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Self-Switching Phenomenon of Synchronization in Globally Coupled Parametrically Forced Logistic Maps

Hironori Kumeno† and Yoshifumi Nishio†

†Tokushima University
2-1,Minami-Josanjima, Tokushima, Japan
Phone/FAX:+81-88-656-7470
Email: {kumeno, nishio}@ee.tokushima-u.ac.jp

Abstract

Various synchronization phenomena in globally coupled parametrically forced logistic maps are observed for several coupling intensity when three maps are coupled. The synchronization phenomena fall into four general categories, which are asynchronous, self-switching of synchronization, synchronization of two among the three maps and synchronization of all the maps. In this study, we investigate the self-switching phenomenon of synchronization in detail. In particular, relationship between sojourn time and the coupling intensity and condition that a synchronization state switch to other synchronization state are investigated.

1. Introduction

Synchronization is one of the fundamental phenomena in nature, and one of typical nonlinear phenomena. Therefore, studies on synchronization phenomena of coupled systems are extensively carried out in various fields, physics [1], biology [2], engineering and so on. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena for future engineering applications. In a simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referred in Refs. [4] and [5]. The network of chaotic elements can be modeled by a system of coupled one-dimensional maps. Behavior generated in coupled system of chaotic one-dimensional map is investigated in Refs. [6]-[8] In particular, Coupled Map Lattice (CML) and Globally Coupled Map (GCM) are well known as mathematical models in discrete-time system. Various kinds of dynamics are observed in their systems. The research into CML and GCM are important for not only modeling of multiple degree of freedom nonlinear systems but also application to biological networks and engineering. In the past we have investigated effects of parametric excitation in coupled van der Pol oscillators [9]. In this study, for more detailed investigation of effect of the parametric excitation on synchronization, we focus on a globally coupled system of simple one-dimensional maps. A typical scheme for global coupling is given by

$$x_i(t+1) = (1-\varepsilon) f[x_i(t)] + \frac{\varepsilon}{N}\sum_{j=1}^{N} f[x_j(t)] \quad (i = 1, 2, \cdots, N,) \tag{1}$$

where ε is the coupling intensity. The globally coupled maps are a scheme that an average number of all the maps affect each of the maps, and similar to the system that we have studied using van der Pol oscillators. The one-dimensional map used in this study is a logistic map, since the map can be described by a simple discrete equation. Mathematically, the logistic map is written as

$$x(t+1) = \alpha x(t)(1 - x(t)). \tag{2}$$

In this study, firstly, we describe the parametrically forced logistic map and synchronization phenomena in the globally coupled parametrically forced logistic maps. After that, we investigate the self-switching phenomenon of synchronization in detail.

2. Parametrically forced logistic map

A parametrically forced logistic map used in this study is described as:

$$x(t+1) = \alpha_f(t) x(t)(1 - x(t)), \tag{3}$$

and

$$\alpha_f(t) = \begin{cases} \alpha_1, & n(\tau_1+\tau_2) < t \le n(\tau_1+\tau_2)+\tau_1 \\ \alpha_2, & n(\tau_1+\tau_2)+\tau_1 < t \le (n+1)(\tau_1+\tau_2) \\ & (n = 1, 2, ...) \end{cases}, \tag{4}$$

where $\alpha_f(t)$ is a term of the parametric force and time-varying. The parametric force operation can be described as follows: in the time interval $n(\tau_1+\tau_2) < t \le n(\tau_1+\tau_2)+\tau_1$, the system is driven by a parameter α_1 during the duration τ_1;

while in the interval $n(\tau_1 + \tau_2) + \tau_1 < t \leq (n + 1)(\tau_1 + \tau_2)$, the system is driven by a parameter α_2 during the duration τ_2. Namely, in this system, two kinds of parameters are replaced alternately by the number of updates. In this study, we assume $\tau_1 = \tau_2 = \tau$ for simplicity. Figure 1 show a example of a return map of the parametrically forced logistic maps. For the original logistic map, two-periodic solution is observed for $\alpha = 3.0$. While, three-periodic solution is observed for $\alpha = 3.83$. These two solutions are periodic, whereas in the logistic map involving parametric force, a solution is chaotic as shown in Fig. 1 when the parameters α_1 and α_2 are set 3.0 and 3.83. Namely, chaotic solution can be observed in the combination of two parameters that generate two kinds of periodic solutions. In the following, the parameter values are fixed as $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$.

