2010 International Workshop on Nonlinear Circuits,
Communication and Signal Processing
NCSP'10, Waikiki, Hawaii, March 3-5, 2010

Switch Coupling of Auto Gain Controlled Oscillators Containing Time Delay

Akiyoshi Ono, Hironori Kumeno and Yoshifumi Nishio

Dept. Electronic Eng.,
Tokushima University, Japan
Email: {akiyoshi, kumeno, nishio}@ee.tokushima-u.ac.jp

Abstract

In this study, we propose two chaotic oscillators coupled via switch. The chaotic oscillators are auto gain controlled oscillators containing time delay. The oscillators have feedback systems which control the gain. The coupling switch is placed between the feedback systems which control both of the oscillators. The coupling switch connects alternately with one subcircuit and the other with a fixed time interval. We carry out computer calculations and circuit experiments for two coupled auto gain controlled oscillators containing time delay and investigate synchronization phenomena. In the computer calculations, we observe coexistence phenomena of in-phase synchronization and opposite-phase synchronization. In the circuit experiments, we observe coexistence phenomena of in-phase quasi-synchronization and opposite-phase quasi-synchronization. Finally, to investigate the difference between the numerical results and the experimental results, we investigate effects of parameter mismatch and noise.

1. Introduction

Synchronization and the related bifurcation of chaotic systems are good methods to describe various high-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena are extensively carried out in various field [1]-[3]. The chaos phenomena are quite dependent on initial values and not periodical and predictable. Moreover, we obtain the synchronization phenomena in coupled oscillators despite of behaviors of one oscillator are complex. Therefore, we are interested in the synchronization phenomena. However, a lot of synchronization phenomena of coupled chaotic oscillators have not been solved yet and we have to investigate them.

In this study, we propose switch coupling system for auto gain controlled oscillators containing time delay. The coupling switch is placed between the feedback systems which control both of the oscillators. The coupling switch connects alternately with one subcircuit and the other with a fixed time interval. The oscillator used in this study is auto gain controlled oscillator containing time delay, whose circuit is shown in Fig. 1. This is introduced in [4]. The oscillator exhibits bifurcation phenomena from one-periodic attractor to chaotic attractor. Figure 2 shows examples of experimental chaotic attractor and corresponding attractor of the numerical simulation. We investigate synchronization phenomena in the coupled auto gain controlled oscillators containing time delay. We observe coexistence phenomena of in-phase synchronization and opposite-phase synchronization in the computer calculations. Moreover, we observe coexistence phenomena of in-phase quasi-synchronization and opposite-phase quasi-synchronization in the circuit experiments. Finally, we investigate effects of parameter mismatch and noise.

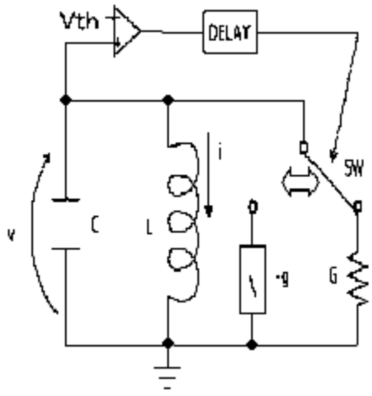

Figure 1: Auto gain controlled oscillator containing time delay.

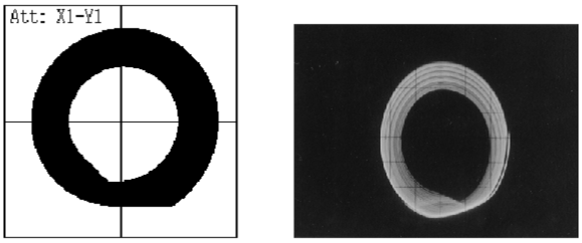

Figure 2: Chaotic attractors in auto gain controlled oscillator containing time delay.

2. Circuit model

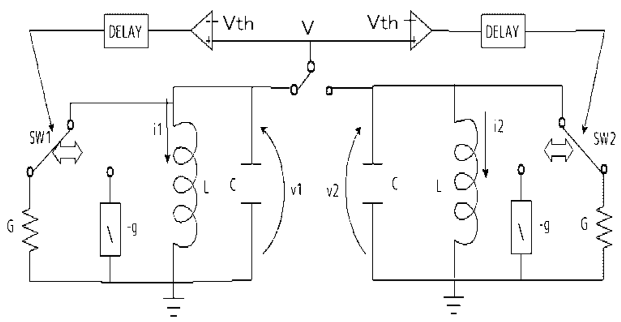

Figure 3: Circuit model.

