2010 International Workshop on Nonlinear Circuits,
Communication and Signal Processing
NCSP'10, Waikiki, Hawaii, March 3-5, 2010

Synchronization Phenomenon in Small World Network of Coupled Cubic Maps

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Abstract

The small world network model is a simple model of the structure of social networks, which possesses characteristics of both regular coupling and random coupling. To structure the small world networks, each map is rewired to other maps chosen at random with a probability. In this study, we investigate synchronization phenomena of small world network model of one hundred coupled cubic maps. As a result, we confirmed that various wave propagations were observed by changing the probability.

1. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields [1][2]. On the other hand, studies of network model are very important, because they help us to understand the basic features and requirements of various systems. In real life, social networks are known to be not completely random network nor completely local. This was modeled in an interesting work as the small world network model [3].

In our past studies [4], we have investigated synchronization phenomena in coupled cubic maps. The cubic map has two attractors located symmetrically with respect to the origin and they merge as increasing a control parameter. By computer simulations, we observe interesting state transition phenomenon. As a result, the two maps are synchronized for the parameters giving periodic solutions.

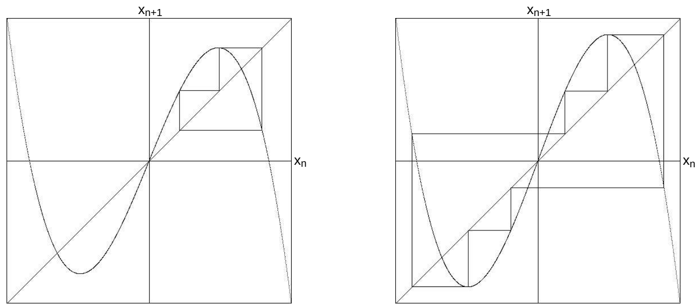
In this study, we propose small world network model of coupled cubic maps. Then, we investigate synchronization phenomena in this model.

2. Coupled cubic map

In our past studies, we investigated two coupled cubic maps. Figure 1 shows return maps of the cubic maps.

The cubic map is described by the following equation:

$$x_{(n+1)} = ax^3_{(n)} - (a+1)x_{(n)}, \qquad (1)$$



(a) (b)

Figure 1: Cubic map. (a) $a = -3.451$. (b) $a = -3.70$.

where n is an iteration and a is a parameter which determines chaotic features. Figure 1(a) shows an asymmetric 3 periodic attractor, and Fig. 1(b) shows a symmetric 6 periodic attractor.

The coupled cubic maps are described by the following equation:

$$\begin{cases} g(x_{(n)}) = ax^3_{(n)} - (a+1)x_{(n)} \\ x_{1(n+1)} = (1-\epsilon)g(x_{1(n)}) + \epsilon(g(x_{2(n)})) \\ x_{2(n+1)} = (1-\epsilon)g(x_{2(n)}) + \epsilon(g(x_{1(n)})), \end{cases} \qquad (2)$$

where ϵ is a coupling parameter. Figure 2 shows examples of time waveforms in two coupled cubic maps. Figure 2(a) shows the time waveform obtained with the parameters giving the periodic solution in Fig. 1(a) and a negative coupling strength. While, Fig. 2(b) shows the time waveform obtained with the parameters giving the periodic solution in Fig. 1(b) and a positive coupling strength. From the coupled cubic maps, we confirmed that the two maps are synchronized for the parameters giving periodic solutions.

3. Small world network model

In this section, we explain a small world network model of coupled cubic maps. The network topology of small world is located between regular graph and random graph. Watts

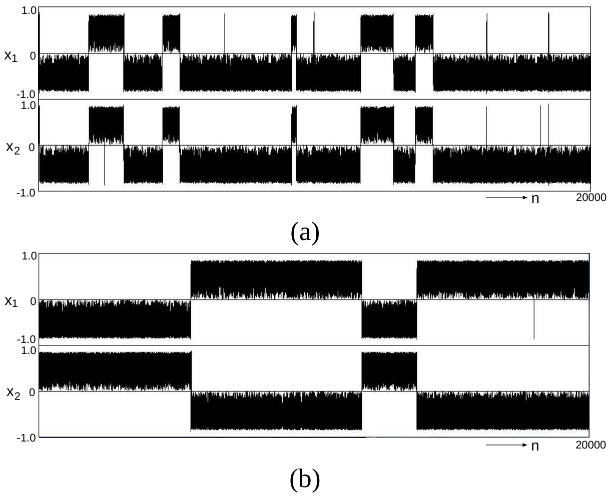


Figure 2: Time waveform of two coupled cubic maps. (a) $\alpha = -3.4505$ and $\epsilon = -0.062$. (b) $\alpha = -3.70$ and $\epsilon = 0.0375$.

