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Investigation of Complicated Phenomenon in Coupled Cubic Maps

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***Abstract*— In this study, we investigate synchronization phenomena in coupled cubic maps. The cubic map has two attractors located symmetrically with respect to the origin and they merge as increasing a control parameter. By computer simulations, we observe interesting state transition phenomenon as similar to the case of the cross-coupled chaotic circuits. Further, we compare the synchronization behaviors of the cross-coupled chaotic circuits and the coupled chaotic maps.**

I. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields [1]-[5]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos.

In our past studies [6]-[8], two Shinriki-Mori circuits [9][10] cross-coupled by inductors are investigated. As a result, we could observe an interesting synchronization phenomenon, i.e. the transitions from/to the positive region to/from the negative region synchronize in in-phase or anti-phase. We consider that this is caused by the symmetry of each circuit.

In this study, we investigate synchronization phenomena in coupled cubic maps. The cubic map has two attractors located symmetrically with respect to the origin and they merge as increasing a control parameter. Such bifurcations are similar to those observed from Shinriki-Mori circuit. We compare the synchronization behaviors of cross-coupled chaotic circuits and the coupled chaotic maps.

II. Cross-coupled chaotic circuits [6]-[8]

In this section, we review the phenomena observed from the two cross-coupled chaotic circuits.

Figure 1 shows the cross-coupled chaotic circuits. In this model, two simple autonomous chaotic circuits called as Shinriki-Mori circuit [9][10] are cross-coupled via inductors

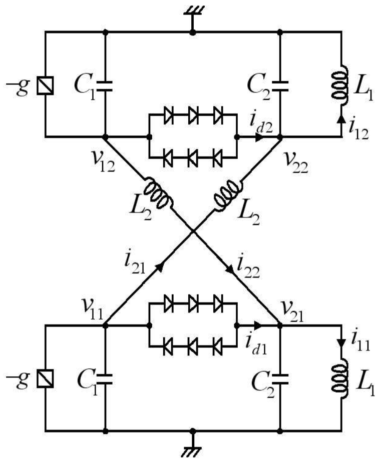

Fig. 1. Circuit model.

L_2. By using the following variables and the parameters,

$$\begin{cases} x_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{1k}}{V}, \quad w_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{2k}}{V}, \\ y_k = \dfrac{v_{1k}}{V}, \quad z_k = \dfrac{v_{2k}}{V}, \quad t = \sqrt{L_1 C_2}\,\tau, \\ \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\,G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}\,g, \\ \delta = \dfrac{L_1}{L_2}, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau} \end{cases} \tag{1}$$

the normalized circuit equations are given as follows where δ is a coupling parameter.

$$\begin{cases} \dot{x}_1 = z_1 \\ \dot{x}_2 = z_2 \\ \dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f\,(y_1 - z_1)\} \\ \dot{y}_2 = \alpha\{\gamma y_2 - w_2 - \beta f\,(y_2 - z_2)\} \\ \dot{z}_1 = \beta\, f(y_1 - z_1) + w_2 - x_1 \\ \dot{z}_2 = \beta\, f(y_2 - z_2) + w_1 - x_2 \\ \dot{w}_1 = \delta(y_1 - z_2) \\ \dot{w}_2 = \delta(y_2 - z_1) \end{cases} \tag{2}$$

where f are the nonlinear functions corresponding to the $v-i$ characteristics of the nonlinear resistors consisting of the diodes and are assumed to be described by the following 3-segment piecewise-linear functions:

$$f(y_k - z_k) = \begin{cases} y_k - z_k - 1 & (y_k - z_k > 1) \\ 0 & (|y_k - z_k| \leq 1) \\ y_k - z_k + 1 & (y_k - z_k < -1) \end{cases} \quad (3)$$

From the circuit in Fig. 1, we could observe interesting state transition phenomenon. Typical examples of the observed phenomena are shown in Figs. 2 and 3. By computer simulations and circuit experiments, we confirmed that the circuits generated many different synchronization states, by changing initial condition.

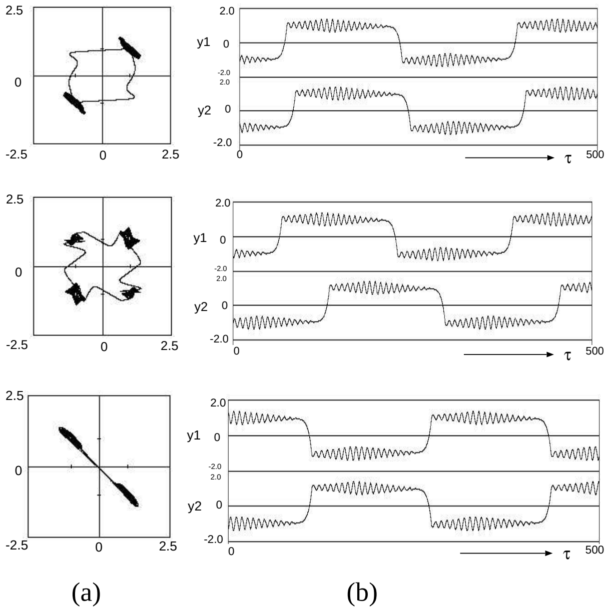

Fig. 2. Some examples of different synchronization states. (computer simulation results). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$. (a) Attractor on $y_1 - y_2$ plane. (b) Timewaveform.