The circuit model used in this study is shown in Fig. 3. In the circuit, two chaotic oscillators are coupled by periodically changing switch SW3. The circuit equations of the oscillator are given as

$$\begin{cases} C\frac{dv}{dt} = gv - i, \quad L\frac{di}{dt} = v \end{cases} \tag{1}$$

$$\begin{cases} C\frac{dv}{dt} = -Gv - i, \quad L\frac{di}{dt} = v. \end{cases} \tag{2}$$

The periodically changing switch SW3 connects alternately with one subcircuit and the other with a fixed time interval (the switching frequency is $2\pi\gamma_t$). The periodically changing switch can be controlled by the following sinusoidal function.

$$k = \sin\gamma_t t. \tag{3}$$

Figure 4 shows operation of the SW3 and information through the SW3. Two comparators of the subcircuits obtain the information of the voltage from the subcircuit connected via SW3 and compare the voltage of the subcircuit with a given threshold V_{th}. Then, two comparators give the information to the delay systems of the subcircuits. The delay systems of the subcircuits delay the information whitch control switches SW1 and SW2.

The periodically changing switch connects to the left subcircuit in Fig. 3 when k is positive, while to the right subcircuit in Fig. 3 when negative. The voltage of the subcircuit connected via SW3 controls SW1 and SW2 at the same time. If the voltage of the subcircuit is lower than the threshold V_{th}, SW1 and SW2 are connected to the negative resistors, while if the voltage of the subcircuit rises above V_{th}, SW1 and SW2 are connected to the positive resistor after the given delay time during the time interval while the voltage of the subcircuit is higher than V_{th}.

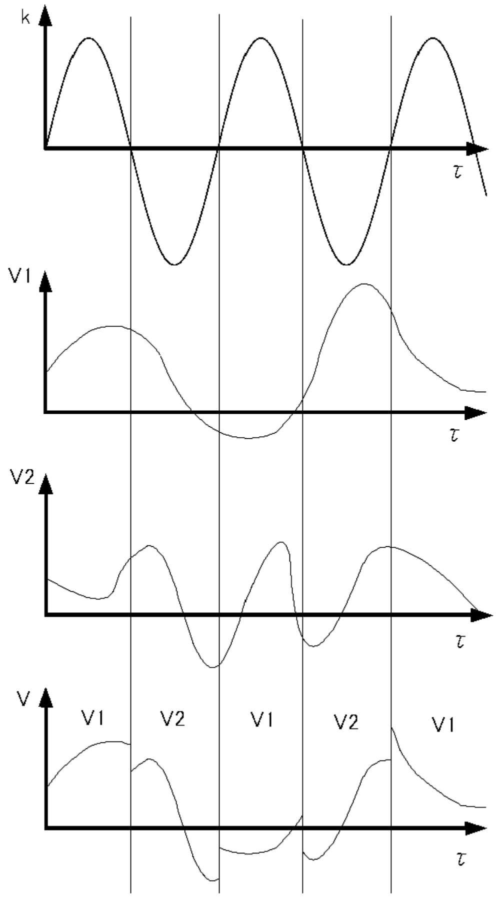

Figure 4: Information V through periodically changing switch SW3.

By using the following normalized variables and parameters,

$$\begin{aligned} &i_i = \sqrt{C/L}V_{th}x_i, \ \ v_i = V_{th}y_i, \\ &t = \sqrt{LC}\tau, \ \ \gamma_t = \gamma/\sqrt{LC}, \\ &g\sqrt{L/C} = 2\alpha, \ \ G\sqrt{L/C} = 2\beta, \end{aligned} \tag{4}$$

the normalized circuit equations are given as:

$$\begin{cases} \dot{x}_i = y_i \\ \dot{y}_i = -x_i + 2\alpha y_i \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}_i = y_i \\ \dot{y}_i = -x_i - 2\beta y_i. \end{cases} \tag{6}$$

When SW1 and SW2 are connected to the negative resistor, the circuit is governed by Eq. (5), while when SW1 and SW2 are connected to the positive resistor, the circuit is governed by Eq. (6).

3. Synchronization phenomena

In this section, synchronization phenomena generated in the coupled auto gain controlled oscillators containing time delay are investigated by computer calculation and circuit experiment.