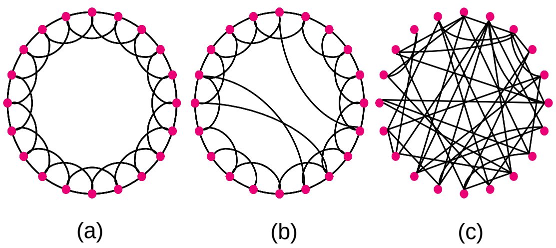


Figure 3: A small world graph. (a) Regular coupling. (b) Small world network. (c) Random coupling.

proposed a method to structure small world network [3]. The small world graph in Fig. 3 shows that the small world network can be obtained by small world couplings between the regular graph and the random graph. In this study, we consider one hundred coupled cubic maps like Fig. 3. Each map is coupled to the first neighbor maps and the second neighbors as shown in Fig. 3(a). To structure the small world network, each coupling is rewired to other maps chosen at random with a probability p. In the case of $p = 0$, any couplings are not rewired. On the other hand, in the case of $p = 1$, all the couplings are rewired as shown in Fig. 3(c) (random coupling). Figure 3(b) shows the small world network. The followings are the equations of one hundred cubic maps for the case of the regular coupling.

$$\begin{cases} g(x_{(n)}) = ax^3_{(n)} - (a+1)x_{(n)} \\ x_{i(n+1)} = (1-\epsilon)g(x_{i(n)}) + \epsilon/4(g(x_{i-2(n)}) \\ \qquad\qquad + g(x_{i-1(n)}) + g(x_{i+1(n)}) + g(x_{i+2(n)})) \\ (i = 1, 2, 3, ..., 100) \end{cases} \tag{3}$$

4. Synchronization phenomenon

In this section, we investigate synchronization phenomenon in small world network of coupled cubic maps. Figure 4 shows the computer simulated results. In the figure, the red (bright) areas represent that x_i are positive. While, the blue (dark) areas represent that x_i are negative. We can see various wave propagations by changing the probability p. In every case, the coupled maps produce complex patterns, however the maps do not synchronize stably.

Figure 5 shows distribution of the frequencies of sojourn time. The sojourn time is the number of iteration during x_1 is continuously in the positive or negative region. The horizontal axis indicates the sojourn time, and the vertical axis indicates the frequency. From these figures, we can see that the sojourn time becomes longer as increasing the probability p except the case of $p = 0.05$. Additionally, the frequencies has the peaks of the sojourn time which are around 5, under all conditions.

5. Conclusions

In this study, we investigated synchronization phenomenon in the small world network of one hundred coupled cubic maps. By computer simulations, we confirmed that various wave propagations were observed by changing probability p. In every case, the coupled maps produce complex patterns, however the maps do not synchronize stably. Then, the sojourn time becomes longer as increasing the probability p. Investigating the mechanism of this synchronization phenomenon is our future research.

