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Investigation of Network-Structured Particle Swarm Optimization

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Abstract

Our previous study has proposed the Network-Structured Particle Swarm Optimizer considering neighborhood relationships (NS-PSO). This study investigates the association between the network structure and the optimization performance by varying the degree of the small-world of NS-PSO.

1. Introduction

Particle Swarm Optimization (PSO) [1] is an algorithm to simulate the movement of flocks of birds. In PSO algorithm, there are no special relationships between particles. Each particle position is updated according to its personal best position and the best particle position among the all particles, and their weights are determined at random in every generation.

On the other hand, the Self-Organizing Map (SOM) [2] is an unsupervised learning and is a simplified model of the self-organizing process of the brain. The map consists of neurons located on a hexagonal or rectangular grid. The neurons self-organize statistical features of the input data according to the neighborhood relationship of the map structure.

In our past study, we have applied the concept of SOM to PSO and have proposed Network-Structured Particle Swarm Optimizer considering neighborhood relationships (NS-PSO) [3]. All particles of NS-PSO are connected to adjacent particles by a neighborhood relation, which dictates the topology of the networks. The particles directly connected on the network share the information of their own past best position. In every generation, we find a winning particle, whose function value is the best among all particles, as SOM algorithm, and each particle is updated depending on the neighborhood distance between it and the winner on the network. NS-PSO can greatly improve the optimization performance from the standard PSO. Furthermore, we applied NS-PSO to the various network topology [4],[5] and found that the circular-topology and the hexagonal-topology are appropriate for the simple unimodal functions and the complex multimodal functions, respectively. However, the relevance between the behaviors of NS-PSO with various topology and its parameters was not completely clear.

In this study, we investigate the association between the network structure and the optimization performance by varying the degree of the small-world of NS-PSO with small-world topology [6]. From results, we confirm that the small-world network is suitable to the unimodal function, and the random graph is suitable to the multimodal functions which including a lot of local optima.

2. Network-Structured PSO with Small-World Topology Considering Neighborhood Relationships (NS-PSO)

In the algorithm of the standard PSO, multiple solutions called “particles” coexist. At each time step, the particle flies toward its own past best position and the best position among all particles. Each particle has two informations; *position* and *velocity*. The position vector of each particle i and its velocity vector are represented by $\boldsymbol{X}_i = (x_{i1}, \cdots, x_{id}, \cdots, x_{iD})$ and $\boldsymbol{V}_i = (v_{i1}, \cdots, v_{id}, \cdots, v_{iD})$, respectively, where $(d = 1, 2, \cdots, D)$, $(i = 1, 2, \cdots, M)$ and $x_{id} \in [x_{\min}, x_{\max}]$.

The standard PSO has no neighborhood relationship. On the other hand, in the algorithm of NS-PSO, the particles are connected to other particles according to the topology of the network and share their local best position with neighbors.

2.1. Small-World Network (WS Model)

In this study, we investigate the behavior of the NS-PSO with small-world topology proposed by Watts and Strogatz (called WS model). The small-world topology is defined on a lattice with M particles and periodic boundary conditions.

(1) Connect each particle i to its k neighbor particles according to the topology of 1-dimensional lattice.

(2) Rewire each particle i to another particle chosen at random with probability p.

When $p = 0$, the network topology is 1-dimensional lattice, and when $p = 1$, it is a random graph.

2.2. Algorithm of NS-PSO with Small-World Topology

This section explains the algorithm of NS-PSO with the small-world topology.

Table 1: Four Benchmark Functions.

Function name	Benchmark Function	Initialization Space
4^{th} De Jong's function;	$f_1(x) = \sum_{d=1}^{D} dx_d^4,$	$x \in [-1.28, 1.28]^D$
Rosenbrock's function;	$f_2(x) = \sum_{d=1}^{D-1} \left(100\left(x_d^2 - x_{d+1}\right)^2 + (1 - x_d)^2\right),$	$x \in [-2.048, 2.048]^D$
Rastrigin's function;	$f_3(x) = \sum_{d=1}^{D} \left(x_d^2 - 10\cos(2\pi x_d) + 10\right),$	$x \in [-5.12, 5.12]^D$
Stretched V sine wave function;	$f_4(x) = \sum_{d=1}^{D-1} (x_d^2 + x_{d+1}^2)^{0.25}\left(1 + \sin^2(50(x_d^2 + x_{d+1}^2)^{0.1})\right),$	$x \in [-10, 10]^D$

