(i, j)$  are shown as follows.

*State equation:*

$$\frac{dv_{xij}}{dt} = -v_{xij} + \sum_{k=i-r}^{i+r} \sum_{l=j-r}^{j+r} A_{(i,j;k,l)} v_{ykl}(t) + \sum_{k=i-r}^{i+r} \sum_{l=j-r}^{j+r} B_{(i,j;k,l)} v_{ukl}(t) + I. \quad (1)$$

Output equation:

$$v_{yij}(t) = \frac{1}{2}(|v_{xij}(t) + 1| - |v_{xij}(t) - 1|) \quad (2)$$

where  $v_x$ ,  $v_y$  and  $v_u$  represent a state, an output and an input of the cell, respectively. In Eq. (1),  $A$  is the feedback template and  $B$  is the control template. These templates and threshold value  $I$  are collectively called the general template.

The  $r$ -neighborhood of  $C(i, j)$  in CNN is defined by

$$Nr(i, j) = \{C(k, l) \mid \max\{|k - i|, |l - j|\} \leq r, \\ 1 \leq k \leq M; \quad 1 \leq l \leq N\}. \quad (3)$$

where  $r$  is a positive integer number. In our study, we fix the value of  $r$  as 1.

### III. HOPFIELD NEURAL NETWORK WORKING AS ASSOCIATIVE MEMORY

Associative memory is a system which retrieves an embedded pattern or its reversed pattern being similar to an inputted pattern. Noisy patterns can be associated or distorted patterns can be recognized by a well-constructed associative memory.

The Hopfield NN is used as an associative memory by exploiting the property that the network has multiple stable states. Namely, if the parameters of the network can be decided in such a way that the patterns to be stored become stable states of the network, the network produces a stored pattern that is similar to an input pattern. The energy function of the Hopfield NN with  $N$  neurons and  $P$  stored binary patterns is defined by the following equation.

$$E = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N w_{ij} x_i x_j + \sum_{i=1}^N \theta_i x_i, \quad (4)$$

where  $w_{ij}$  is the weight between  $i$ -th neuron and  $j$ -th neuron, and  $\theta_i$  is the threshold of the  $i$ -th neuron. The weight  $w_{ij}$  is given as follows.

$$w_{ij} = \begin{cases} \sum_{p=1}^P x_i^{(p)} x_j^{(p)} & (i \neq j) \\ 0 & (i = j). \end{cases} \quad (5)$$

The weight  $w_{ij}$  is 0 when  $i = j$ , because all the units have combined with all the others except themselves. The states of the neurons are asynchronously updated due to the following difference equation:

$$x_i(t+1) = \text{sgn}\left(\sum_{i \neq j} w_{ij} x_j(t)\right), \quad (6)$$

where  $\text{sgn}$  is an output function as follows:

$$\text{sgn}(a) = \begin{cases} 1 & (a \geq 0) \\ -1 & (a < 0). \end{cases} \quad (7)$$

Figure 1 shows the neuron model of the conventional Hopfield NN model.

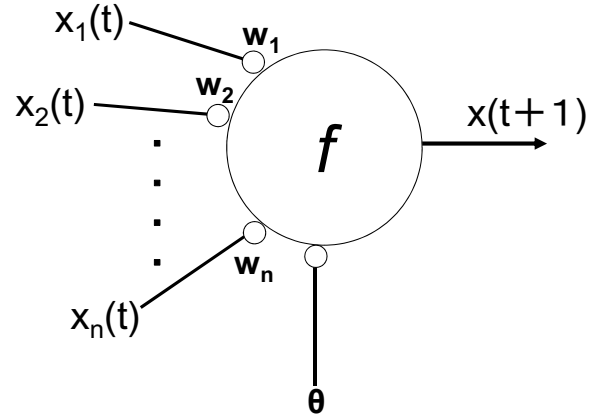


Fig. 1. Neuron model of conventional Hopfield NN model.

### IV. SPACE-VARYING CELLULAR NEURAL NETWORKS DESIGNED BY HOPFIELD NEURAL NETWORK

In this section, we describe the space-varying CNN by Hopfield NN. The proposed CNN is a space-varying system, and it is designed by using the ability of the associative memory of Hopfield NN. In general, a design of the space-varying system needs to solve complicated differential equations. However, in the proposed design method, we do not have to do it.

