

Phase Synchronization in a Coupled Non-Uniform Network of Multi-State Chaotic Oscillators

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Abstract—This paper presents several phase synchronization modes of multi-state chaotic oscillators coupled as a non-uniform network system. They have been known several phase patterns in several oscillators. Each chaotic circuit which used in this paper can individually behave both chaotic and two periodic oscillations in the same parameters asynchronously. In this study, such the coupled chaotic circuits are proposed and classifications of phase synchronization modes are investigated. In numerical simulation, many types of phase synchronization modes could be confirmed.

1. Introduction

Nonlinear dynamics on coupled chaotic oscillators is considerable interesting for a wide variety of systems in several scientific fields and applications. Many types of coupled systems have been widely studied in order to clarify inherent features and many researchers have already proposed and investigated them. Coupled chaotic systems are as one of them which have several varieties of interesting behavior with emergent properties. The dynamics of chaotic multimode oscillations or chaotic itinerancy on several coupled systems is still considerable interest from the viewpoint of both natural scientific fields and several applications. They have been confirmed in several systems; e.g., coupled van der Pol oscillators [1], coupled chaotic systems [2], and so on. Phase synchronization and pattern dynamics are also interesting for several engineering applications. On the other hand, many types of chaotic systems and circuits have already been proposed and investigated in detail. As interesting phenomena, there are famous chaotic attractors such a double-scroll family [3], n -double scroll [4]–[6] and scroll grid attractors [7]. If the active elements including in the systems have complexity constructed by compound some nonlinear elements, it can be easily considered that they yield several interesting features. The circuit which can individually behave both chaotic or periodic oscillations in the same parameters had been shown [8]. This type of circuit was called a Multi-State Chaotic Oscillator/Circuit (abbr. MSCO/MSCC). Multimode oscillations in coupled two or more multi-state chaotic circuits had also been investigated [9]–[11]. Furthermore, we have

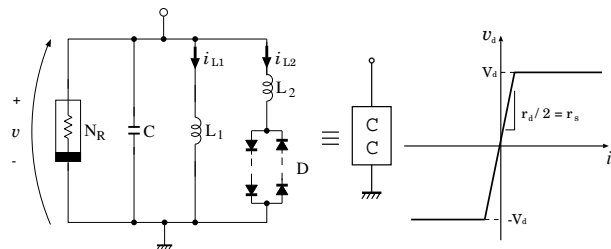


Figure 1: Multi-state chaotic oscillator with piecewise linear resistors N_R and v - i characteristic of the diode D

been reported several phase synchronization on a uniform network system [12]. However they have been treated only a uniform network of a large scale system. It is very important to investigate a non-uniform system of the coupled network in the sense of solving the several phenomena of the natural world.

In this study, several phase patterns and multimode asynchronous oscillations on the coupled MSCOs are investigated. There is a typical three dimensional autonomous chaotic system, which consists of three memory elements, some diodes and designed negative resistors. We substitute a symmetrical continuous segments piecewise linear resistor for the negative active resistor including in the original chaotic circuit. This proposed circuit can behave both chaotic and periodic oscillations in the same parameters when we supply with different initial conditions. Several phase synchronization and classification of several phase patterns in some MSCOs coupled as a non-uniform network are investigated. It will be shown that several types of phase synchronization modes can be confirmed asynchronously, but all parameter settings of each circuit are the same.

2. Model Description

The circuit shown in Fig. 1 is modified chaotic circuit from a well-known three dimensional chaotic circuit [13]. The original circuit consists of three memory elements, some diodes and designed negative resistors. It is well known that it can behave as Rössler type chaotic motions. We substitute a symmetrical piecewise linear resistor for the negative active resistor including in the original chaotic

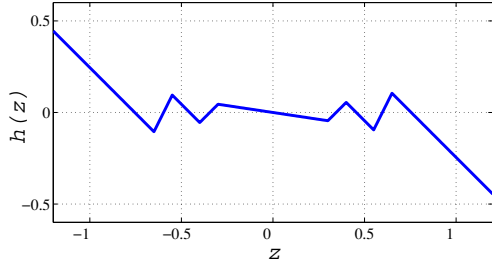


Figure 2: Designed sawtooth nonlinear resistor N_R in the circuit. $h(z): \{p_1, p_2, p_3, p_4\} = \{0.65, 0.55, 0.40, 0.30\}$, $\{m_0, m_1, m_2, m_3, m_4\} = \{-1.0, 2.0, -1.0, 1.0, -0.15\}$

circuit. Further this circuit possesses another symmetrical piecewise nonlinear resistor with respect to the origin.