(Step1) (Initialization) Let a generation step $t = 0$. Randomly initialize the particle position $\boldsymbol{X}_i$, initialize its velocity $\boldsymbol{V}_i$ for each particle i to zero, and initialize $\boldsymbol{P}_i = (p_{i1}, p_{i2}, \cdots, p_{iD})$ with a copy of $\boldsymbol{X}_i$. Evaluate the fitness $f(\boldsymbol{X}_i)$ for each particle i and find $\boldsymbol{P}_g$ with the best fitness among all the particles. Define a particle g as the winner c. Connect all the particles according to the subsection 2.1. Find $\boldsymbol{L}_i = (l_{i1}, l_{i2}, \cdots, l_{iD})$ with the best fitness among the directly connected particles, namely own neighbors.
(Step2) Evaluate the fitness $f(\boldsymbol{X}_i)$ and find a winning particle c with the best fitness among the all particles at current generation t;

$$c = \arg\min_i \{f(\boldsymbol{X}_i(t))\}. \tag{1}$$

For each particle i, if $f(\boldsymbol{X}_i) < f(\boldsymbol{P}_i)$, update the personal best position (called *pbest*) by $\boldsymbol{P}_i = \boldsymbol{X}_i$.

Let $\boldsymbol{P}_g$ represents the best position with the best fitness among all particles so far (called *gbest*). If $f(\boldsymbol{X}_c) < f(\boldsymbol{P}_g)$, update *gbest* by $\boldsymbol{P}_g = \boldsymbol{X}_c$, where $\boldsymbol{X}_c$ is the position of the winner c.
(Step3) Find each local best position (called *lbest*) $\boldsymbol{L}_i$ among the particle i and its neighborhoods which are directly connected with i on the network. For each particle i, update *lbest* $\boldsymbol{L}_i$, if needed.
(Step4) Update $\boldsymbol{V}_i$ and $\boldsymbol{X}_i$ of each particle i depending on its *lbest*, the winner's position $\boldsymbol{X}_c$ and the distance on the network between i and the winner c, according to

$$\begin{aligned} \boldsymbol{V}_i(t+1) &= w\boldsymbol{V}_i(t) + c_1\mathbf{Rand}(\cdot)\left(\boldsymbol{L}_i - \boldsymbol{X}_i(t)\right) \\ &\quad + c_2 h_{c,i}\left(\boldsymbol{X}_c - \boldsymbol{X}_i(t)\right), \\ \boldsymbol{X}_i(t+1) &= \boldsymbol{X}_i(t) + \boldsymbol{V}_i(t+1), \end{aligned} \tag{2}$$

where w is the inertia weight determining how much of the previous velocity of the particle is preserved. c_1 and c_2 are two positive acceleration coefficients, generally $c_1 = c_2$, and $\mathbf{Rand}(\cdot) = (\mathrm{rand}_1, \mathrm{rand}_2, \cdots, \mathrm{rand}_\mathrm{D})$ is an uniform random number vector from $U(0,1)$. $h_{c,i}$ is the fixed neighborhood function defined by

$$h_{c,i} = \exp\left(-\frac{dis(c,i)}{\sigma^2}\right), \tag{3}$$

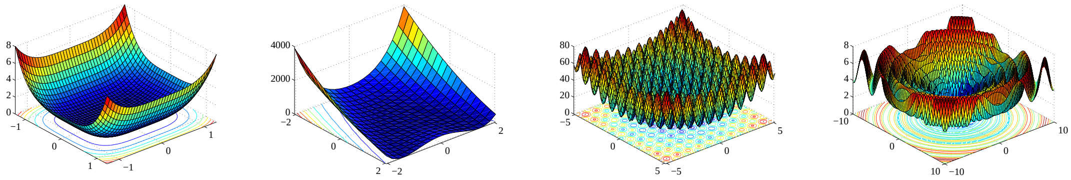
where $dis(c,i)$ is the shortest-path distance between the winner c and the particle i on the network and is called *neighborhood distance*. The fixed parameter σ corresponds to the width of the neighborhood function. Therefore, large σ strengthens particles' spreading force to the whole space, and small σ strengthens their convergent force toward the winner.
(Step5) Let $t = t + 1$ and go back to (Step2).