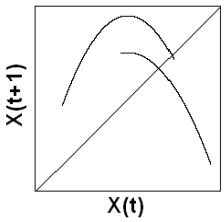

Figure 1: Return maps of parametrically forced logistic maps for $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$.

3. Synchronization

Various synchronization phenomena in the globally coupled parametrically forced logistic map are observed for one control parameter ε which is a coupling intensity when the number of coupling is three. Figure 2 shows examples of synchronization phenomena. In Fig. 2, upper figures show the return maps and lower figures show the phase differences between the maps. The synchronization phenomena fall into four general categories, which are asynchronous, self-switching of synchronization as shown in Fig. 2(a), synchronization of two among the three maps as shown in Fig. 2(b) and synchronization of all the maps as shown in Fig. 2(c).

4. Self-switching phenomenon

The self-switching phenomenon of synchronization is observed when the ε is set around 0.04. The phenomenon is that two among the three maps are synchronized and the combination which maps are coupled changes with time. Figure 3 shows time series of differences of $x(t)$ between two maps. Areas where the amplitudes of the time series are small correspond to in-phase synchronization in the figure. Additionally, the difference between synchronized maps is smaller than 0.1.

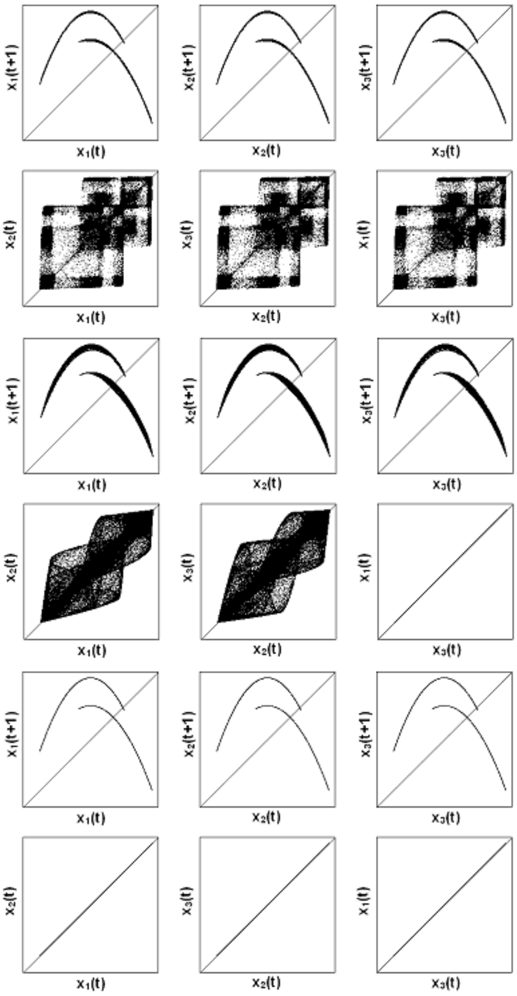

Figure 2: Synchronization of three chaos. $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$. (a) $\varepsilon = 0.040$. (b) $\varepsilon = 0.170$. (c) $\varepsilon = 0.200$.

In Fig. 3(a), firstly, map 1 and map 2 are synchronized. However, after a time, the synchronous state breaks up and map 1 and map 3 are synchronized. As seen above, the synchronous states switch with time in sequence.

4.1. Sojourn time

A sojourn time of the self-switching is related to ε. The sojourn time is short when ε is small, whereas the sojourn time is long when ε is big as shown Figs. 3(a) and (b). Figure 4 shows sojourn time of the self-switching between $\varepsilon = 0.04$ and $\varepsilon = 0.0405$. The sojourn time increases exponentially with ε.

