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Complex Pattern in a Chain of Coupled Maps Based on Neuron Model with Space-Varying Coupling

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***Abstract*— Biological neurons are able to exhibit spiking and bursting behavior. The interesting phenomena is ensembles of neurons by synchronization. We study synchronization phenomena in a chain of 2-dimensional maps based on neuronal model with a space-varying coupling. We realize the space-varying coupling by normal distribution function. In the case of the space-varying coupling, wave propagation and complex pattern can be observed.**

I. Introduction

Generally, complex dynamical phenomena can be observed in networks formed by many elements with nonlinearity. Coupled Map Lattice (CML) has proposed by Kaneko and Bunimovich [1]-[5], to use as general models for the complex high-dimensional dynamics, such as biological systems, networks in DNA, economic activities, neural networks, and evolutions. We can observed the spatio-temporal patterns in CML. Moreover, coupled oscillatory systems can also produce interesting phase patterns, including wave propagation, clustering, and complex phase patterns [6]-[9]. It is very important to make clear this mechanism of these spatio-temporal patterns for understanding complex patterns observed in natural science. Usually, the chaotic maps are used for CML and many interesting spatio-temporal patterns were observed.

Recently, a discrete map for spiking-bursting neural behavior was proposed by Rulkov [10], [11]. Rulkov map in the form of a two-dimensional map can be useful for understanding the dynamical mechanism of oscillators in the large scale networks. And Rulkov map produce spiking-bursting behavior such as real neurons. In this study, we use Rulkov maps applying for CML to study synchronization in large scale networks. Furthermore, we assume that the coupling strength of between the neurons is not simple and the coupling strength plays important role for whole system. We consider that the coupling strength is changed with space. Space-varying coupling is realized by using normal distribution function.

In this study, we study synchronization in a chain of coupled Rulkov maps with space-varying coupling. When the space-varying coupling has positive value, irregular wave propagation can be observed. While, in the case of the negative space-varying coupling, complex patterns are observed.

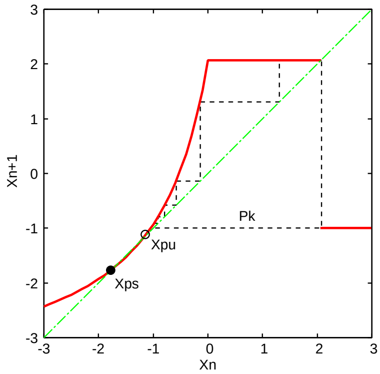


Fig. 1. Rulkov map. The dashed line illustrates a superstable cycle P_k. The stable and unstable fixed points of the map are indicated by x_{ps} and x_{pu}, respectively.

II. Two Coupled Rulkov Maps

Consider the two coupled Rulkov maps [10] as following equation.

$$\begin{aligned} x_{i,n+1} &= f(x_{i,n}, y_{i,n} + \beta_{i,n}), \\ y_{i,n+1} &= y_{i,n} - \mu(x_{i,n}+1) + \mu\sigma_i + \mu\sigma_{i,n}, \end{aligned} \tag{1}$$

$$f(x_n, y) = \begin{cases} \alpha/(1-x_n) + y, & x_n \leqslant 0 \\ \alpha + y, & 0 < x_n < \alpha + y \textit{ and } x_n \leqslant 0 \\ -1, & x_n \geqslant \alpha + y \textit{ or } x_{n-1} > 0, \end{cases} \tag{2}$$

where x and y are the fast and slow dynamical variables, respectively. The coupling between the cells is provided by the current flowing from one cell to the other. This coupling is modeled by

$$\begin{aligned} \beta_{i,n} &= g\beta^e(x_{j,n} - x_{i,n}), \\ \sigma_{i,n} &= g\sigma^e(x_{j,n} - x_{i,n}), \end{aligned} \tag{3}$$

where g denotes the coupling strength. In the numerical simulations the values of the coefficients are set to be equal: $\beta^e = 1.0$ and $\sigma^e = 1.0$. The other parameters has the following values: $\mu = 0.001$, $\alpha = 5.0$ and $\sigma = 0.24$. The coupling between the maps is symmetrical, i.e., $g_{ji} = g_{ij} = g$.