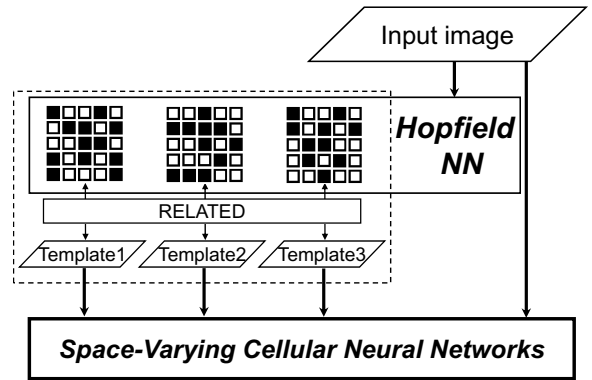


Fig. 2. Design method by Hopfield NN.

Figure 2 shows the design method of the space-varying CNN by Hopfield NN. Each step of the design method is explained as follows.

**Step 1:** We prepare some two-dimensional patterns for the memory of Hopfield NN and some existing templates associated with these patterns, respectively. By using existing templates, we are able to shorten the time of design. Example of two dimensional patterns and existing templates are shown in Fig. 3.

**Step 2:** Hopfield NN memorizes the two-dimensional patterns 1, 2 and 3 as Fig. 3 by using Eq. (5). In this article,

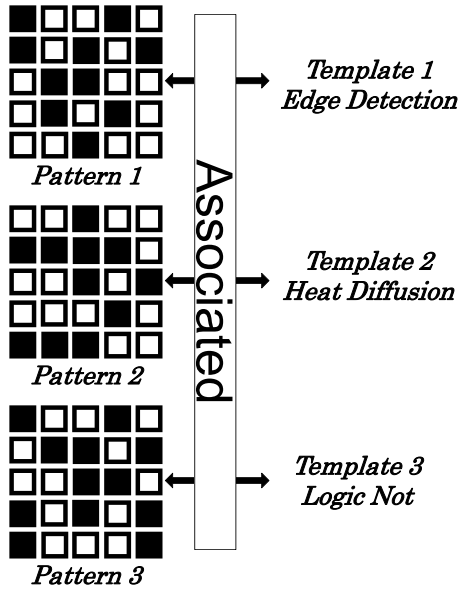


Fig. 3. Example of patterns associated with templates.

the two-dimensional patterns are only binary images.

**Step 3:** Figure 4 shows the template decision procedure by the associated patterns. In the example of Fig. 4, after the input pattern is inputted, some two-dimensional patterns and pattern of  $C(i, j)$  and its two-neighborhood pixels are compared. Hopfield NN retrieves one of the embedded two-dimensional patterns which is similar to the pattern of  $C(i, j)$  and its two-neighborhood pixels. In this example, “Pattern 2” in Fig. 3 is retrieved from the inputted pattern. Namely, the local feature of the input image is classified into one of the embedded images.

**Step 4:** From the retrieved pattern in Step 3, template of cell which is represented  $C(i, j)$  is decided the “Heat Diffusion” template in Fig. 3. Namely, the state equation and output equation of  $C(i, j)$  are shown as follows.

*State equation:*

$$\begin{aligned} \frac{dv_{xij}}{dt} = & -v_{xij} + \sum_{k=i-r}^{i+r} \sum_{l=j-r}^{j+r} A_{2(i,j;k,l)} v_{ykl}(t) \\ & + \sum_{k=i-r}^{i+r} \sum_{l=j-r}^{j+r} B_{2(i,j;k,l)} v_{ukl}(t) + I_2. \end{aligned} \quad (8)$$

*Output equation:*

$$v_{yij}(t) = \frac{1}{2}(|v_{xij}(t) + 1| - |v_{xij}(t) - 1|). \quad (9)$$

In Eq. (8),  $A_2$ ,  $B_2$  and  $I_2$  represent the “Heat Diffusion” template in Fig. 3. Additionally, the output equation is the same as the conventional CNN.

