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Investigation of Synchronization Phenomena in Coupled Simultaneous Oscillators

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Abstract

There have been many investigations of the mutual synchronization of oscillators, because mutual coupled oscillators have various phase states. We paid our attentions to simultaneous oscillator, which consists of two resonators with different frequencies. In this study, we investigate synchronization phenomena in coupled two simultaneous oscillators with hard nonlinearities. By computer calculations and circuit experiments, we observe various synchronization phenomena.

1. Introduction

In our surroundings, there are a lot of synchronous phenomena. The synchronous luminescence of firefly group, cell of heart producing pulses at equal intervals and revolution of the moon etc. are good examples in which a synchronous phenomenon is comprehensible. Namely, synchronization is common phenomenon in the field of natural science. We feel that it is very interesting but the mystery. Because, it is often difficult to analyze these phenomena.

There have been many investigations of the mutual synchronization of oscillators, because mutual coupled oscillators have various phase states. We consider that it is very important to analyze synchronization phenomena of coupling oscillators. We can analyze the synchronization phenomena of coupling oscillators more easily than that of firefly group by using computer calculations and circuit experiments. Through these research, we expect that investigation of simple synchronization phenomena is the key that arrives at complicated synchronization phenomena in real natural field.

J. Schaffner confirmed that an oscillator with two degree of freedom can oscillate simultaneously at two different frequencies when the nonlinear characteristics is fifth-power [1]. However, after their pioneering work, there have not been many researches on simultaneous oscillator except [2][3].

In this study, we investigate synchronization phenomena in two coupled simultaneous oscillators with hard nonlinearities. By computer calculations and circuit experiments, we observe various synchronization phenomena.

2. Circuit Model

The circuit model is shown in Fig. 1. Two simultaneous oscillators are coupled by a resistor R. The simultaneous oscillator consists of nonlinear resistor and two resonators. We apply fifth-power nonlinearity to the nonlinear resistor. The oscillator with hard nonlinearity has multiple stable states and causes complicate phenomenon. In order to change the frequencies of resonators, we change the value of L.

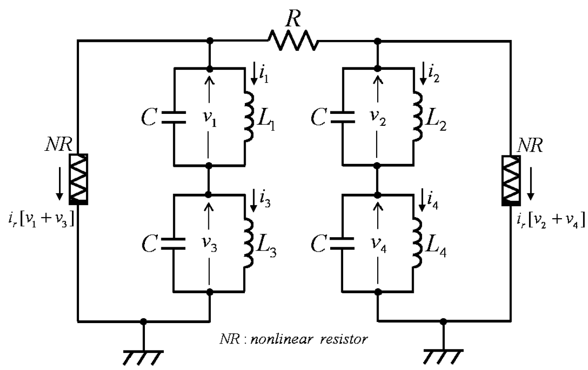


Figure 1: Circuit model.

3. Circuit Equations

The circuit equations are described as follows,

$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt},\ v_2 = L_1 \dfrac{di_2}{dt},\ v_3 = L_2 \dfrac{di_3}{dt},\ v_4 = L_2 \dfrac{di_4}{dt}, \\ C\dfrac{dv_1}{dt} = -i_1 - \dfrac{1}{R}(v_1 + v_3 - v_2 - v_4) - i_r[v_1 + v_3], \\ C\dfrac{dv_2}{dt} = -i_2 + \dfrac{1}{R}(v_1 + v_3 - v_2 - v_4) - i_r[v_2 + v_4], \\ C\dfrac{dv_3}{dt} = -i_3 - \dfrac{1}{R}(v_1 + v_3 - v_2 - v_4) - i_r[v_1 + v_3], \\ C\dfrac{dv_4}{dt} = -i_4 + \dfrac{1}{R}(v_1 + v_3 - v_2 - v_4) - i_r[v_2 + v_4]. \end{cases} \tag{1}$$

where $i_r[v_k]$ indicates the $v-i$ characteristics of the nonlinear resistor and is approximated by Eq. (2). For circuit experiments, the fifth-power nonlinear resistor is realized as shown in Fig. 2.

