Peak Search for Frequency Analysis of Nonlinear Circuits
Hiroshige KATAOKA, Yoshihiro YAMAGAMI and Yoshifumi NISHIO
Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: hiroshige, yamagami, nishio@ee.tokushima-u.ac.jp

Abstract
For designing PCBs (printed circuit boards), it is very important to find out the locations and the frequencies giving large peak values of the voltages. The case frequency is very large, we have to treat the wire line in the PCBs as transmission line. We apply the nonlinear elements for that transmission line. In that case, the frequency characteristics become distorted, compared with linear circuits. If the distortion becomes large, the characteristics has unstable region. In this article, the SPICE-oriented algorithm based on HB (harmonic balance) method, which has been proposed by the authors, is applied to some LRC ladder circuits including nonlinear capacitors in order to show the applicability of the algorithm and to investigate complex frequency characteristics of the circuits.

1. Introduction
In our algorithm, we apply the sine-cosine circuit [1][2] based on HB (harmonic balance) method. The HB method is well-known method for the frequency domain analysis. The Fourier transformation circuit [3] is used to obtain the response of nonlinear elements. Although they may be found by the standard transient analysis of SPICE, it is difficult to find the exact peaks when the quality factor \( Q \) is very large. Because we may pass over them if we choose a large step size. In order to avoid this problem, we apply the differentiator and the nonlinear limiter [4]. Furthermore, the frequency characteristics of nonlinear circuits often have unstable regions. Such regions cannot be obtained the standard methods using SPICE. In order to avoid this problem, we apply the STC (solution trace circuit) [5][6]. By combining the sine-cosine circuits, the Fourier transformation circuit, the nonlinear limiter and the solution tracing circuit, we can analyze the frequency response even if the curve has unstable region and obtain more exact peaks compared with common analysis of SPICE [7].

In this article, we apply the SPICE-oriented algorithm, which has been proposed in [7], to some LRC ladder circuits including nonlinear capacitors in order to show the applicability of the algorithm and to investigate complex frequency characteristics of the circuits. Computer simulation results show that the circuits have several peaks on the frequency characteristics curves depending on the number of the nonlinear elements.

2. Simulation Results of Frequency Response
In this article we set the value of resistor, inductor and linear capacitor as \( R = 0.01[\Omega], L=0.1[H], C_1 = C_2 = C_3 = C_4=1[F] \). We set the characteristics of nonlinear capacitor \( C'_1 = C'_2 = C'_3 = C'_4 \) as \( v(q) = q + 0.8q^3 \). The value of \( R = 0.01[\Omega] \) is enough to be high \( Q \) circuits in our models. The results of frequency response in the followings are voltages of \( C_4 \) in all examples.

2.1. Ladder circuit with one nonlinear capacitor
First, we analyze LRC ladder circuit including only one nonlinear capacitor as shown in Fig. 1. Note that four circuits in Fig. 1 have nonlinear capacitors in different positions. Figure 2 shows the simulation results of peak search and tracing curve of the frequency characteristics for four ladder circuits in Fig. 1. The simulation results show that each circuit has at least one resonance. By combining nonlinear limiter in the algorithm, we can analyze exact peaks even when the resonance is too sharp. There are not unstable regions in these curves.

2.2. Ladder circuit with two nonlinear capacitors
Next, we analyze LRC ladder circuit including two nonlinear capacitors as shown in Fig. 3. The simulation results are shown in Fig. 4. In this case, the curves have two resonances at least. Although it is unclear in the figure, the resonance occurs between 5\( [rad/s] \) and 12\( [rad/s] \) of \( \omega \) in Fig. 4(c). All curves in Fig. 4 have unstable regions. It is interesting that two different types of resonances exist in the curve. Namely, almost resonance curves have positive second derivative to reach the peaks, while some have negative like the second resonance in Fig. 4(e).
2.3. Ladder circuit with three nonlinear capacitors

Next, we analyze LRC ladder circuit including three nonlinear capacitors as shown in Fig. 5. The simulation results are shown in Fig. 6. In this case, the curves have three resonances at least. In Fig. 6(b), the resonance occurs between $8 \text{rad/s}$ and $10 \text{rad/s}$ of $\omega$. It is very interesting that the circuit in Fig. 5(b) has very complex frequency characteristics curve like Fig. 6(b).

3. Conclusions

We analyzed 14 patterns of fourth order LRC ladder circuits including some nonlinear capacitors by using the SPICE-oriented peak search and curve tracing algorithm. By including nonlinear limiter, we find some small resonances which we can not see at first sight. For example, Fig. 4(c) and Fig. 6(b) have the region in which points gather near $V = 0[V]$. We investigate such regions from this reason and can find some small resonances.

From the analysis, we obtained several interesting phenomena described in Sec. 2. The analysis of unstable regions of the frequency curves in detail is our future research.

References


Figure 3: LRC ladder circuit with two nonlinear capacitors. (a) \(C_1\) and \(C_2\) are nonlinear capacitors. (b) \(C_1\) and \(C_3\) are nonlinear capacitors. (c) \(C_1\) and \(C_4\) are nonlinear capacitors. (d) \(C_2\) and \(C_3\) are nonlinear capacitors. (e) \(C_2\) and \(C_4\) are nonlinear capacitors. (f) \(C_3\) and \(C_4\) are nonlinear capacitors.

Figure 4: Simulation results for ladder circuit in Fig 3. (a) Results of Fig 3(a). (b) Results of Fig 3(b). (c) Results of Fig 3(c). (d) Results of Fig 3(d). (e) Results of Fig 3(e). (f) Results of Fig 3(f).


Figure 5: LRC ladder circuit with three nonlinear capacitors. (a) $C_1$, $C_2$ and $C_3$ are nonlinear capacitors. (b) $C_1$, $C_2$ and $C_4$ are nonlinear capacitors. (c) $C_1$, $C_3$ and $C_4$ are nonlinear capacitors. (d) $C_2$, $C_3$ and $C_4$ are nonlinear capacitors.

Figure 6: Simulation results for ladder circuit in Fig 5. (a) Results of Fig 5(a). (b) Results of Fig 5(b). (c) Results of Fig 5(c). (d) Results of Fig 5(d).
