Peak Search Algorithm of Frequency Characteristics with Unstable Region

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***Abstract*—For designing PCBs (printed circuit boards), it is very important to find out the locations and the frequencies giving large peak values of the voltages. Electrostatic capacity exists between the wire lines of PCBs. The characteristics of the capacitors depend on the distance between the wire lines. From this reason, we have to analyze circuits including nonlinear capacitors. In that case, the frequency characteristics become distorted, compared with linear circuits. If the distortion becomes large, the characteristics has unstable region. In this article, we propose a SPICE-oriented algorithm to analyze frequency characteristics with unstable region and to find the peaks of the frequency characteristics.**

I. INTRODUCTION

In this study, we propose a SPICE-oriented algorithm to analyze frequency characteristics of nonlinear circuits and to find more exact peak voltages of the frequency characteristics. Although they may be found by the standard transient analysis of SPICE, it is difficult to find the exact peaks when the quality factor (Q) is very large. In the SPICE, analysis is every fixed spacing. Because of this, we may pass over them if we choose a large step size. Furthermore, the frequency characteristics of nonlinear circuits often have unstable regions. Such regions cannot be obtained the standard methods using SPICE. In our algorithm, we derive the sine-cosine circuit [1][2] from the nonlinear circuit. Next, the Fourier transformation circuit [3] is used to obtain the response of nonlinear elements. When we analyze this circuit with the transient analysis of SPICE, we may pass over the exact peaks. In order to avoid this problem, we apply the differentiator and the nonlinear limiter [4]. Finally, we apply the STC (solution trace circuit) [5] to obtain the frequency characteristics even when the curve has unstable regions.

Section 2 shows how to use the sine-cosine circuits and the Fourier transformation circuit. Section 3 explains the peak search algorithm with the differentiator and the nonlinear limiter. Section 4 explains the tracing of the frequency characteristics curve by using STC. Illustrated example of the proposed algorithm is shown in Sec. 5 and Sec. 6 concludes the article.

II. SPICE-ORIENTED HARMONIC BALANCE ALGORITHM

A. Sine-cosine transformation

Sine-cosine transformation based on the HB (harmonic balance) method such that the determining equation is solved by transient analysis of SPICE. We discuss the sine-cosine circuit corresponding to the determining equation of the HB method. If we set the voltage through a capacitor C

$$v_C = V_{CS}\sin\omega t + V_{CC}\cos\omega t, \tag{1}$$

the current i_C is given by

$$i_C = C\frac{dv_C}{dt} = -\omega C V_{CC}\sin\omega t + \omega C V_{CS}\cos\omega t. \tag{2}$$

Thus, the coefficients of $\sin\omega t$ and $\cos\omega t$ are described by

$$I_{CS} = -\omega C V_{CC}, \quad I_{CC} = \omega C V_{CS}. \tag{3}$$

Namely, the capacitor is replaced by coupled voltage-controlled current sources in the sine-cosine transformation of the HB method. In the same way, let the current through an inductor L be

$$i_L = I_{LS}\sin\omega t + I_{LC}\cos\omega t. \tag{4}$$

Then, the voltage v_L is given by

$$v_L = L\frac{di_L}{dt} = -\omega L I_{LC}\sin\omega t + \omega L I_{LS}\cos\omega t. \tag{5}$$

Thus, the coefficients of $\sin\omega t$, $\cos\omega t$ are described by

$$V_{LS} = -\omega L I_{LC}, \quad V_{LC} = \omega L I_{LS}. \tag{6}$$

Namely, the inductor is replaced by coupled current-controlled voltage sources in the sine-cosine transformation.

As an example, Fig. 1 shows a LRC ladder circuit and Fig. 2 shows the corresponding sine-cosine circuits.

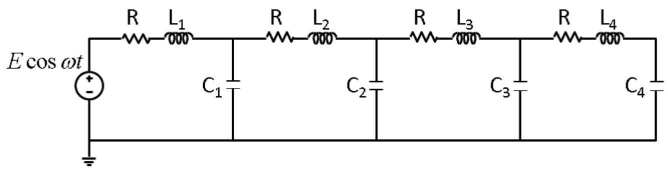

Fig. 1. LRC ladder circuit.

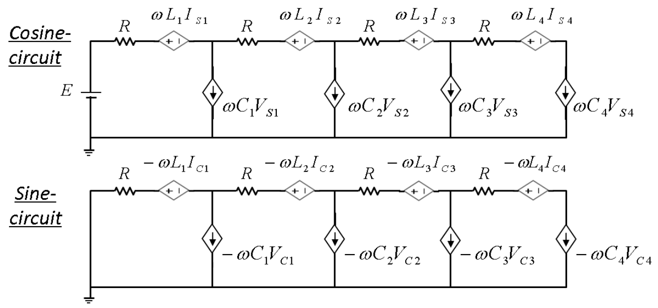

Fig. 2. Sine-cosine circuit for Fig.1.