References

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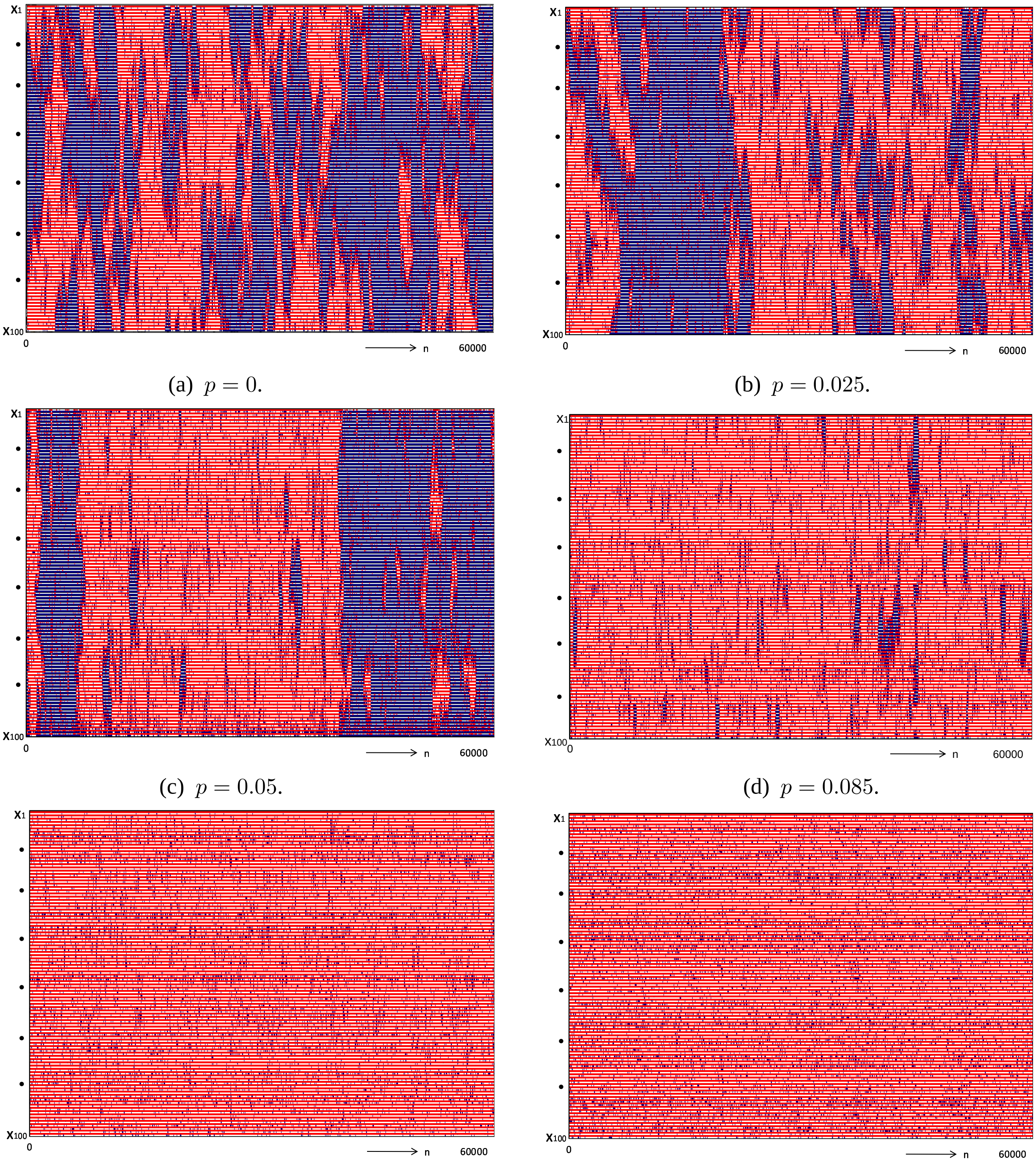


(a) $p = 0$.

(b) $p = 0.025$.

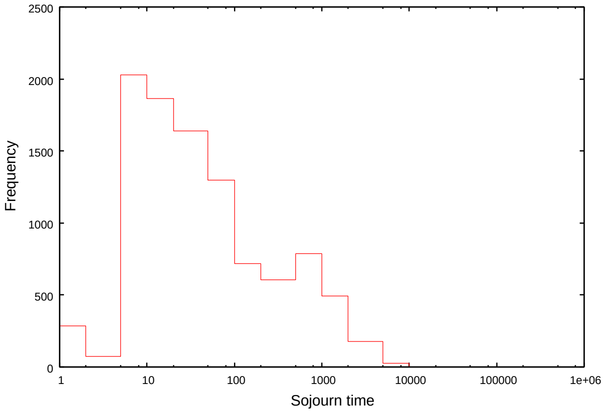
(c) $p = 0.05$.

(d) $p = 0.085$.

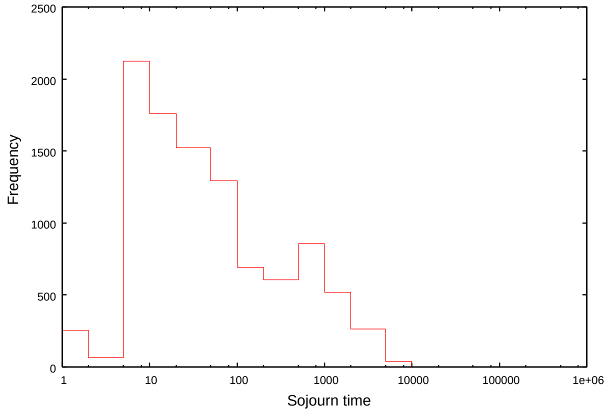
(e) $p = 0.46$.