**Step 5:** Hopfield NN carries out the Steps 3 and 4 for the all cells of CNN. Finally, the space-varying CNN is designed

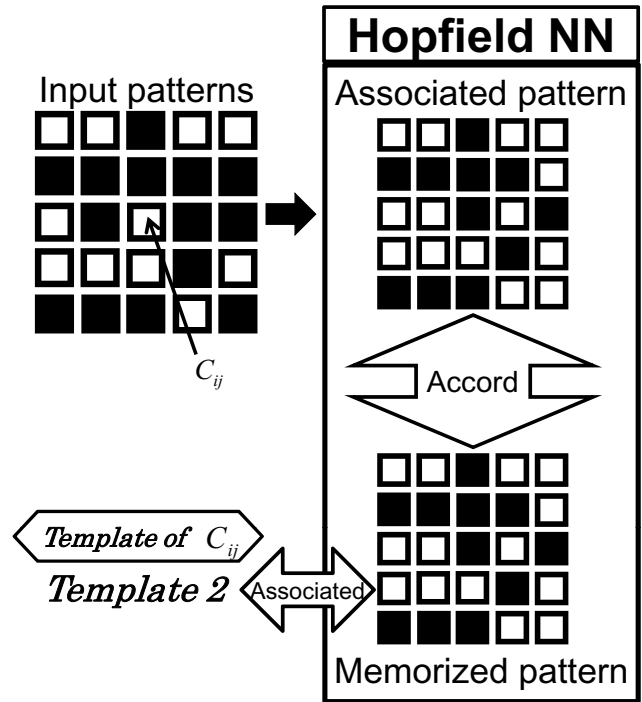


Fig. 4. Template decision procedure by associated patterns.

through a series of steps as Fig. 2.

Hence, the input image is processed using each suitable template of the space-varying CNN designed by Hopfield NN.

## V. SIMULATION RESULTS

In this section, we show some simulation results using the designed space-varying CNN. All existing templates are found in [8].

### A. Task 1: Texture segmentation

In this task, we prepare three patterns as Fig. 5 and three existing templates as Eqs. (10), (11) and (12).

The three patterns show the three directions like vertical, horizontal and diagonal line, respectively. The three existing templates have completely different performance of the image processing. If Hopfield NN retrieves the pattern 1, 2 or 3 in the certain pixel of an input image, a cell corresponding to the pixel has “White Filler” template, “Heat Diffusion” template or “Logic Not” template, respectively.

*Existing templates associated with patterns in Fig. 5:*

*Pattern 1 : “White Filler” template:*

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = 0, \quad I = -4. \quad (10)$$

This template makes image white all around.

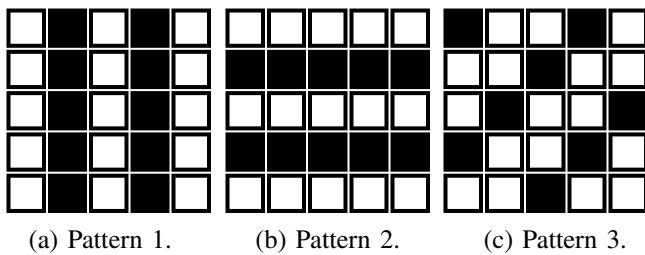


Fig. 5. Three patterns for the Task 1.

*Pattern 2 : “Heat Diffusion” template:*

$$A = \begin{bmatrix} 0.1 & 0.15 & 0.1 \\ 0.15 & 0 & 0.15 \\ 0.1 & 0.15 & 0.1 \end{bmatrix}, B = 0, I = -1. \quad (11)$$

This template gradates the objects to uniform gray scale.

*Pattern 3 : “Logic Not” template:*

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, I = 0. \quad (12)$$

In binary images, this template turns over white and black.

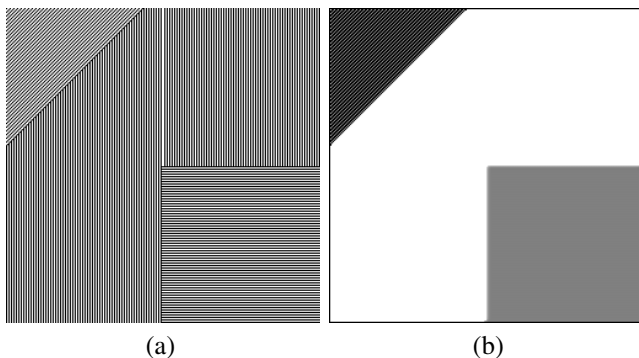


Fig. 6. Texture segmentation result using the designed space-varying CNN. (a) Texture input image. (b) Texture segmentation result for Fig. 6(a).

