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Synchronization in Coupled van der Pol Oscillators Involving Periodically Forced Capasitors

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Abstract—In this study, synchronization phenomena observed from coupled van der Pol oscillators involving periodically forced capacitors are investigated. Firstly, we confirm effects of parametric excitation on synchronization. Next, we investigate behavior of parametric excitation in coupled system. By carrying out computer calculations for two to five subcircuits cases, various interesting synchronization phenomena of chaos are confirmed.

1. Introduction

Synchronization is one of the fundamental phenomena in nature, and one of typical nonlinear phenomena. Therefore, studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. However, issues that should be investigated for synchronization remain in existence in spite of many researching. In particular, it is necessary to investigate synchronization phenomena in special conditions. There is parametric excitation that increases amplitude of oscillation by periodic changing of a parameter in the system. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In the past we have investigated synchronization phenomena in coupled van der Pol oscillators involving time-varying inductors [1]. In this study, for specify the effect of parametric excitation on synchronization, we focus on a parametric excitation which is generated by a capacitor periodically forced. Then, we investigate synchronization phenomena observed from coupled van der Pol oscillators involving periodically forced capacitors. By carrying out computer calculations for two to five subcircuits cases, various interesting synchronization phenomena of chaos are confirmed.

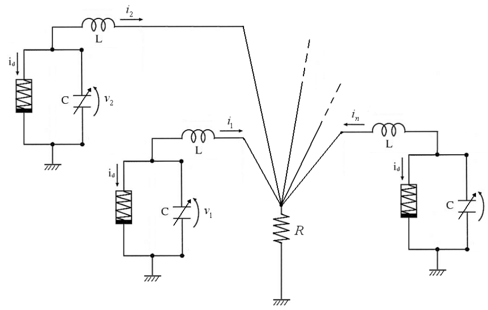

Figure 1: Circuit model.

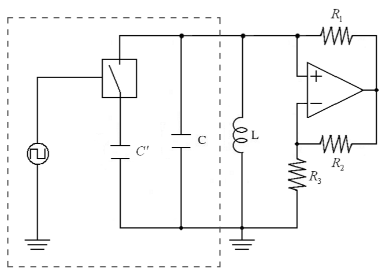

Figure 2: Subcircuit corresponds to van der Pol oscillator involving periodically forced capasitor.

2. Circuit model

The circuit model used in this study is shown in Fig 1. In our system n identical parametrically excited van der Pol oscillators are coupled by one resistor R. The subcircuit which is parametrically excited van der Pol oscillator consists of an inductor, a nonlinear resistor and a time-varying capacitor, which is periodically forced, and realized as Fig. 2. This circuit exhibits bifurcation phenomena whose diagram is shown in Fig. 3. Figure 4 shows examples of attractors obtained from the subcircuit. In Fig. 2, the dashed-line box area corresponds to the time-varying capacitor which exhibits periodic rectangular characteristics by an externally switch which gives periodic force. The characteristics of the time-varying capacitor are given as following equation.

$$C = C_0\gamma(t). \qquad (1)$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 5, and its amplitude and angular frequency are termed α and

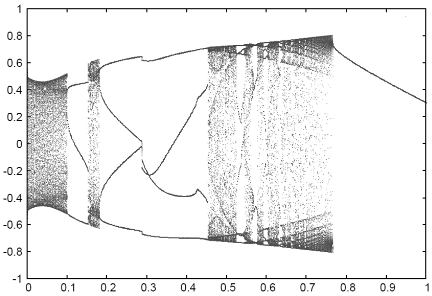

Figure 3: One-parameter bifurcation diagram for $\alpha = 0.8$, $\omega = 1.1$ and varying ε.

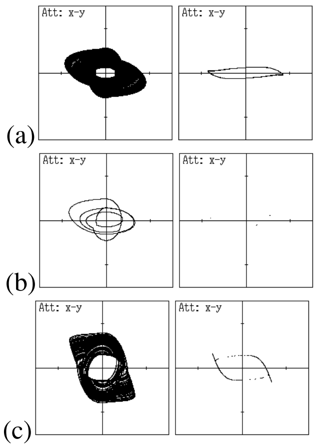

Figure 4: Attractors and Poincaré maps. $\alpha = 0.8$ and $\omega = 1.1$. (a) $\varepsilon = 0.10$. (b) $\varepsilon = 0.12$. (c) $\varepsilon = 0.75$.