$$i_r[v] = g_1 v - g_3 v^3 + g_5 v^5 \quad (g_1, g_3, g_5 > 0) \tag{2}$$

By using the following variables and parameters,

$$\begin{cases} v_k = \sqrt[4]{\dfrac{g_1}{5g_5}}\, x_k, \quad i_k = \sqrt{\dfrac{C}{L}}\sqrt[4]{\dfrac{g_1}{5g_5}}\, y_k, \\ \alpha = R\sqrt{\dfrac{C}{L_1}}, \quad \varepsilon = g_1\sqrt{\dfrac{L_1}{C}} \quad \phi = \dfrac{3g_3}{g_1}\sqrt{\dfrac{g_1}{5g_5}}, \\ \beta = \dfrac{L_1}{L_2}, \quad \gamma = \dfrac{L_1}{L_3}, \quad \delta = \dfrac{L_1}{L_4}, \\ t = \sqrt{L_1 C}\,\tau, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau}, \end{cases} \tag{3}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{y}_1 = x_1, \ \ \dot{y}_2 = \beta x_1, \ \ \dot{y}_3 = \gamma x_3, \ \ \dot{y}_4 = \delta x_4, \\ \dot{x}_1 = -y_1 - \alpha(x_{13} - x_{24}) - \varepsilon\left(x_{13} - \dfrac{1}{3}\phi x_{13}^3 + \dfrac{1}{5}x_{13}^5\right), \\ \dot{x}_2 = -y_2 + \alpha(x_{13} - x_{24}) - \varepsilon\left(x_{24} - \dfrac{1}{3}\phi x_{24}^3 + \dfrac{1}{5}x_{24}^5\right), \\ \dot{x}_3 = -y_3 - \alpha(x_{13} - x_{24}) - \varepsilon\left(x_{13} - \dfrac{1}{3}\phi x_{13}^3 + \dfrac{1}{5}x_{13}^5\right), \\ \dot{x}_4 = -y_4 + \alpha(x_{13} - x_{24}) - \varepsilon\left(x_{24} - \dfrac{1}{3}\phi x_{24}^3 + \dfrac{1}{5}x_{24}^5\right). \end{cases} \tag{4}$$

$$(\ x_{kl} = x_k + x_l\ )$$

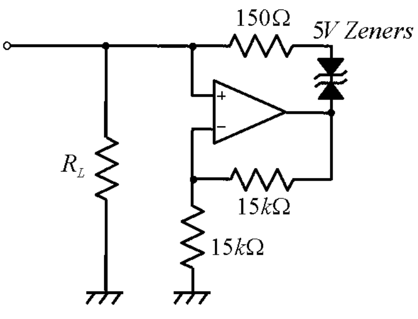


Figure 2: Nonlinear resistor (fifth-power).

4. Synchronization Phenomena

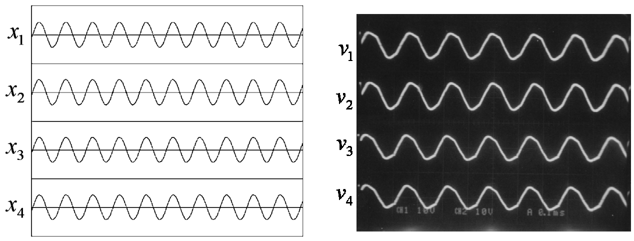
Synchronization phenomena for changing frequencies of four resonators of two oscillators are classified below. The frequencies of x_1, x_2, x_3, x_4 are corresponding to $\omega_1, \omega_2, \omega_3, \omega_4$, respectively.

4.1. Same frequencies

First, we consider the case of $\omega_1 = \omega_2 = \omega_3 = \omega_4$.

- $\omega_1 = \omega_2 = \omega_3 = \omega_4$

Figure 3 shows the computer simulation results and the corresponding circuit experimental results. As shown in Fig. 3, we are able to observe in-phase oscillation of $x_1 - x_2 - x_3 - x_4$. This is the most basic synchronization of the circuit.