3. Numerical Experiments

In order to evaluate the performance of NS-PSO with the small-world topology and to investigate its behavior, we use four benchmark optimization problems summarized in Table 1. f_1 is an unimodal function, and f_2, f_3 and f_4 are multimodal functions with numerous local optima. The optimum solution x^* of Rosenbrock's function f_2 is $[1, 1, \ldots, 1]$, and x^* of the other functions are all $[0, 0, \ldots, 0]$. The optimum values $f(x^*)$ of all the functions are 0. All the functions have $D = 50$ variables.

The population size M is set to 36 in PSO, and the network size is 36 in NS-PSO. For all PSOs, the parameters are set as $w = 0.7$ and $c_1 = c_2 = 1.6$. The neighborhood radius σ of NS-PSO is 2 for f_4 and 1.5 for other functions. To generate the small-world topology, the neighborhood parameter k is set to 2. We carry out the simulations repeated 100 times for all the optimum functions with 3000 generations.

We consider the behavior according to the network topology in terms of the clustering coefficient C [6] and the average shortest-path length L. Figure 2 shows the network average clustering coefficient C and the average shortest-path



(a) (b) (c) (d)

Figure 1: Landscape of the four benchmark functions with two variables. (a) 4^{th} De Jong's function. (b) Rosenbrock's function. (c) Rastrigin's function. (d) Stretched V sine wave function.

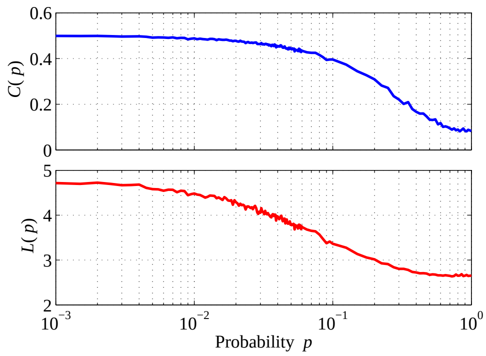


Figure 2: Network average clustering coefficient C and the average shortest-path length L of WS network with varying the probability p. $M = 36$. $k = 2$.

length L of WS network with the population size 36 which is used in this simulation. From this figure, we can confirm that the network has the small-world property which satisfies the small $L(p)$ and the large $C(p)$ when $p \in [0.03, 0.1]$.

3.1. Comparison results of PSO and NS-PSO

Experimental results of the standard PSO with no connections and NS-PSO with small-world topology by using various the probability p are summarized in Table 2. $p = 0$ and $p = 1$ mean that the networks are the regular network and the random graph, respectively. We skip comparison with the results of PSO with small-world topology because our previous study [5] has shown that its results are worse than the standard PSO. From this table, we can confirm that the standard PSO has never obtained the better results than any NS-PSOs on all the benchmarks. In addition, NS-PSOs significantly improve the optimization efficiency for all the functions.

Next, let us note the probability p and its performance. At the unimodal function as f_1, NS-PSO with the regular network (as $p = 0$) obtains better results than the random graph network (as $p = 1$), and NS-PSO with the small-world network as $p = 0.03$ and $p = 0.1$ can obtain effective results than other networks. On the other hand, at all the multimodal functions as f_2, f_3 and f_4, the results of the network with some irregularities as $p > 0$ are better than the regular network as $p = 0$. Furthermore, when the network is the random graph as $p = 1$, the performances are the best.

Let us consider the association between the probability and its performance in detail. Simulation results of NS-PSO with small-world topology by varying the probability p are shown in Fig. 3. It should be noted that the characteristic is different between for the unimodal function as f_1 and the multimodal functions. For the unimodal function as Fig. 3(a), the regular network can obtain the acceptable results, and the performance achieves the best result when the network is the small-world. However, as the network becomes the random graph with large p, the optimization result grows worse than the regular network. In contrast, for the unimodal function as Figs. 3(b)–(c), the results of the regular network are the worst, and the performance grows better as the network becomes the random graph. These results are because that the diversity of the particles is different between the regular network and the random graph. On the regular network and the network with small p, the shortest path length L and the number of particles in local neighbor N_l are almost same for each particle. From these effects, it is easy to transmit the information of *lbest* to the whole particles, therefore, the network with small p is effective for the unimodal function which is simple. However, the premature communication produces the premature convergence, then, the regular network easily goes into local optima in the multimodal functions. On the other hand, NS-PSO with small-world topology. whose p is large, contains various kinds of particles which has different shortest path length and different size of local neighbors. Because these effects produce the diversity of the particles and avert the premature convergence, the particles of NS-PSO with the random-network can easily escape from the local optima.