ω, respectively. The $v - i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \quad (2)$$

By changing the variables and the parameters,

$$\begin{aligned} t &= \sqrt{L_0 C}\tau, \ \ v_k = \sqrt{\frac{g_1}{g_3}} x_k, \ \ \delta = \sqrt{\frac{C}{L_0}} R, \\ i_k &= \sqrt{\frac{g_1}{g_3}} \sqrt{\frac{C}{L_0}} y_k, \ \ \varepsilon = g_1 \sqrt{\frac{L_0}{C}}, \end{aligned} \quad (3)$$

the normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_k}{d\tau} = \dfrac{1}{\gamma(\tau)} \left\{ \varepsilon (x_k - x_k^3) - y_k \right\} \\ \dfrac{dy_k}{d\tau} = x_k - \delta \displaystyle\sum_{j=1}^{n} y_j. \end{cases} \quad (4)$$

In the following computer calculations, the parameter values are fixed as $\varepsilon = 0.75$, $\alpha = 0.80$ and $\omega = 1.10$ and (4) is calculated by using the Runge-Kutta method with step size $\Delta t = 0.01$.

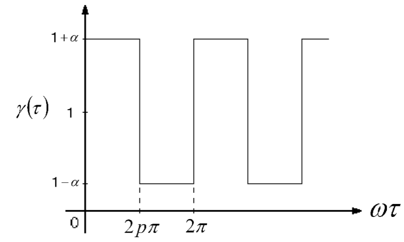

Figure 5: Function relating to parametrically excitation.

3. Synchronous effects of parametric excitation

Before investigating synchronization phenomena in the coupled parametrically excited van der Pol oscillators, it is necessary to investigate synchronization phenomena in non-coupled chaotic circuits which are identically forced. Namely, it is necessary to investigate synchronous effects that all capacitors of the chaotic circuits are changed at the same time. In this section, for investigate the synchronous effects we focus on the two identical chaotic circuits. Figure 6 shows chaotic attractors, time series and rectangular wave corresponding to the periodically forced capacitor. In Fig. 6, a chaotic attractor, shown as Fig. 6(a), consists of upper rotationg orbit (Fig. 6(b)) and lower rotating orbit (Fig. 6(c)) that corresponding to the region (b') and the region (c') in time series, respectively. From this figure, a period of the chaotic subcircuit is determined by the periodic force and according to its period. Additionally, the upper rotating orbit and the lower rotating orbit are symmetric about the coordinate origin. From the above things, the periodically forced chaotic circuits can be synchronized at the in phase or at the opposite phase or be self-switching at these in and opposite phase and it is decided by overlapping of the upper rotating areas and lower rotating areas between the two chaotic subcircuits. It means, sampling synchronization states every periods, the chaotic subcircuits which the capacitors are governed by one force seem to be synchronized at the in-phase or at the opposite-phase.

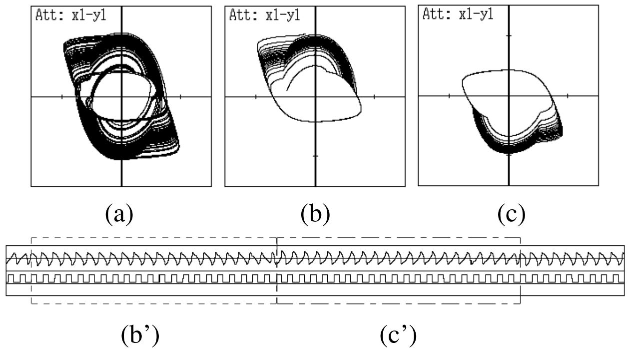

Figure 6: Attractors, time series and rectangular wave relating to parametrically excitation $\varepsilon = 0.75$, $\alpha = 0.80$ and $\omega = 1.10$.

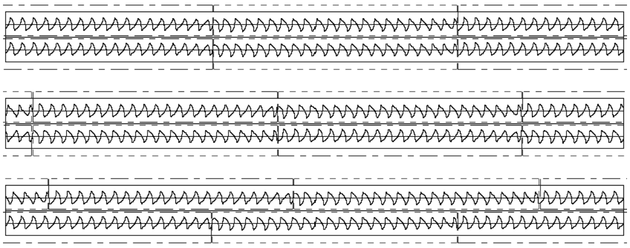

Figure 7: Three types of synchronization by parametric excitation.

4. Synchronization phenomena

In this section, we investigate synchronization phenomena by carrying out computer calculations for two to five subcircuits cases.

4.1. Two subcircuits case

In this subsection, we consider the case of $N = 2$, namely only two van der Pol oscillators involving periodically forced capacitors are coupled by one resistor. In this case, opposite-phase synchronization of two subcircuits is observed. Figure 8 shows computer calculated results. As shown in the figure, with γ increases, chaotic signals obtained from two subcircuits become to be synchronized at the opposite-phase. When $\gamma = 0.01$, two chaotic signals seem to be completely synchronized at the opposite-phase.