B. Fourier transformation circuit

In this study, we use the Fourier transformation circuit in order to realize the characteristics of nonlinear capacitors. Figure 3 shows the Fourier transformation circuit. Suppose the input and output waveforms as follows:

$$\begin{cases} i(t) = I_1\cos\omega t + I_2\sin\omega t \\ v(t) = V_1\cos\omega t + V_2\sin\omega t \end{cases}. \tag{7}$$

The characteristics of the electric current which flows through a capacitor can be indicated as

$$i = dq/dt = (\partial q/\partial v)(dv/dt). \tag{8}$$

From Eq. (8), the coefficients for electric charge $q(t)$ of $\sin\omega t$, $\cos\omega t$ are described by

$$q(t) = -\frac{1}{\omega}I_2\cos\omega t + \frac{1}{\omega}I_1\sin\omega t. \tag{9}$$

From this, input of Fourier transformation circuit model $i(t)$ is changed as

$$q(t) = Q_1\cos\omega t + Q_2\sin\omega t, \tag{10}$$

and from Eq. (8), the characteristics of a nonlinear capacitor is expressed with an equation using $q(t)$ and v as $v = G(q)$. We expand $G(q)$ to Fourier series,

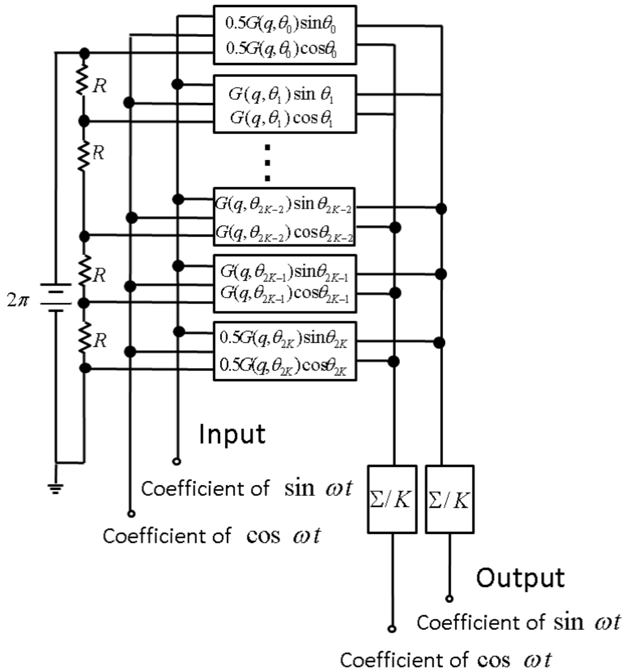

Fig. 3. Fourier transformation circuit.

and obtain the coefficients of the voltages by using the trapezoidal formula as follows.

$$\begin{aligned} V_1 &= \frac{1}{\pi}\int_0^{2\pi}(G(q)\cos\omega t)d(\omega t) \\ &= \frac{1}{2K}(G_0 + G_{2K}) + \frac{1}{K}(G_1\cos\frac{\pi}{K} \\ &\quad + G_2\cos\frac{2\pi}{K} + \ldots + G_{2K-1}\cos\frac{(2K-1)\pi}{K}), \end{aligned} \tag{11}$$

$$\begin{aligned} V_2 &= \frac{1}{\pi}\int_0^{2\pi}(G(q)\sin\omega t)d(\omega t) \\ &= \frac{1}{K}(G_1\sin\frac{\pi}{K} + G_2\sin\frac{2\pi}{K} \\ &\quad + \ldots + G_{2K-1}\sin\frac{(2K-1)\pi}{K}). \end{aligned} \tag{12}$$

where

$$\begin{aligned} \int_a^b G(q)d(\omega t) &= \frac{h}{2}(G_0 + G_{2K}) \\ &\quad + h(G_1 + G_2 + \ldots + G_{2K-1}), \\ h &= \frac{\pi}{K}, G_i = G(q(t_i)), \\ &\quad \omega t_i = 0, \pi/K, ..., (2K-1)\pi/K, 2\pi. \end{aligned} \tag{13}$$

K is a number which divides the region of a to b.