An MSCO which is modified from the original circuit as shown in Fig. 1 is a basic circuit as a subcomponent of a coupled network model. In this study, we substitute a symmetrical continuous piecewise linear resistor for the negative active resistor including in the original chaotic circuit. The piecewise linear resistor can be easily constructed by combining some components in parallel [8].

By changing the following variables and parameters as follows

$$\begin{aligned} i_{L1} &= \sqrt{\frac{C}{L_1}} V_d x, \quad i_{L2} = \sqrt{\frac{C}{L_1}} V_d y, \\ v &= V_d z, \quad t = \sqrt{L_1 C} \tau, \quad \text{“.”} = \frac{d}{d\tau}, \\ \beta &= \frac{L_1}{L_2}, \quad \gamma = g \sqrt{\frac{L_1}{C}}, \quad \delta = r_s \sqrt{\frac{C}{L_1}} \end{aligned} \quad (1)$$

where g is a linear negative conductance value of N_R if we consider the negative resistor as an ideal element. Consider that the part of negative resistance N_R in Fig. 1 replaces to the function $h(z)$ represented by a voltage source z as canonical form with 9-segments as shown in Fig. 2. When we chose the threshold voltage V_d for a normalized parameter, then the circuit equations can be normalized and rewritten as follows.

$$\begin{cases} \dot{x} = z \\ \dot{y} = \beta \{z - f(y)\} \\ \dot{z} = -(x + y) - h(z) \end{cases} \quad (2)$$

where

$$f(y) = \frac{1}{2} \{ |\delta y + 1| - |\delta y - 1| \} \quad (3)$$

and

$$h(z) = m_0 \gamma^* z + \frac{\gamma^*}{2} \left\{ \sum_{k=0}^K (m_k - m_{k+1}) \{ |z - p_{k+1}| - |z + p_{k+1}| \} \right\} \quad (4)$$

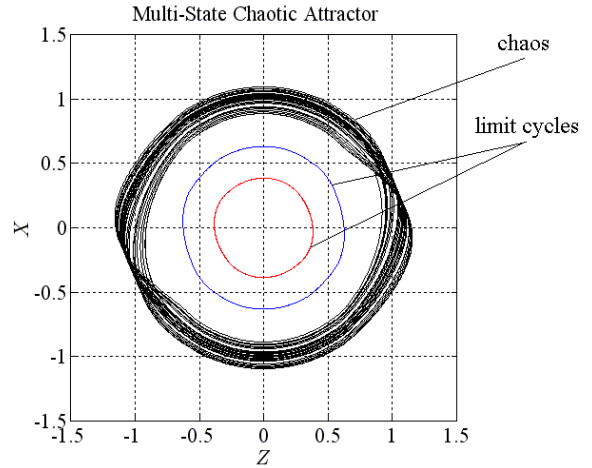


Figure 3: Attractor drawing onto the $z - x$ plane for the parameters $\beta = 10.0$, $\gamma^* = 0.78$ and $\delta = 100$

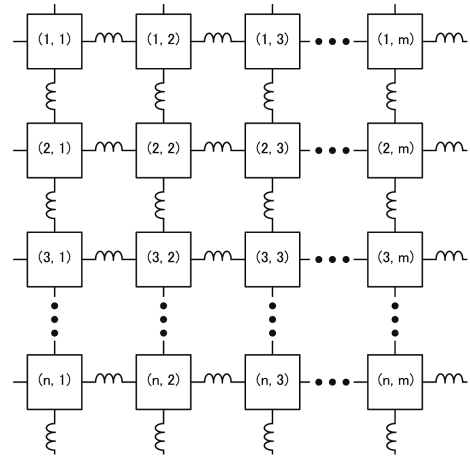


Figure 4: A coupled network model which each circuit is connected to four neighbors

$f(y)$ is a function of the current y and $h(z)$ is a function of the voltage z , respectively. The function $h(z)$ which is designed for several segment piecewise linear as symmetric with respect to the origin. The parameter γ^* is used for a basic common value, hence the values $m_k (k = 0, 1, 2, \dots, K)$ mean magnitude of the slope to the ratio for γ^* .

Figure 3 also shows a typical chaotic attractor obtained for the parameters $\beta = 10.0$, $\gamma^* = 0.78$, $\delta = 100$, with piecewise linear characteristics realized by breakpoints $p_1 = 0.65$, $p_2 = 0.55$, $p_3 = 0.40$, $p_4 = 0.30$, slopes $m_0 = -1.0$, $m_1 = 2.0$, $m_2 = -1.0$, $m_3 = 1.0$ and $m_4 = -0.15$. We can confirm that chaotic and two periodic attractors coexist in the circuit. This means coexistence of both chaos and two different size of limit cycles in the same parameters.