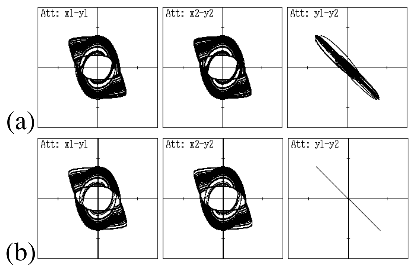

Figure 8: Synchronization of two chaos. $\varepsilon = 0.75$, $\alpha = 0.80$ and $\omega = 1.10$. (a) $\gamma = 0.001$. (b) $\gamma = 0.01$.

4.2. Three subcircuits case

In this subsection, we consider the case of $N = 3$. In this case, self-switching phenomena of in-phase synchronization and opposite-phase synchronization can be observed. Figure 9 shows the computer calculated result. In Fig. 9, the upper figure, the middle figure and the lower figure show attractors of the subcircuits, phase differences between subcircuits and time series of the subcircuits, respectively. As shown in time series of Fig. 9, the synchronization states which subcircuits are synchronized at the in-phase or opposite-phase are switching with time.

Three subcircuits generate the same shape of chaotic attractors in Fig. 9 for long time observation. However, in short time observation during synchronization states are steady, the attractors are not the same. Two attractors synchronized at the in-phase are smaller than the other one. Figure 10 shows a computer calculated simulation for observed attractors in some steady synchronization states when initial values of two subcircuits are set as the same values and initial values of the rest subcircuit is set as the almost inverse about the coordinate origin. In the figure, two attractors synchronized at the in-phase are also smaller than the other one.

The coupled chaotic circuits used in this study are expected to minimize the current through the coupling resistor. Additionally, subcircuits are synchronized at the in-phase or the opposite-phase by the effect of parametric excitation. Thus, the two attractors synchronized at the in-phase are small for minimizing current through the coupling resistor. However, chaos attractor does not have periodic orbit. Additionally, three subcircuits can not be synchronized completely without the special case that all initial values are same. Thus sometimes two solutions of the subcircuits synchronized at the opposite-phase approach each other. In that time, synchronization states switch to other synchronization state.

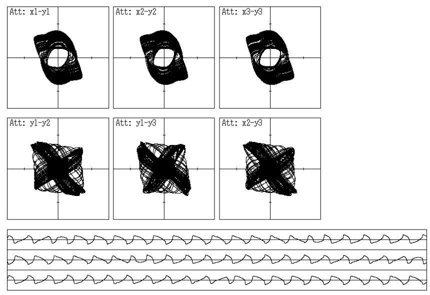

Figure 9: Self-switching phenomena of in-phase synchronization and opposite-phase synchronization. $\varepsilon = 0.75$, $\alpha = 0.80$, $\omega = 1.10$ and $\gamma = 0.10$.

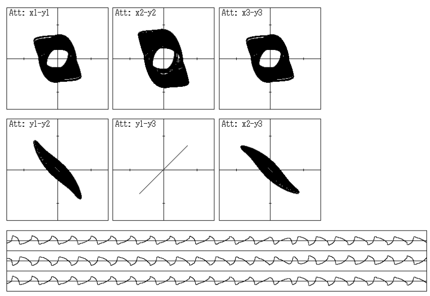

Figure 10: Synchronization of three chaos when initial values of two subcircuits are set as same values. $\varepsilon = 0.75$, $\alpha = 0.80$, $\omega = 1.10$ and $\gamma = 0.10$.

4.3. Four subcircuits case

In this subsection, we consider the case of $N = 4$. In this case, two pairs of the opposite-phase synchronization can be observed. Figure 11 shows the computer calculated results. In Fig. 11, (a) and (b) are obtained by using different initial values. In Fig. 11(a), two of four subcircuits are synchronized at the opposite phase. Also the remain two subcircuits are synchronized at the opposite phase. Then, two opposite-phase synchronizations switch between in and opposite phases. While, in Fig. 11(b), the synchronization state between two opposite-phase synchronizations is steady. For instance, subcircuits 2 and 3 seem to be synchronized at the in-phase. The synchronization states of four coupled subcircuits become either the above two state because of parametric excitation that all subcircuits are synchronized at in-phase or opposite-phase locally.

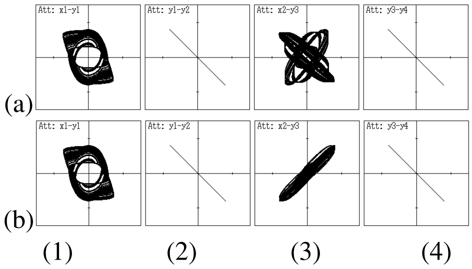

Figure 11: Synchronization of four chaos. $\varepsilon = 0.75$, $\alpha = 0.80$, $\omega = 1.10$ and $\gamma = 0.70$. (1) x_1 versus y_1. (2) y_1 versus y_2. (3) y_2 versus y_3. (4) y_3 versus y_4.