4.2. Second iterate of the map

To investigate the self-switching phenomenon of synchro-

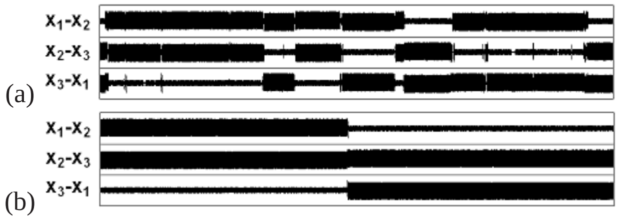

Figure 3: Time series of differences between two maps. $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$. (a) $\varepsilon = 0.0400$. (b) $\varepsilon = 0.0415$.

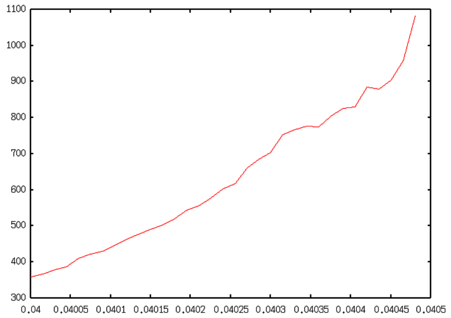

Figure 4: Sojourn time of the self-switching. $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$. Horizontal axis: ε, vertical axis: the mean value of the sojourn time.

nization, a second iterate of the map is used. $x_i(1)$ is calculated with $\alpha_f(0) = \alpha_1$ in the first update. After that, $x_i(2)$ is calculated with $\alpha_f(1) = \alpha_2$. The above calculations are alternately repeated. Thus, a second iterate of the map for each odd number of t is different from a second iterate of the map for each even number. Here, The two kinds of the second iterate of the map are termed as:

MAP-A: the second iterate of the map for each even number of t,

MAP-B: the second iterate of the map for each odd number of t.

Return maps of the MAP-A are shown in Fig. 5(a). From the figure, it can be seen $x_i(2t)$ is projected in $0.41 < x_i(2t) < 0.96$. On the other hand, return maps of the MAP-B are shown in Fig. 5(b). From the figure, it can be seen that $x_i(2t-1)$ is projected in $0.12 < x_i(2t-1) < 0.75$.

4.3. Switching values

In this subsection, condition that synchronization state switches to other synchronization states in the self-switching are investigated. When map 1 and map 2 are synchronized, the synchronization state change to other synchronization states, synchronization of map 2 and map 3 or synchronization of map 1 and map 3, for combination of x_1, x_2 and x_3. Here, we assume that $x_i(0)$ is $x_i(t)$ before the switching for convenience. Fixing difference between $x_1(0)$ and $x_2(0)$, combinations of $x_1(0)$ and $x_3(0)$ that the synchronization state switch to other synchronization states are shown in Fig. 6. If the difference between $x_1(0)$ and $x_2(0)$ is smaller than 0.05, the switching of synchronization state does not occur. In Fig. 6, the upper figures show combinations of $x_1(0)$ and $x_3(0)$ that the switching occurs on MAP-A. While, the right figures show the combinations of $x_1(0)$ and $x_3(0)$ on MAP-B. In the figures, the green area shows combinations of $x_1(0)$ and $x_3(0)$ that map 1 and map 3 are synchronized after the switching. While the red area shows combinations of $x_1(0)$ and $x_3(0)$ that map 2 and map 3 are synchronized after the switching. The switching is easy to occur in MAP-B, by comparison with the case of MAP-A. Moreover, with increasing the difference between x_1 and x_2, the combinations of x_1 and x_3 that the switching occurs is increased.

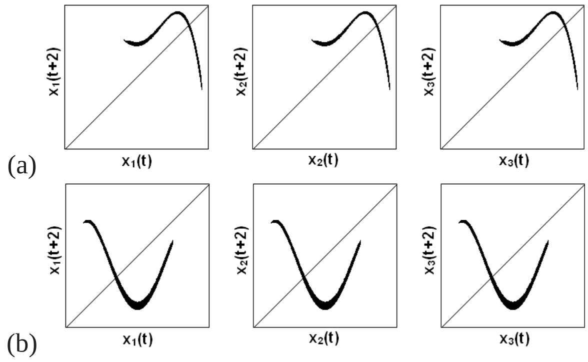

Figure 5: Return maps of the second iterate of the map . $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$. (a) Second iterate of the map for each even number of t. (b) Second iterate of the map for each odd number of t

5. Conclusion

In this study, the self-switching phenomenon of synchronization in globally coupled parametrically forced logistic maps was investigated. In the investigation of the relationship between sojourn time and the coupling intensity, it was observed that the sojourn time increases exponentially with the coupling intensity. In the investigation of the condition that a synchronization state switches to other synchronization states, it was observed that the combinations which the switching of synchronization state occurs is increased with increasing a difference between synchronized maps.