3. Simulation for coupled MSCOs

In this section, the model of MSCOs coupled by inductors as a non-uniform network system are investigated. For

example of a coupled network, now let us consider the coupled MSCOs model which combined number of $N \times M$ chaotic circuits are connected by inductors to neighbors' circuit as a network structure shown in Fig. 4. In this study, the coupling strength of the network is a non-uniform that it is different of the coupling strength between chaotic oscillators in the previous study [12]. The circuit index is defined as (i, j) . It is note that every chaotic circuit is composed by all the same parameters and connected to four neighbors circuits. The circuit on the edge of this coupled network is connected to an opposite side circuit, it seems like a distribution on surface of the torus structure. By the similar way described before, the circuit equations of coupled MSCOs can be normalized by the variables (1) with a new basic parameter $\alpha = L_1/L_0$. Here, as we consider a non-uniform network, we can change the parameter α to $\alpha_{(i,j)}^*$. Therefore, the whole circuit equations can be rewritten as follows.

$$\left\{ \begin{array}{l} \dot{x}_{(i,j)} = z_{(i,j)} \\ \dot{y}_{(i,j)} = \beta(z_{(i,j)} - f(y_{(i,j)})) \\ \dot{z}_{(i,j)} = \left(\begin{array}{l} \alpha_{(i,j)}^w (x_{(i,j-1)} - x_{(i,j)}) \\ + \alpha_{(i,j)}^n (x_{(i+1,j)} - x_{(i,j)}) \\ + \alpha_{(i,j)}^e (x_{(i,j+1)} - x_{(i,j)}) \\ + \alpha_{(i,j)}^s (x_{(i-1,j)} - x_{(i,j)}) \\ - (x_{(i,j)} + y_{(i,j)}) - h(z_{(i,j)}) \end{array} \right) \end{array} \right. \quad (5)$$

where functions $f(y)$ and $h(z)$ are similar to (3) and (4), respectively. The parameter $\alpha_{(i,j)}^*$ is the coupling strength of between both oscillators. The symbol * means a target which corresponds to left(=w), upper(=n), right(=e) and lower(=s), respectively.

We show some computer calculation results by using 4-th order Runge–Kutta method with time step size $\Delta t = 0.001$ for the circuit equations in some cases of (N, M) as follows. The parameters of each circuit are the same in the Sec. 2. The initial conditions of each circuit are given at random. Figure 5 shows some typical results obtained from computer simulation in some cases of coupled number (N, M) . Figure shows both uniform and non-uniform network of coupling strength. The matrices as shown in Fig. 6 also show all parameters of coupling strength α which was made to be added a uniform random distributed number of a section $[-0.3, 0.3]$.

We can confirm complex and chaotic synchronization phenomena on the coupled system. Because it cannot be shown all simulation results, we can several types of complex and interesting synchronization phenomena in the same parameters, i.e., in-phase synchronization, anti-phase synchronization, clustering of phase synchronization, phase locking and other types. Several phase synchronization modes are coexisting in spite of the same parameters. In the camera-ready paper, we will show that stochastic analysis for these several synchronization modes are investigated.

4. Conclusions

In this study, we have investigated several synchronization modes in coupled multi-state chaotic oscillators on a non-uniform network system. Coexistence of several types oscillation modes have been confirmed in the coupled MSCOs. Several interesting chaotic phenomena of spatio-temporal behavior have been observed in the coupled network system. On a large scale of coupled chaotic oscillators such a small-world and a scale-free network, we consider that several types of complex behavior are expected to yield novel applications and inherent emergent properties in the natural systems.

Acknowledgment

This work was partly supported by The Hiraio Taro Foundation of the Konan University Association for Academic Research, Japan.