Figure 4 shows how the sojourn times between the state transitions change as the coupling parameter δ changes. The horizontal axis is the coupling parameter δ and the vertical axis is the average lengths of τ. From this figure, we can see that the sojourn time between the transitions becomes shorter as increasing the coupling parameter δ.

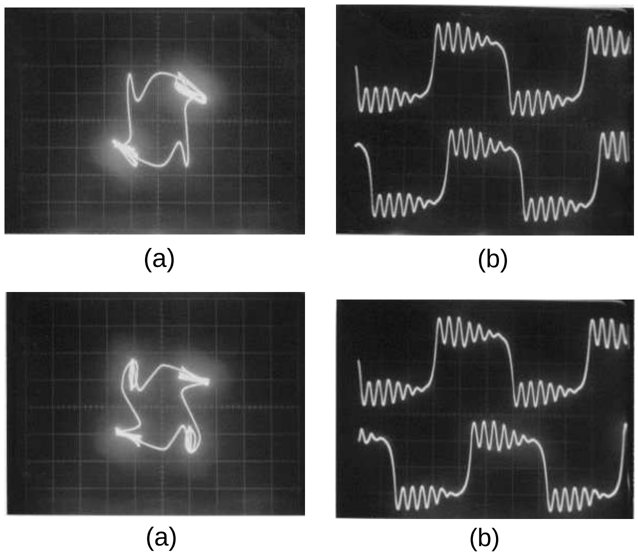

Fig. 3. Two examples of different synchronization states. (circuit experimental results). L_1 = 10.56mH, L_2 = 1.28H, C_1=33.3nF, C_2=49.5nF and g=515mS. (a) Attractor on $v_{11} - v_{12}$ plane. Horizontal and vertical: 5 V/div. (b) Time waveform v_{11} and v_{21}. Horizontal 0.5 ms/div and vertical: 5 V/div.

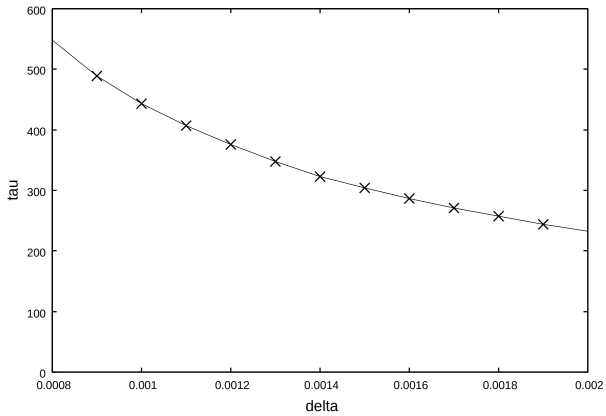

Fig. 4. Sojourn time between state transitions versus coupling parameter δ. $\alpha = 2.0$, $\beta = 4.0$, and $\gamma = 0.1$.

III. Cubic Map

In this study, we consider two coupled cubic maps. Figure 5 shows the cubic map.

The cubic map is described by the following equation:

$$x_{(n+1)} = ax_n^3 - (a+1)x_n \quad (4)$$

where n is an iteration and a is a parameter which determines the chaotic feature. We can easily confirm that the cubic map generates various periodic solutions and chaos. Figure 5(a) shows asymmetric 3 periodic solution and Fig. 5(b) shows symmetric 6 periodic solution.

The followings are the equations of the coupled cubic maps where ϵ is a coupling parameter.

$$\begin{cases} g(x_{(n)}) = ax_n^3 - (a+1)x_n \\ x_{1(n+1)} = (1-\epsilon)g(x_{1(n)}) + \epsilon(g(x_{2(n)})) \\ x_{2(n+1)} = (1-\epsilon)g(x_{2(n)}) + \epsilon(g(x_{1(n)})) \end{cases} \quad (5)$$

Figure 6 shows examples of the computer simulated results of Eq. (5). Figure 6(a) shows the timewaveform obtained with the parameters giving the periodic solution in Fig. 5(a) and a negative coupling strength. In the same way, Fig. 6(b) shows the timewaveform obtained with the parameters giving the periodic solution in Fig. 5(b) and a positive coupling strength. From the coupled cubic maps, we can observe interesting state synchronization phenomena. We could confirm that the two maps synchronize for the parameters giving periodic solutions.