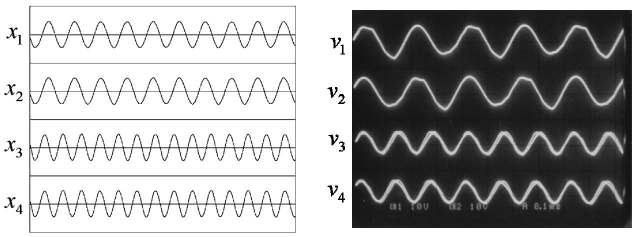
(a) (b)

Figure 3: Time waveform. (a) Numerical result (β=γ=δ=1.0, α=0.1). (b) Experimental result (L_1=L_2=L_3=L_4=10mH, C=47nF, R_L=6.45kΩ, R=2.17kΩ).

4.2. Two different frequencies

- $\omega_1 = \omega_2 \neq \omega_3 = \omega_4$

As shown in Fig. 4, we can observe two pairs of in-phase oscillations of $x_1 - x_2$ and $x_3 - x_4$. It is interesting that the oscillators synchronize with simultaneously oscillation. x_1 and x_3 do not synchronize. x_3 and x_4 synchronize although they are not directly connected each other.

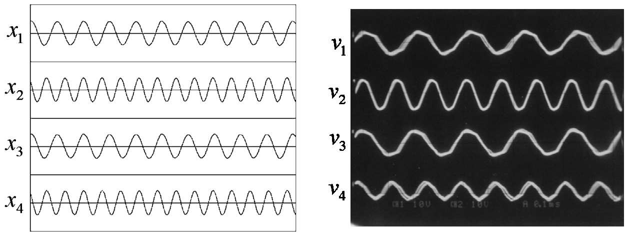


(a) (b)

Figure 4: Time waveform. (a) Numerical result (β=1.0, γ=δ=2.0, α=0.1). (b) Experimental result (L_1=L_2=20mH, L_3=L_4=10mH, C=47nF, R_L=6.45kΩ, R=2.17kΩ).

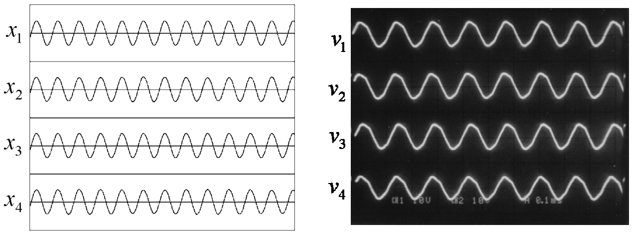
- $\omega_1 = \omega_3 \neq \omega_2 = \omega_4$

Figure 5 shows two pairs of in-phase oscillations of $x_1 - x_3$ and $x_2 - x_4$ when α is small. α is corresponding to the coupling strength. On the other hand, as shown in Fig. 6, when α is large, we are able to observe in-phase oscillation of $x_1 - x_2 - x_3 - x_4$. It seems that the frequency for this case is between the frequency of $x_1 - x_3$ in Fig. 5 and the frequency of $x_2 - x_4$ in Fig. 5.



(a) (b)

Figure 5: Time waveform. (a) Numerical result (β=2.0, γ=1.0, δ=2.0, α=0.01). (b) Experimental result (L_1=L_3=20mH, L_2=L_4=10mH, C=47nF, R_L=6.45kΩ, R=12kΩ).



(a) (b)

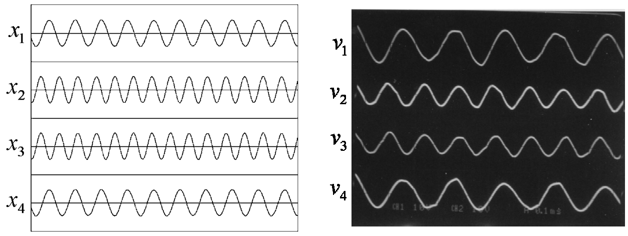
Figure 6: Time waveform. (a) Numerical result (β=2.0, γ=1.0, δ=2.0, α=8.0). (b) Experimental result (L_1=L_3=20mH, L_2=L_4=10mH, C=47nF, R_L=6.45kΩ, R=200Ω).

- $\omega_1 = \omega_4 \neq \omega_2 = \omega_3$

Figure 7 shows two pairs of in-phase oscillations of $x_1 - x_4$ and $x_2 - x_3$. x_1 and x_2 do not synchronize. Although two different frequencies are set to different positions, the oscillators can synchronize. For this case, synchronous state does not change for different α.