3.1. Computer calculations

We carry out computer calculations for the coupled auto gain controlled oscillators containing time delay. We observe coexistence phenomenon of in-phase synchronization and opposite-phase synchronization. Figure. 5 shows examples of the observed attractors and phase differences at the parameters $\alpha = 0.0186$, $\beta = 0.87$ and $\gamma = 2510$. The two subcircuits are synchronized at in-phase as shown in Fig. 5(a), while the two subcircuits are synchronized at opposite-phase as shown in Fig. 5(b). These states are observed for the same parameters, although initial values are different. Thus, the in-phase synchronization and the opposite-phase synchronization coexist.

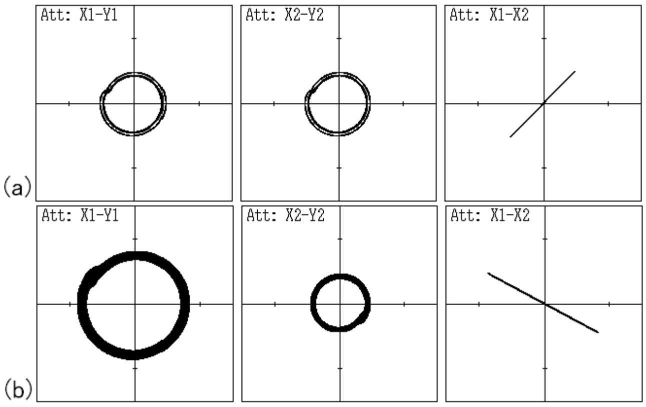

Figure 5: Attractors and phase differences in numerical results for $\alpha = 0.0186$, $\beta = 0.87$ and $\gamma = 2510$.

3.2. Circuit experiments

We carry out circuit experiments for the coupled oscillators. We observe coexistence phenomenon of in-phase quasi-synchronization and opposite-phase quasi-synchronization as shown in Fig. 6. Two periodic attractors are quasi-synchronized at in-phase in Fig. 6(a). Two attractors are quasi-synchronized at opposite-phase in Fig. 6(b). Two chaotic attractors are quasi-synchronized at in-phase in Fig. 6(c). These states are observed for the same parameters, although initial values are different. These results is different from the computer calculations. Figure 6 show the phase difference and the attractor on unstable quasi-in-phase synchronization.

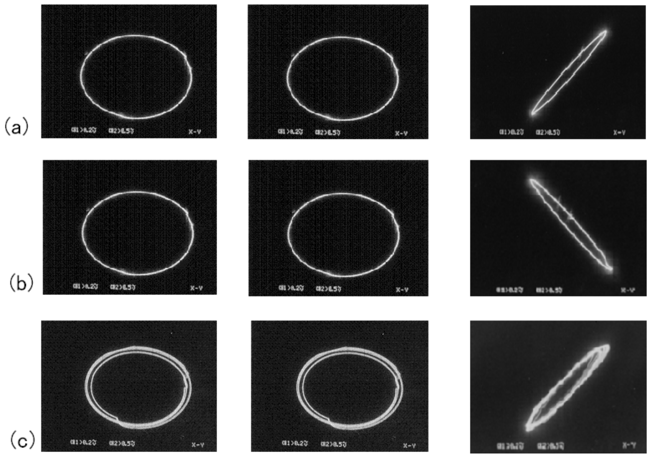

Figure 6: Attractors and phase differences in experimental results. (a) In-phase quasi-synchronization of periodic attractors. (b) Opposite-phase quasi-synchronization of periodic attractors. (c) Quasi-synchronization of chaotic attractors.

4. Effect of noise and parameter mismatch

The synchronization phenomena observed in the experimental results are slightly different from the synchronization phenomena observed in the computer calculation, although coexistence phenomena of in-phase synchronization and opposite-phase synchronization. Synchronization in the practical experiment will be affected by two factor: noise and parameter mismatch between two oscillators. On one hand it is impossible to make the parameters of one subcircuit to be completely consistent with the other. On the other hand, noise inevitably exists in the any realistic physical system. These perturbations may kick the motion off the synchronization manifold. We consider the influence from the noise and the difference of parameter mismatch cause the difference between the numerical results and the experimental results. Then, carry out computer calculation for the coupled circuits containing the noise and changing the parameter mismatch. Normalized circuit equations containing the noise and the parameter mismatch are described as:

$$\begin{cases} \dot{x}_1 = y_1 \\ \dot{y}_1 = -x_1 + 2\alpha y_1 + \xi_1 \end{cases} \tag{7}$$

$$\begin{cases} \dot{x}_1 = y_1 \\ \dot{y}_1 = -x_1 - 2\beta y_1 + \xi_2, \end{cases} \tag{8}$$