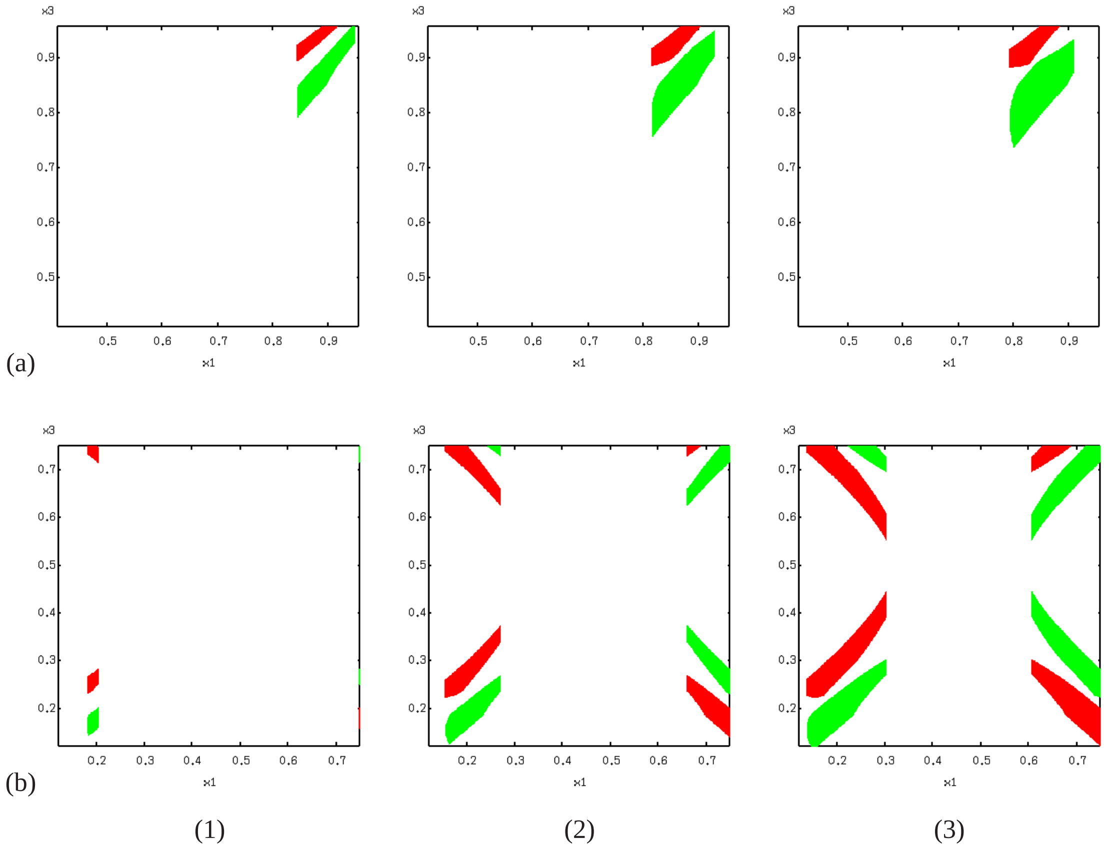

Figure 6: Combinations of x_1 and x_3 that switching of synchronization state occurs for $\alpha_1 = 3.0$, $\alpha_2 = 3.83$ and $\tau = 1$. (a) t is even number. (b) t is odd number. (1) $x_2(0) - x_1(0) = 0.05$. (2) $x_2(0) - x_1(0) = 0.07$. (3) $x_2(0) - x_1(0) = 0.09$.

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