Figure 6 shows the texture segmentation result using the space-varying CNN designed by Hopfield NN based on three patterns as Fig. 5 and three existing templates Eqs. (10), (11) and (12). Figure 6(a) shows the texture input image having three types of texture corresponding vertical, horizontal and diagonal line, respectively. From Fig. 6(b), we can say that the cells of each texture region having vertical, horizontal and diagonal line are associated with the patterns 1, 2 and 3, respectively. Since “White Filler” template is associated with the vertical lines, the region with vertical lines converges to white. Also, since “Heat Diffusion” template is associated with the horizontal lines, the region with horizontal lines is converged on uniform gray scale. Additionally, since “Logic Not” template is associated with the diagonal lines, the region with diagonal lines becomes reversal of white and black. In Fig. 6(b), we can see that each texture can

be segregated because each texture region having different template converges to different value. Namely, we consider that each image processing is carried out at the same time in each texture region by the designed space-varying CNN.

*B. Task 2: Multiple direction lines detector with noise removal*

In this task, we prepare three patterns as Fig. 7 and two existing templates as Eqs. (13) and (14). The three patterns show the horizontal line, vertical line and noises pattern, respectively. The two existing templates have completely different performance of image processing. If Hopfield NN retrieves the pattern 1 or 2 in the certain pixel of an input image, a cell corresponding to the pixel has “Edge Detection” template. Similarly, if Hopfield NN retrieves the pattern 3 in the certain pixel of an input image, a cell corresponding to the pixel has “Small Object Remover” template.

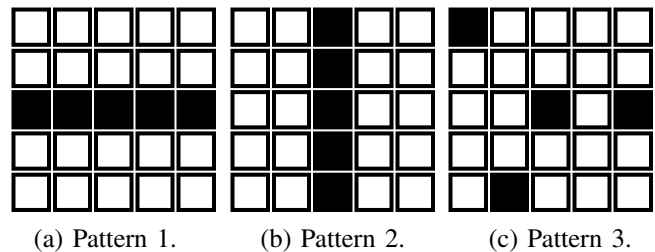


Fig. 7. Three patterns for Task 2.

*Existing templates associated with patterns in Fig. 7:*

*Patterns 1 and 2 : “Edge Detection” template:*

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}, I = -1. \quad (13)$$

This template can detect the edge of objects.

*Pattern 3 : “Small Object Remover” template:*

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}, B = 0, I = 0. \quad (14)$$

This template removes the small objects.

Figure 8 shows the multiple direction lines detector with noise removal result using the space-varying CNN designed by Hopfield NN based on the three patterns in Fig. 7 and the two templates Eqs. (13) and (14).

Figure 8(a) shows the binary input image having a circle and many noises. From Fig. 8(b), we can say that Hopfield NN retrieves the pattern 3 in the pixels of noise. Additionally, Hopfield NN retrieves the pattern 1 or 2 in the pixels of multiple direction lines which are similar to the vertical or horizontal line. Therefore, multiple direction lines which are similar to the vertical or horizontal line can be detected although diagonal line can not be detected. In this simulation,

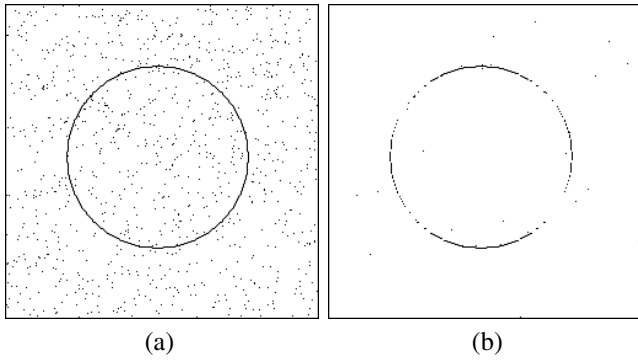


Fig. 8. Multiple direction lines detector with noise removal result using the designed space-varying CNN. (a) Input image. (b) Some directions line detector with noise removal result for Fig.8(a).

noises of the input image can be removed at the same time by the effect of “Small Object Remover” template. Namely, we consider that two different image processings are carried out at the same time by the designed space-varying CNN.