First, we investigate basic synchronization phenomena when the normal coupling without changing the value of coupling is used. When two Rulkov maps are coupled with positive value $g = 0.029$, two bursting waves are synchronized at the in-phase as shown in Fig. 2. While, introduction of negative coupling $g = -0.029$, in this regime of synchronization shows anti-phase (see. Fig. 3). For comparison between the in-phase and the anti-phase states, the oscillation frequency of the anti-phase is faster than the in-phase state.

When three maps are coupled with negative coupling, we observe the three-phase synchronization as shown in Fig. 4.

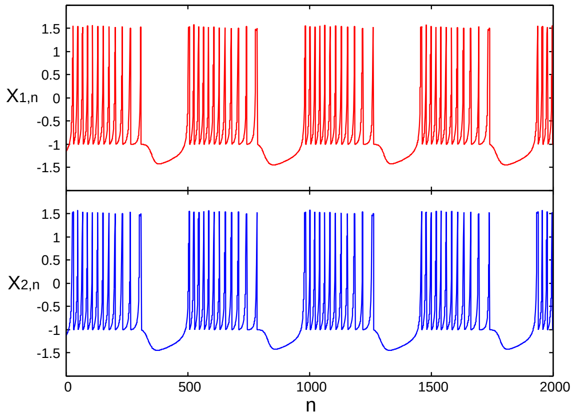


Fig. 2. In-phase synchronization ($g = 0.029$).

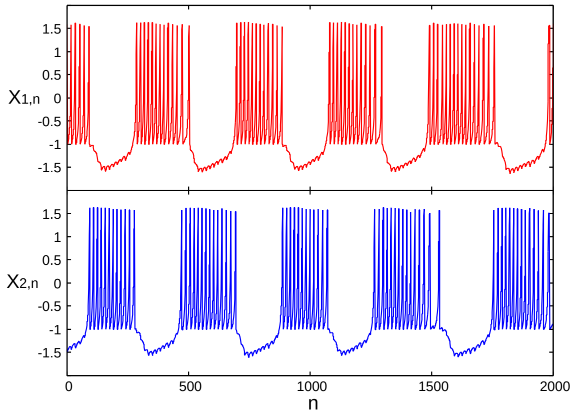


Fig. 3. Anti-phase synchronization ($g = -0.029$).

III. Synchronization in a Chain of Coupled Maps

We consider a chain of coupled maps (see. Fig. 5):

$$\begin{aligned} x_{i,n+1} = \;& f(x_{i,n} x_{i,n-1}, y_{i,n}) \\ & + \frac{1}{2} g(x_{i+1,n} - 2x_{i,n} + x_{i-1,n}), \\ y_{i,n+1} = \;& y_{i,n} - \mu(x_{i,n} + 1) + \mu\sigma_i \\ & + \mu\frac{1}{2} g(x_{i+1,n} - 2x_{i,n} + x_{i-1,n}), \\ & i = 1, ..., N, \end{aligned} \tag{4}$$

where x and y are the fast and slow dynamical variables, respectively. $\mu = 10^{-3}$ and σ_i are the parameters of the individual map and g is the coupling. The function $f()$ has the following form:

$$f(x_n, y_n) = \begin{cases} \alpha/(1 - x_n) + y_n, & x_n \leq 0, \\ \alpha + y_n, & 0 < x_n < \alpha + y_n \\ & and \;\; x_{n-1} \leq 0, \\ -1, & x_n \geq \alpha + y_n \;\; or \;\; x_{n-1} > 0, \end{cases} \tag{5}$$