4.4. Five subcircuits case

In this subsection, we consider the case of $N = 5$. Figure 12 shows the computer calculated results. In this case, two kinds of synchronization phenomena are observed. The one is self-switching phenomena of all subcircuits (see Fig. 12(a)). It can be observed when coupling intensity γ is small. In Fig. 12(a), two subcircuits are synchronized at the in-phase, while the remain three subcircuits are synchronized at the in-phase. Then, the above two groups are synchronized at the opposite-phase. Though, as time advances one of the subcircuits which belongs to the three subcircuits group switch to another group. By the way, increasing coupling intensity, another synchronization can be observed as shown in Fig. 12(b). In Fig. 12 (b), subcircuits 1 and 5 are synchronized at the in-phase and subcircuits 2 and 4 are also synchronized at the in-phase. Then, the above two pairs are synchronized at the opposite-phase. Besides, synchronization states between remain subcircuit and others switch from the in-phase to the opposite-phase or vice versa with time.

5. Conclusions

In this study, we have investigated synchronization phenomena in coupled van der Pol oscillators involving periodically forced capacitors. Firstly, we confirm effects of parametric excitation on synchronization that all subcircuits are synchronized at in-phase or opposite-phase locally. Next, we investigate behavior of parametric excitation in coupled system. By carrying out computer calculations for two to five subcircuits case, various interesting synchronization phenomena of chaos are confirmed. In two subcircuits case, opposite-phase synchronization is observed. In three subcircuits case, self-switching of in-phase and opposite-phase synchronization is observed. In four subcircuits case, two pairs of opposite-phase synchronization is observed. In five subcircuits case, two different types of synchronization phenomena are observed. One of the synchronization phenomena is self-switching of all subcircuits. Another one is in and opposite phase synchronization and self-switching.

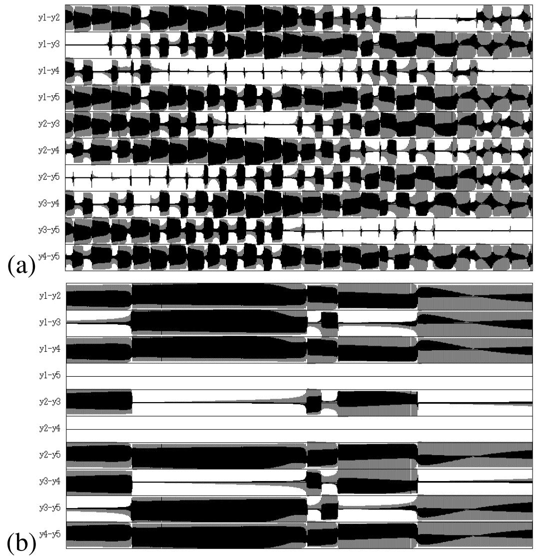

Figure 12: Synchronization of five chaos. $\varepsilon = 0.75$, $\alpha = 0.80$ and $\omega = 1.10$. (a) $\gamma = 0.17$. (a) $\gamma = 0.50$.

Acknowledgments

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References

[1] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, "Synchronization and graph topology," Int. J. Bifurcation and Chaos, vol.15, no.11, pp.3423-3433, Nov. 2005.

[2] J. Cosp, J. Madrenas, E. Alarcon, E. Vidal and G. Villar, "Synchronization of nonlinear electronic oscillators for neural computation," IEEE Trans. Neural Networks, vol.15, no.5, pp.1315-1327, Sep. 2004.

[3] C. Hayashi, "Nonlinear Oscillations in Physical Systems," Chap. 11, McGraw-Hill, New York (1964).

[4] C. Hayashi, M. Abe, K. Oshima and H. Kawakami, "The Method of Mapping As Applied To the Solution for Certain Types of Nonlinear Differential Equations," Proc. of the Ninth International Conference on Nonlinear Oscillations, pp. 1-8, 1981.

[5] M. Inoue, "A Method of Analysis for the Bifurcation of the Almost Periodic Oscillation and the Generation of Chaos in a Parametric Excitation Circuit," Trans. of IEICE, vol. J68-A, no. 7, pp. 621-626, 1985.

[6] H. Kumeno and Y. Nishio, "Synchronization Phenomena in Coupled Parametrically Excited van der Pol Oscillators," Proceedings of International Symposium on Nonlinear Theory and its Applications (NOLTA'08), pp. 128-131, Sep. 2008.