- $\omega_1 = \omega_3 = \omega_4 \neq \omega_2$ or $\omega_1 = \omega_2 = \omega_3 \neq \omega_4$

Figure 8 shows in-phase oscillations of $x_1 - x_3 - x_4$ and $x_1 - x_2 - x_3$ when α is small. The amplitude of x_4 is about twice of x_1 or x_3, as shown in Fig. 8(a). Similarly, the amplitude of x_2 is about twice of x_1 or x_3, as shown in Fig. 8(c). On the other hand, Fig. 9(a) shows the distorted oscillations when α is large. However the top of the oscillators $(x_1 + x_3)$ and $(x_2 + x_4)$ synchronize in-phase oscillation, as shown in Fig. 9(b).



(a) (b)

Figure 7: Time waveform. (a) Numerical result (β=2.0, γ=2.0, δ=1.0, α=0.1). (b) Experimental result (L_1=L_4=20mH, L_2=L_3=10mH, C=47nF, R_L=6.45kΩ, R=2.17kΩ).

4.3. Three different frequencies

- $\omega_1 = \omega_2 \neq \omega_3 \neq \omega_4$ or $\omega_1 \neq \omega_2 \neq \omega_3 = \omega_4$

Figures 10 and 11 show three types of synchronizations. Figure 10 shows waveforms for $\omega_1 = \omega_2 \neq \omega_3 \neq \omega_4$, Fig. 11 shows waveforms for $\omega_1 \neq \omega_2 \neq \omega_3 = \omega_4$. In both conditions, we are able to observe three types of synchronizations to change α. As increasing α, we are able to observe independent oscillations, oscillation death, and distorted oscillations.

4.4. Four different frequencies

- $\omega_1 \neq \omega_2 \neq \omega_3 \neq \omega_4$

Shown in Fig. 12(a), we are able to observe four independent oscillations when α is small. On the other hand, when α is large, the oscillations are distorted as shown in Fig. 12(b) or some resonators stop to oscillate.

5. Conclusions

In this article, we have presented synchronization phenomena in coupled simultaneous oscillators. We focused on the oscillators with the fifth-power nonlinear characteristics, and investigated synchronization phenomena with coupled multiple frequencies. We investigated synchronization phenomena by computer calculations and circuit experiments. By changing the frequencies of the resonators and the coupling strength, we observed various synchronous states.

References

[1] J. Schaffner, “Simultaneous Oscillations in Oscillators,” IRE Trans. on Circuit Theory, vol.1, pp.2-8, June. 1954.

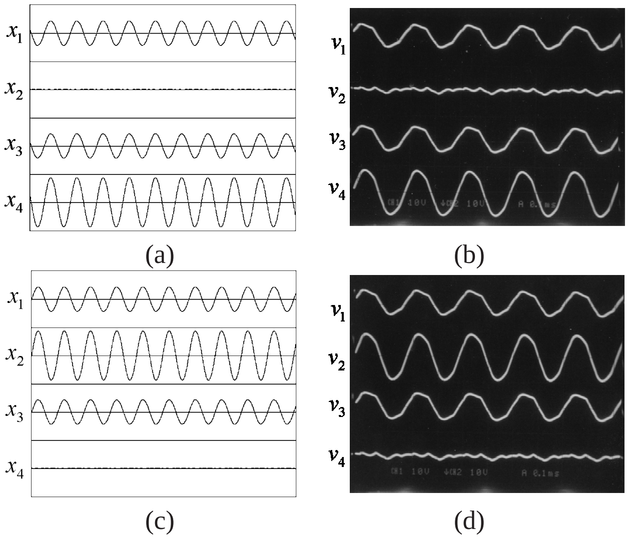


Figure 8: Time waveform. (a) Numerical result (β=0.5, γ=1.0, δ=1.0, α=0.1). (b) Experimental result (L_1=L_3=L_4=10mH, L_2=20mH, C=47nF, R_L=6.45kΩ, R=2.17kΩ). (c) Numerical result (β=1.0, γ=1.0, δ=0.5, α=0.1). (d) Experimental result (L_1=L_2=L_3=10mH, L_4=20mH, C=47nF, R_L=6.45kΩ, R=2.17kΩ).