$$\begin{cases} \dot{x}_2 = \sigma_1 y_2 \\ \dot{y}_2 = -x_2 + 2(1 + \sigma_2)\alpha y_2 + \xi_1 \end{cases} \tag{9}$$

$$\begin{cases} \dot{x}_2 = \sigma_3 y_2 \\ \dot{y}_2 = -x_2 - 2(1+\sigma_4)\beta y_2 + \xi_2, \end{cases} \tag{10}$$

where σ correspond to the parameter mismatch, and ξ is the noise. Figure 7 shows simulation results for influences of the parameter mismatch and the noise. Criterial attractors and phase differences without the parameter mismatch and the noise are shown in Fig. 7(a). Attractors and phase differences containing only the noise are shown in Fig. 7(b). Attractors and phase differences containing only the parameter mismatch are shown in Fig. 7(c). Attractors and phase differences containing both of the noise and the parameter mismatch in Fig. 7(d).

The parameter mismatch and the noise perturb the synchronization. From the results, it seem that the parameter mismatch largely affect the synchronization to be perturbed in the experimentation.

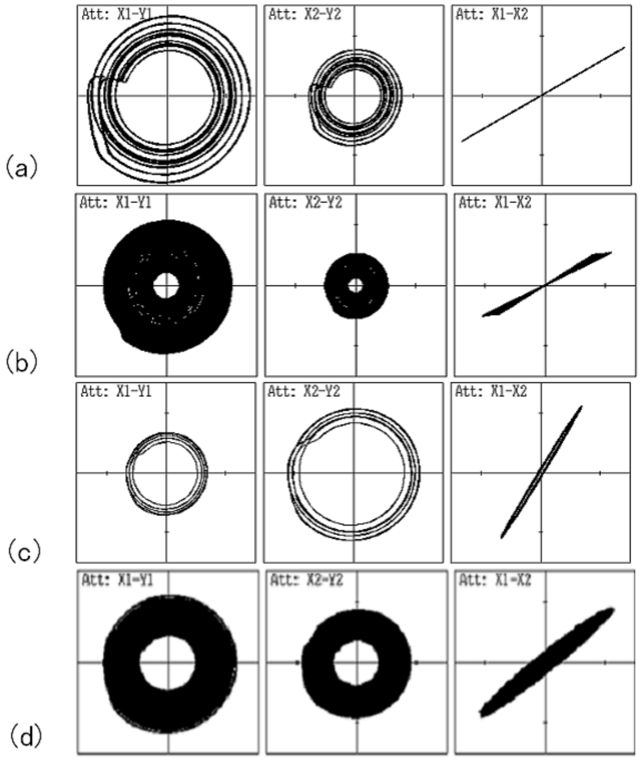

Figure 7: Attractors and phase differences in numerical results including noise and parameter mismatch. (a) Without the noise and the parameter mismatch. (b) Containing only the noise. (c) Containing only the parameter mismatch. (d) Containing both of the noise and the parameter mismatch.

5. Conclusions

In this study, we have proposed the switch coupling system for auto gain controlled oscillators containing time delay, and investigated the synchronization phenomena generated in the coupled oscillators. couped auto gain controlled oscillators containing time delay. We observed coexistence phenomena of in-phase synchronization and opposite-phase synchronization on the computer calculations for this circuit. On the other hands, we observed coexistence phenomena of in-phase quasi-synchronization and opposite-phase quasi-synchronization on the experiments for the circuit.

Our future research is to make clear the mechanism of the generation of the coexistence phenomenon and the cause of that we observed quasi-synchronization.

Acknowledgment

This work was partly supported by CASIO Science Promotion Foundation.

References

[1] G. Abramson, V. M. Kenkre and A. R. Bishop, "Analytic solutions for nonlinear waves in coupled reacting systems," *Physica A: Statistial Mechanics and its Applications,* vol. 305, no. 3–4, pp. 427–436, 2002.

[2] C. M. Gray, "Synchronous oscillators in neural systems: mechanisms and functions," *J. Computational Neuroscience,* vol. 1, pp. 11–38, 1994.

[3] Y. Zhang, G. H. Du and J. J. Jiang, "Synchronizing Chaos by Impulsive Feedback Method," *International Journal of bifurcation and Chaos,* vol. 11, no. 8, pp. 2233–2243, 2001.

[4] T. Maruyama, N. Inaba, Y. Nishio and S. Mori, "Chaos in Auto Gain Controlled Oscillator Containing Time Delay," Transactions of IEICE, vol. J72-A, no. 11, pp. 1814–1820, 1989.