III. Peak Search Algorithm

We search the peaks by using differentiator and nonlinear limiter. The frequencies ω at the peak voltages satisfy

$$\frac{d|v(\omega)|}{d\omega} = 0, \tag{14}$$

on the characteristics curve. Hence, $|v(\omega)|$ need to be firstly differentiated by a differentiator. In order to find the exact peak points, the output is limited and expanded with a nonlinear limiter, which consists of a limiter and nonlinear diode as shown in Fig. 4.

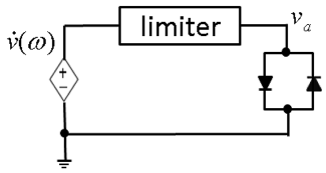

Fig. 4. Nonlinear limiter.

We suppose the characteristics of the limiter as follows:

$$v_a = \begin{cases} -V_{max} & : \quad \text{for} \quad v_{in} < -V_L \\ k\, v_{in} & : \quad \text{for} \quad -V_L \le v_{in} \le V_L \\ V_{max} & : \quad \text{for} \quad V_L < v_{in} \end{cases}, \qquad (15)$$

The output of the nonlinear limiter is given by

$$i_{out} = \begin{cases} I_s \exp(\lambda v_a) & : \quad \text{for} \quad v_a > 0 \\ -I_s \exp(-\lambda v_a) & : \quad \text{for} \quad v_a < 0 \end{cases}, \qquad (16)$$

as shows in Fig. 5.

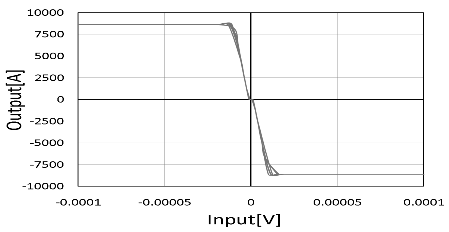

Fig. 5. Characteristics of nonlinear limiter.

The expansion factor of k is large enough. In order to complicate the analysis, we include nonlinear diodes in nonlinear limiter. Thus, the analysis near the zero points (an input of Fig. 5) is executed with a very small step size. In our algorithm, the nonlinear limiter is connected to differentiator. Input of the nonlinear limiter is equal to the slope of the voltage wave. From this reason, we can analyze the curve finely around peak voltages.

IV. Trace for unstable region

Since we set time as frequency, we can not analyze unstable region. In this section, we explain STC (solution trace circuit) for change a horizontal axis into a voltage v_ω from time (namely, frequency).

STC is based on the arc-length method [6][7]. Those voltages are differentiated with respect to the time t

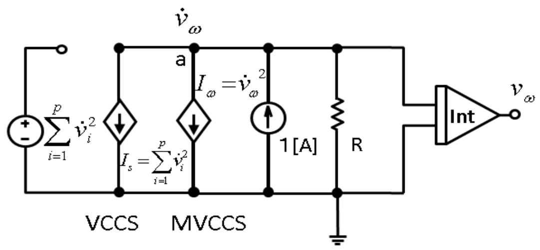

Fig. 6. STC (Solution trace circuit).

instead of the arc-length s by using differentiators. They are transformed to the corresponding voltage sources with current controlled voltage source (CCVS). Next, each voltage is squared and transformed to the current source. We have

$$I_s = \sum_{i=1}^{p} \left(\frac{dv_i}{dt} \right)^2 \qquad (17)$$

as shown in Fig. 6. If we set the voltage of node a as $\dot{v}_\omega$, $I_\omega = \dot{v}_\omega^2$ can be obtained by multiplier and voltage controlled current source VCCS (MVCCS). Thus, the additional constant current source in Fig. 6 realize to satisfying the arc-length method by Eq. (17). Then, the node voltage $\dot{v}_\omega$ is integrated to obtain v_ω. Note that R in Fig. 6 is a very large resistance used only to avoid the L-J cut-set.

In this study, v_ω in Fig. 6 is equal to ω in the main circuit. The value of new v_ω depends on voltages of main circuit, and the voltages depend on the last v_ω. Repeating the calculations, the STC traces the value of ω.

V. Illustrative example

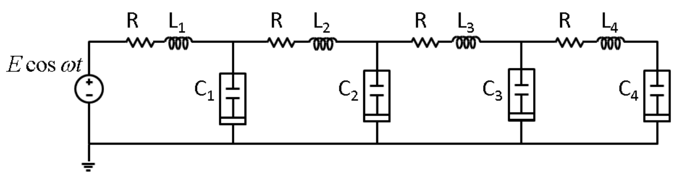

Fig. 7. LRC ladder circuit including nonlinear capacitor.