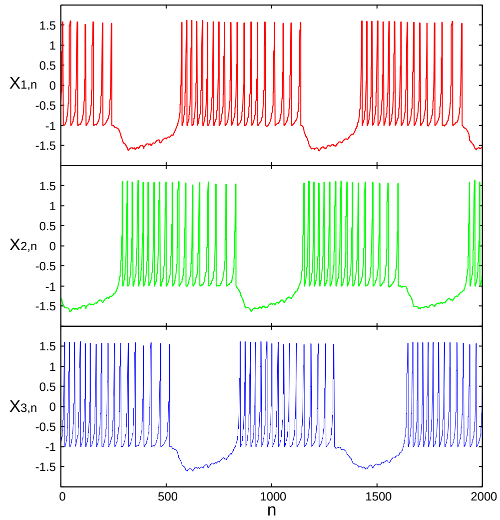


Fig. 4. Three-phase synchronization ($g = -0.029$).

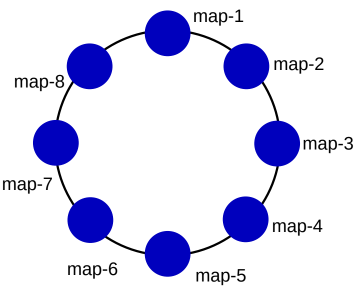


Fig. 5. Conceptual chain of coupled maps. ($N = 8$)

In this simulations, we take $\alpha = 3.5$ and σ_i is set for randomly distributed in the interval [0.15:0.16]. The number of coupled map is set to 100.

A. Normal Coupling

For comparison, we show synchronization of chain of the Rulkov maps when the coupling strength is constant with space. The simulation results of the space-time plot of positive and negative constant coupling are shown in Figs. 6, 7, respectively. The horizontal axis is iteration time n and the vertical axis is space i. The wave propagation can be observed when the space-varying coupling has positive value. In the case of negative coupling, complex pattern is generated.

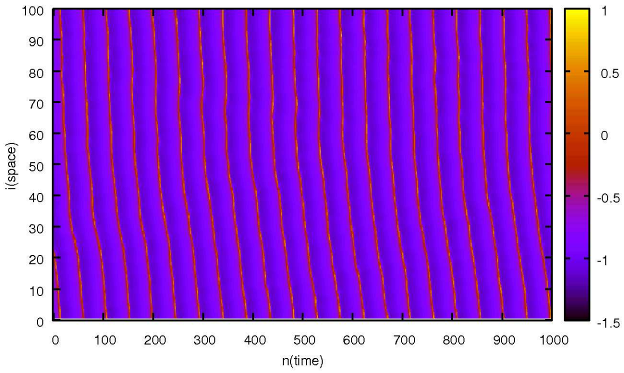


Fig. 6. Space-time plots of x_i with positive constant coupling. ($g = 0.38$)

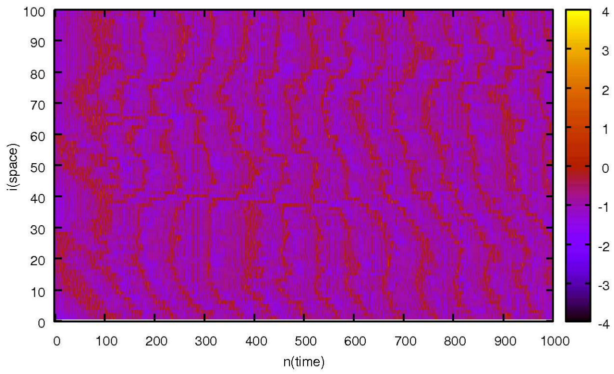


Fig. 7. Space-time plots of x_i with negative constant coupling. ($g = -0.38$)

B. Space-Varying Coupling

In this study, the space-varying coupling is realized by using a normal distribution function. The function for a normal distribution is given by the formula

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{(z-\mu)^2}{2\sigma^2}), \tag{6}$$

where μ is the mean, σ is the standard deviation. One example of space-varying coupling is shown in Fig. 8