[2] M. Matsuki, S. Mori, “Asynchronous Simultaneous Oscillations in Negative Resistance Oscillatory Circuit Containing Periodically Operating Analog Switch,” IEICE technical report, Circuits and systems, vol.101, pp.81-87, June. 1993.

[3] M. Matsuki, S. Mori, “Asynchronous Excitation Phenomena in Negative Resistance Oscillatory Circuit Containing Periodically Operating Analog Switch,” IEICE technical report, Nonlinear problems, vol.94, pp.51-58, May. 1994.

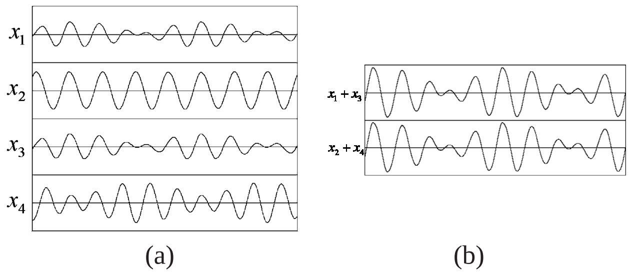


Figure 9: Time waveform. (a) Numerical result (β=0.5, γ=1.0, δ=1.0, α=0.1). (b) Top of the oscillator (x_1 + x_3) and (x_2 + x_4)

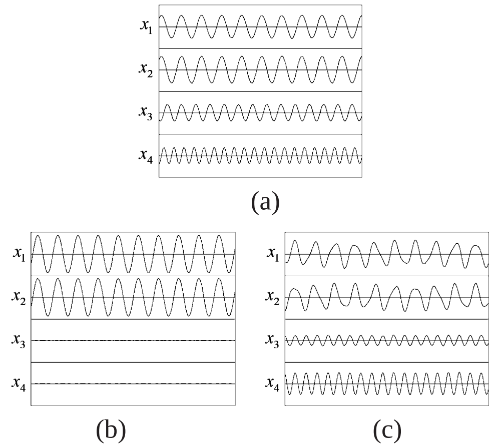


Figure 10: Change in waveform by changing α. (β=1.0, γ=2.0, δ=4.0, α=0.01). (a) Independent oscillation (α=0.01). (b) Oscillation death (α=0.1). (c) Distorted oscillation (α=8.0).

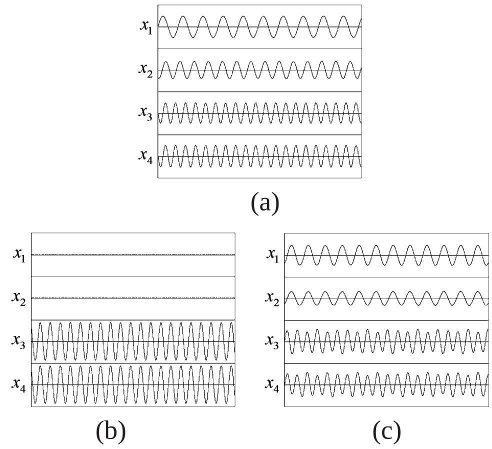


Figure 11: Change in waveform by changing α. (β=0.5, γ=4.0, δ=4.0). (a) Independent oscillations (α=0.01). (b) Oscillation death (α=0.1). (c) Distorted oscillations (α=8.0).

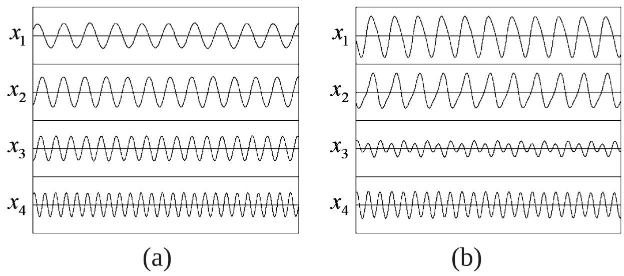


Figure 12: Time waveform. (β=1.5, γ=2.0, δ=4.0). (a) Independent four oscillations (α=0.01). (b) Distorted oscillations (α=8.0).