### C. Task 3: Reversed edge detection

In this task, we prepare three patterns as Fig. 9 and two templates as Eqs. (13) and (15). The three patterns show the vertical edge, horizontal edge and background region, respectively. The two existing templates have completely different performance of image processing. If Hopfield NN retrieves the pattern 1 or 2 in the certain pixel of an input image, a cell corresponding to the pixel has “Logic Not” template. Similarly, if Hopfield NN retrieves the pattern 3 in the certain pixel of a input image, a cell corresponding to the pixel has “Black Propagation” template.

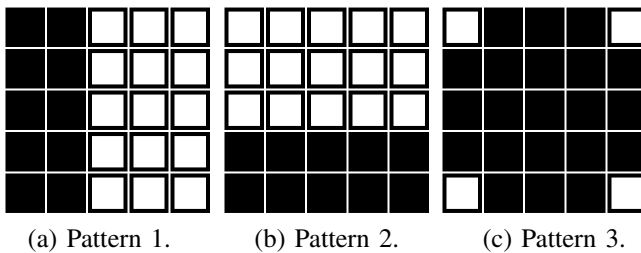


Fig. 9. Three patterns for Task 3.

Existing templates associated with patterns in Fig. 7:

Patterns 1 and 2 : “Edge Detection” template:

Existing templates associated with patterns in Fig. 9:

Pattern 1 and 2 : “Logic Not” template as Eq. (12):

Pattern 3 : “Black Propagation” template:

$$A = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 3 & 0.25 \\ 0.25 & 0.25 & 0.25 \end{bmatrix}, B = 0, I = 3.75. \quad (15)$$

This template propagates the inputted black.

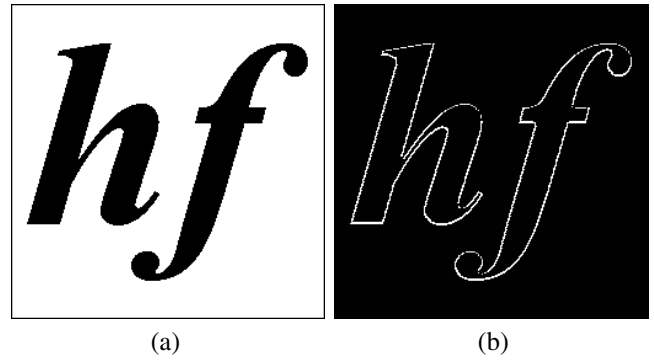


Fig. 10. Reversed edge detection results using the designed space-varying CNN. (a) Binary input image. (b) Reversed edge detection results for Fig.10(a).

Figure 10 shows the reversed edge detection result using the space-varying CNN designed by Hopfield NN based on the three patterns in Fig. 9 and the two existing templates Eqs. (12) and (15). Figure 10(a) shows the binary input image. From Fig. 10(b), by setting the three patterns of Fig. 9, cells corresponding to the periphery of edges of input image are associated with the patterns 1 or 2, and has “Logic Not” template. On the other hand, other cells are associated with the pattern 3, and have “Black Propagation” template. By the “Logic Not” template, the color of the edge of alphabet and its circumference turns over. In fact, the edge of alphabet in input image changes to white, and its circumference changes to black. The other cells turn black by effect of “Black Propagation” template. As a result, white edge equals in the original edge. Namely, we consider that the each image processing is carried out at the same time in the circumference of the edges and other region by the designed space-varying CNN.

## VI. CONCLUSIONS

In this study, we have proposed a space-varying CNN designed by using ability of associative memory of Hopfield NN. Some simulation results showed the basic properties of the proposed CNN and its effectiveness. According to our simulation, we consider that the setting of the prepared patterns is the most important for the performance of image processing using the proposed space-varying CNN. In other words, the proposed CNN can perform various image processing by using different patterns and templates. The proposed space-varying CNN has a possibility that a new direction of CNN is developed. In the future work, we would like to carry out in gray-scale images and more complicated image processing.

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