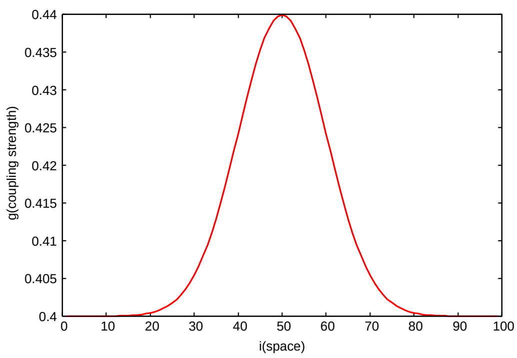
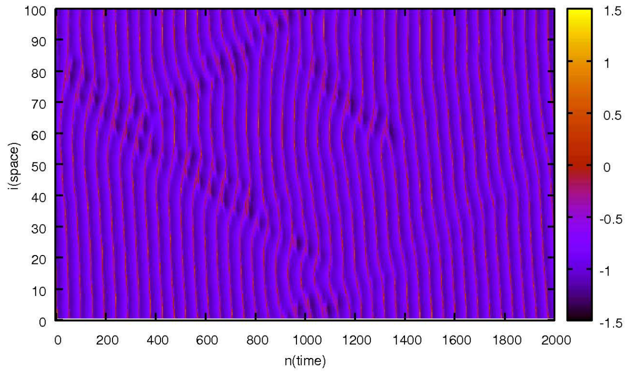
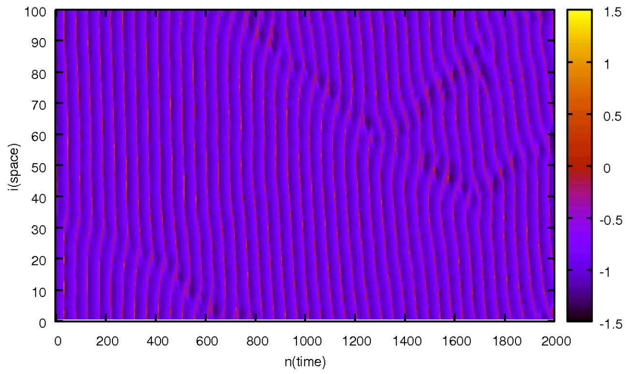


Fig. 8. Space-varying coupling. ($\mu = 50$, $\sigma = 10$)

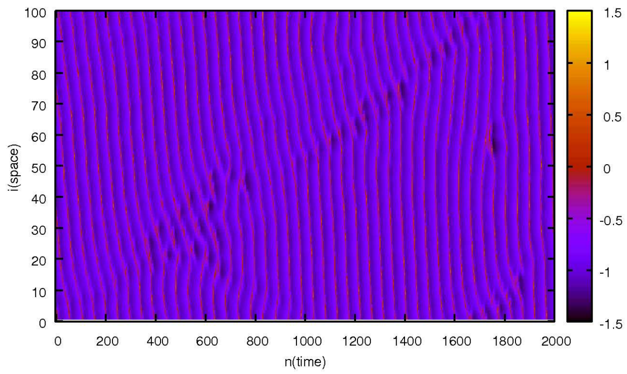
First, we consider that the space-varying coupling has positive value. The simulation results by changing initial condition



(a) Example 1.



(b) Example 2.



(c) Example 3.

Fig. 9. Examples of regular wave propagation with irregular wave propagation. ($\mu = 50$, $\sigma = 10$)

of the maps are shown in Fig. 9. From these figures, we can see that irregular wave propagation can be occurred on the wave propagation. The every form of the irregular wave propagation is different by the initial condition of the maps.

Next, we investigate synchronization of maps when the space-varying coupling has negative value. In this case, we change σ of the normal distribution function to study influence of form of the space-varying coupling, Figure 10 shows the space-varying coupling with negative value by changing σ. The space-time plots are shown in Fig. 11. We confirmed that the high density of waves when coupling is high amplitude.