4. Conclusions

In this study, we have investigated the association between the network structure and the optimization performance by varying the degree of the small-world of Network-Structured

Table 2: Comparison results of PSO and NS-PSO with small-world topology on 4 test functions.

f		PSO	NS-PSO with small-world topology			
			$p = 0$	$p = 0.03$	$p = 0.1$	$p = 1$
f_1	Mean	1.58e-35	4.41e-58	1.87e-59	**5.48e-61**	9.45e-57
	Minimum	9.86e-42	2.66e-65	6.06e-69	3.38e-71	4.60e-67
f_2	Mean	55.24	38.11	37.23	37.22	**35.67**
	Minimum	36.74	28.05	25.38	23.31	23.81
f_3	Mean	148.31	82.87	74.91	67.51	**66.77**
	Minimum	94.52	48.75	35.82	39.80	33.83
f_4	Mean	65.62	30.62	29.78	21.40	**16.13**
	Minimum	39.36	12.64	7.26	4.94	4.37

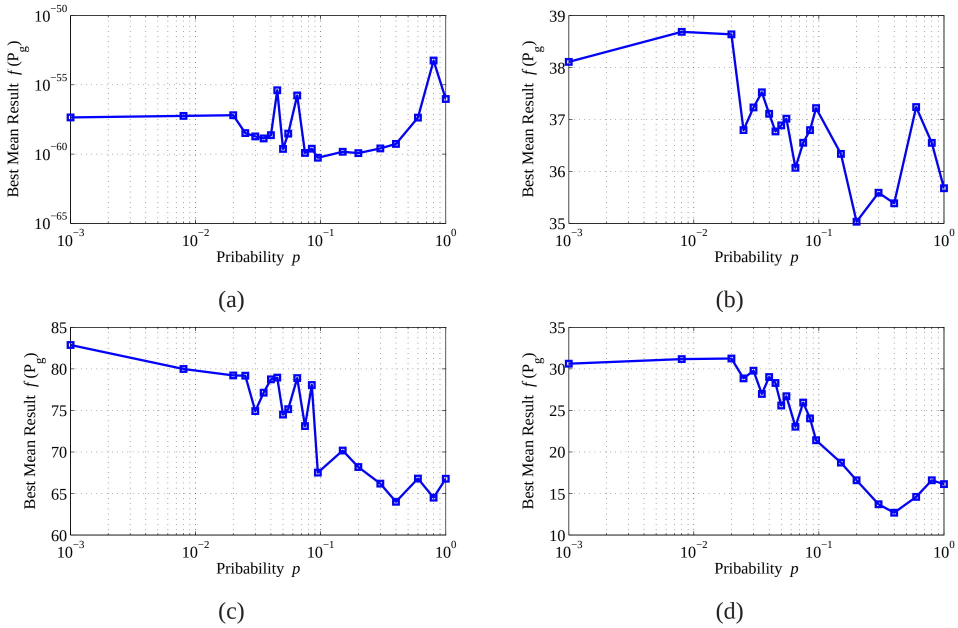


Figure 3: Results by varying the probability p. (a) 4^{th} De Jong's function f_1. (b) Rosenbrock's function f_2. (c) Rastrigin's function f_3. (d) Stretched V sine wave function f_4.

Particle Swarm Optimization. The simulation results have shown that NS-PSO can greatly improve the optimization performance from the original PSO. we have found that the small-world network is suitable to the unimodal function, and the random graph is suitable to the multimodal functions which including a lot of local optima. From these results, we can say that NS-PSO with small-world topology can obtain effective results flexibly by rewiring the network structure for various optimization problems without changing the algorithm structure.

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