(f) $p = 1$.

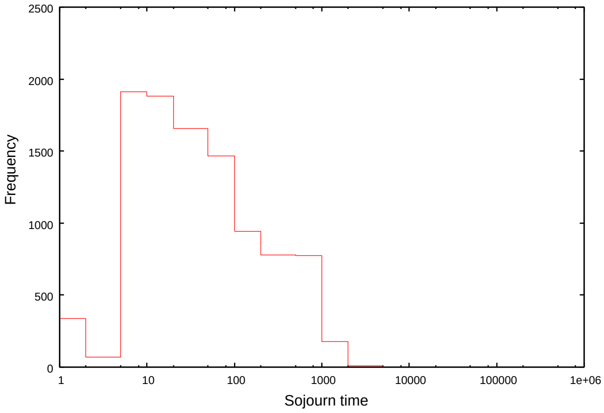
Figure 4: Time waveform of coupled cubic maps. $\alpha = -3.4505$ and $\epsilon = -0.07$. (a) $p = 0$. (b) $p = 0.025$. (c) $p = 0.05$. (d) $p = 0.085$. (e) $p = 0.46$. (f) $p = 1$.



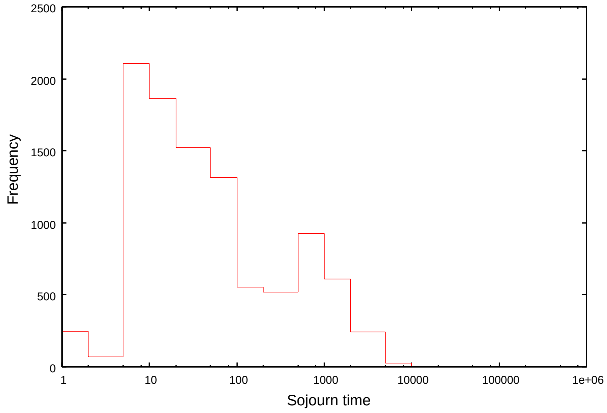
(a) $p = 0$.



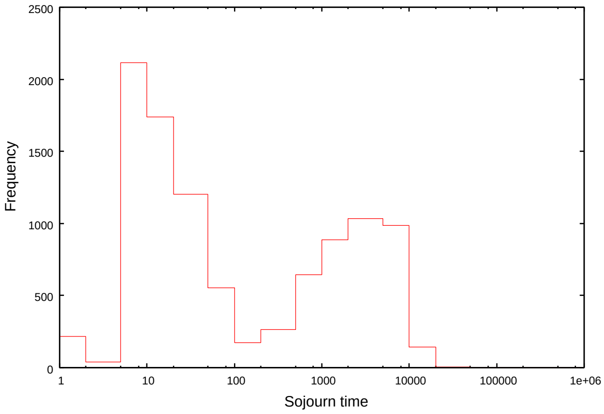
(b) $p = 0.025$.



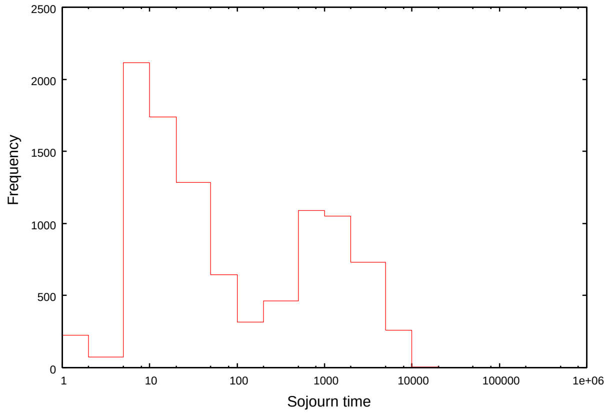
(c) $p = 0.05$.



(d) $p = 0.085$.



(e) $p = 0.46$.



(f) $p = 1$.

Figure 5: Distribution of the frequencies of sojourn time. $\alpha = -3.4505$ and $\epsilon = -0.07$. (a) $p = 0$. (b) $p = 0.025$. (c) $p = 0.05$. (d) $p = 0.085$. (e) $p = 0.46$. (f) $p = 1$.