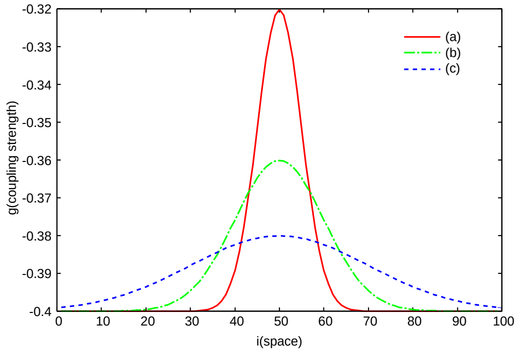
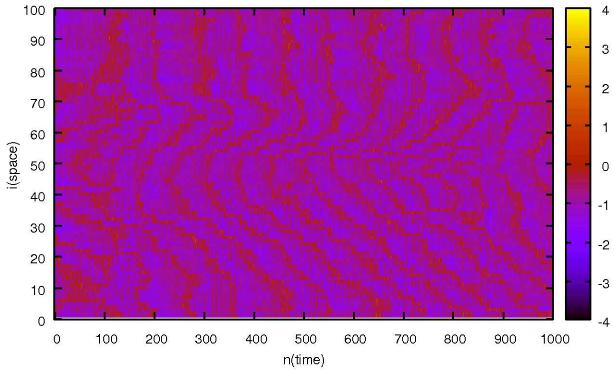
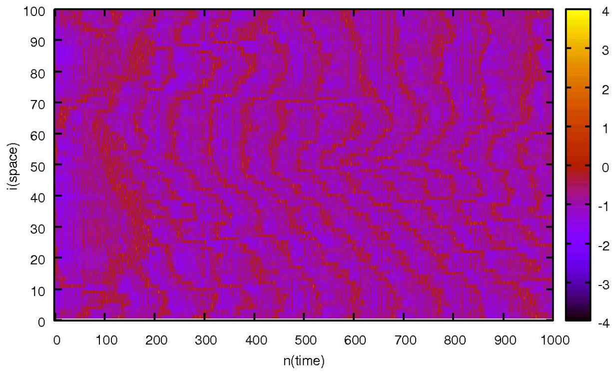


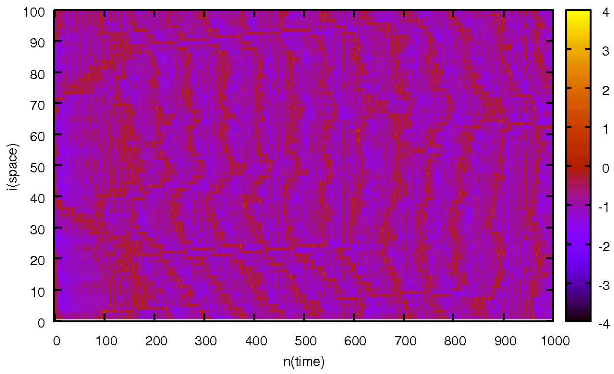
Fig. 10. Space-varying coupling with negative value by changing σ. (a) $\sigma = 5$, (b) $\sigma = 10$, (c) $\sigma = 20$.



(a) $\sigma = 5$.



(b) $\sigma = 10$.



(c) $\sigma = 20$.

Fig. 11. Complex patterns with space-varying coupling (negative value).

IV. Conclusions

In this study, we have studied synchronization in a chain of coupled Rulkov maps with space-varying coupling. We have realized the space-varying coupling by using normal distribution function. When the space-varying coupling has positive value, irregular wave propagation could be observed. While, in the case of the negative space-varying coupling, complex patterns were observed.

In the future works, we study these complex phenomena by applying statistical analysis, and we investigate synchronization of maps by the other coupling methods such as the chemical and diffusion coupling.

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