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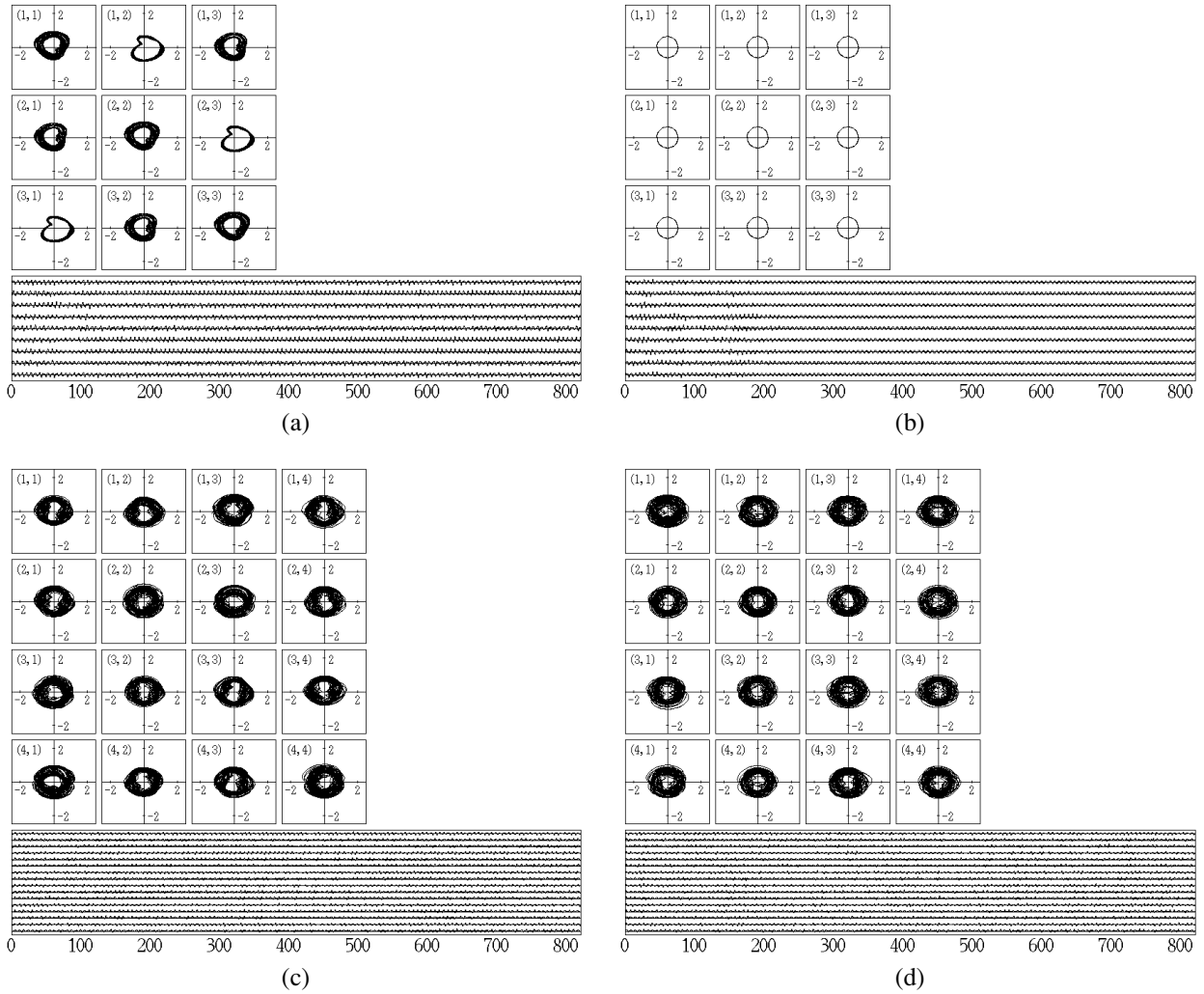


Figure 5: Examples of simulation results obtained from coupled MSCOs for $\alpha = 0.50$ (in case of the uniform network), $\beta = 10.0$, $\gamma^* = 0.78$, $\delta = 100$. $h(z): \{p_1, p_2, p_3, p_4\} = \{0.65, 0.55, 0.40, 0.30\}$, $\{m_0, m_1, m_2, m_3, m_4\} = \{-1.0, 2.0, -1.0, 1.0, -0.15\}$. Size (N, M) of the network: (a) uniform network of (3,3), (b) non-uniform network of (3,3), (c) uniform network of (4,4) and (d) non-uniform network of (4,4) with waveform of each oscillator

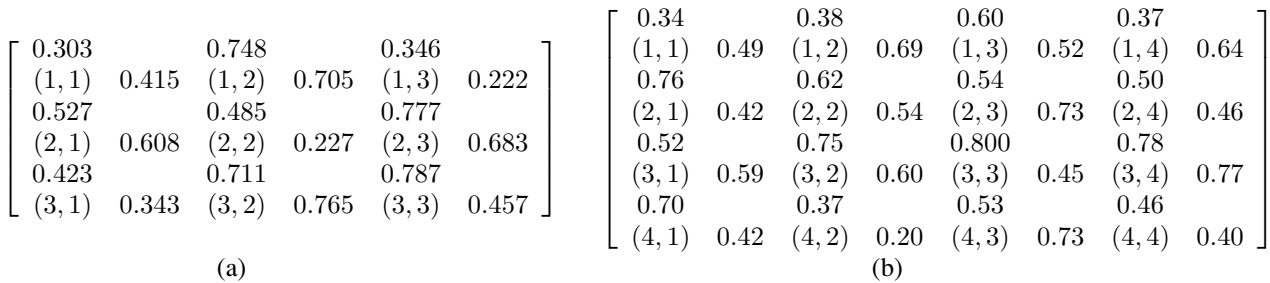


Figure 6: Matrices of the coupling strength for (a) 3×3 and (b) 4×4 in the simulation results of Figure 5