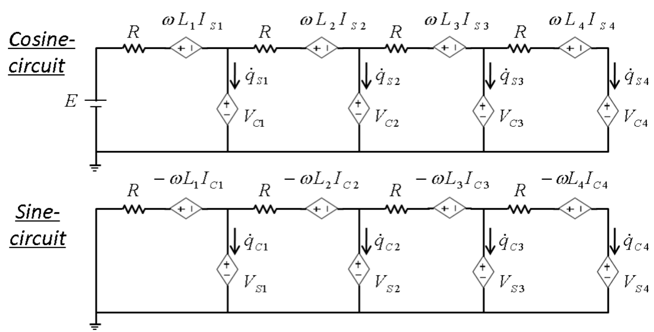

Fig. 8. Sine-cosine circuit for Fig. 7.

As an example we consider the LRC ladder circuit including nonlinear capacitors as Fig. 7. Figure 8 is the sine-cosine circuit for Fig. 7. In Fig. 7, $L_1 = L_2 = L_3 = L_4 = 0.1H$, and the nonlinear characteristics of C_1, C_2, C_3, C_4 are the same. We set $K = 10$ and the characteristics of the nonlinear capacitors are $G(q) = q+0.8q^3$. The voltages of the CCVS in Fig. 7 are inputted to the STC. We set v_ω in Fig. 6 as ω for the frequency characteristics.

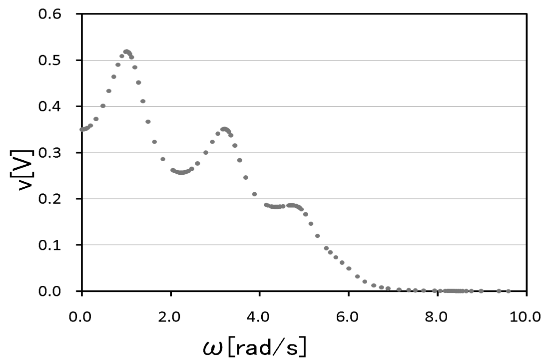

Fig. 9. Plot of frequency characteristics for R=0.1Ω.

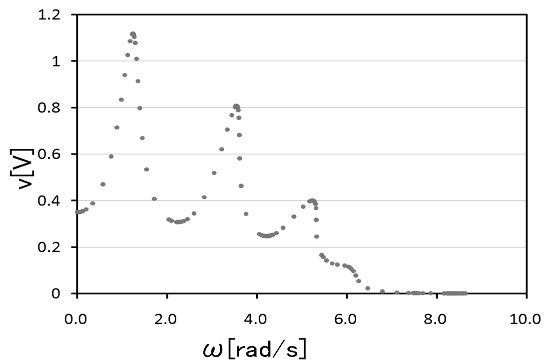

Fig. 10. Plot of frequency characteristics for R=0.05Ω.

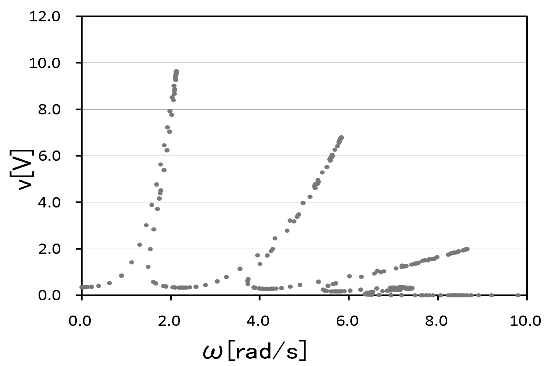

Fig. 11. Plot of frequency characteristics for R=0.01Ω.

The simulated results are shown in Figs. 9, 10 and 11 for the cases of $R = 0.1\Omega$, 0.05Ω and 0.01Ω, respectively. The horizontal axis is ω and the vertical axis is the voltage through the nonlinear capacitor C_4. We name the peaks of the curves as peak1, peak2, peak3 and peak4 from the left. We can see that the peaks of the curves become inclined as reducing the resistance (this corresponds to increase the effect of the nonlinearity). Although the unstable region appears in the case of $R = 0.01\Omega$, we could trace the curve successfully as shown in Fig. 11. We can also notice that the step size around the peak becomes smaller by the effect of the nonlinear limiter. We should mention that each frequency characteristics curve could be obtained by a single run of the transient analysis of SPICE.

VI. CONCLUSIONS

We have proposed a SPICE-oriented algorithm to analyze frequency characteristics with unstable region and to find the peaks of the frequency characteristics. By combining the sine-cosine circuits, the Fourier transformation circuit, the nonlinear limiter and the solution tracing circuit, the frequency characteristics curve can be obtained even if the curve has unstable region. The simulation results of the LRC ladder circuit with nonlinear capacitors showed the efficiency of the proposed method. The analysis of printed circuit boards using our proposed method is our future research.

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