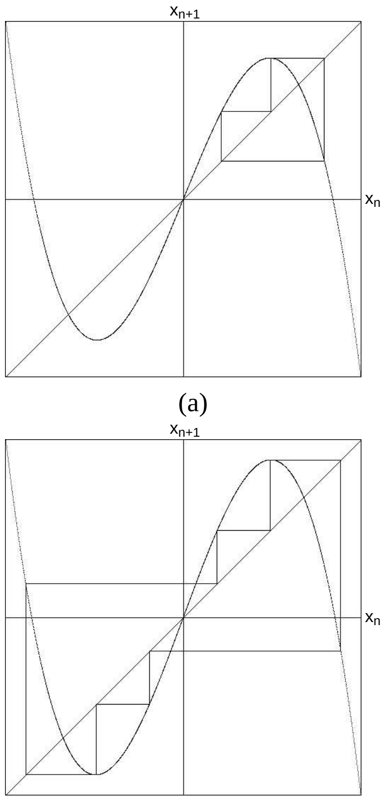

Fig. 5. Cubic map. (a) $a = -3.451$. (b) $a = -3.70$.

However, we do not understand the relationship between the obtainable synchronization states and symmetry property of the original solutions.

Figure 7 shows how the sojourn times between the state transitions change as the coupling parameter ϵ changes. The parameter a in Fig. 7(a) is the same value as Fig. 5(a). While, the parameter a in Fig. 7(b) is the same as Fig. 5(b). The horizontal axis is the coupling parameter ϵ and the vertical axis is the average lengths of the iterations n between the transitions. From these figures, we can see that the sojourn time between the transitions becomes longer as increasing the coupling parameter ϵ.

IV. Comparison of Synchronization Behaviors

In this section, we compare the synchronization behaviors of the cross-coupled chaotic circuits and coupled chaotic maps.

First, for the cross-coupled chaotic circuits, we can observe several synchronization phenomena by changing initial conditions. On the other hand, for the coupled chaotic maps, we can observe anti-phase synchronization for the parameters giving asymmetric periodic solutions and a negative coupling strength. Further, we can observe in-phase synchronization for the parameters giving symmetric periodic solutions and a positive coupling strength. However, we do not understand the relationship between the obtainable synchronization states and symmetry property of the original solutions.

Secondly, we compare the sojourn time between the state transitions for the cross-coupled chaotic circuits and the coupled chaotic maps. For the cross-coupled chaotic circuits, we confirmed that the sojourn time between the transitions becomes shorter as increasing the coupling parameter as shown in Fig. 4. In contrast, for the coupled chaotic maps, we confirmed that the sojourn time between the transitions becomes longer as increasing the coupling parameter as shown in Fig. 7.

V. Conclusions

In this study, we have investigated the synchronization phenomena of two coupled cubic maps. Also, we have compared the synchronization behaviors of the cross-coupled chaotic circuits and the chaotic maps. We could observe interesting synchronization phenomenon. We confirmed that the two maps synchronized for the parameters giving periodic solutions. For the cross-coupled chaotic circuits, the sojourn time between the transitions became shorter as increasing the coupling parameter. In contrast, for the coupled chaotic maps, the sojourn time between the transitions becomes longer as increasing the coupling parameter. Clarifying the mechanism of the observed phenomena is our future work.

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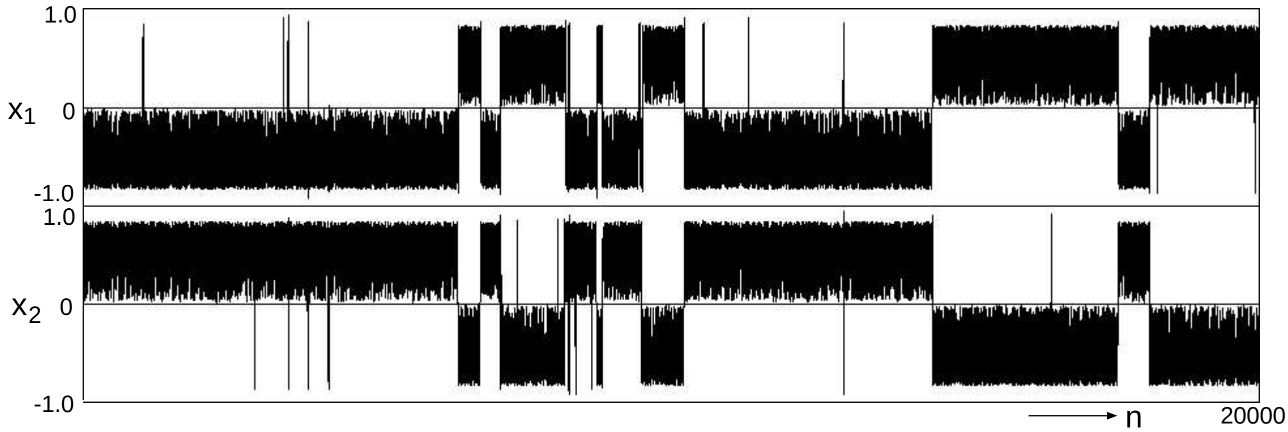
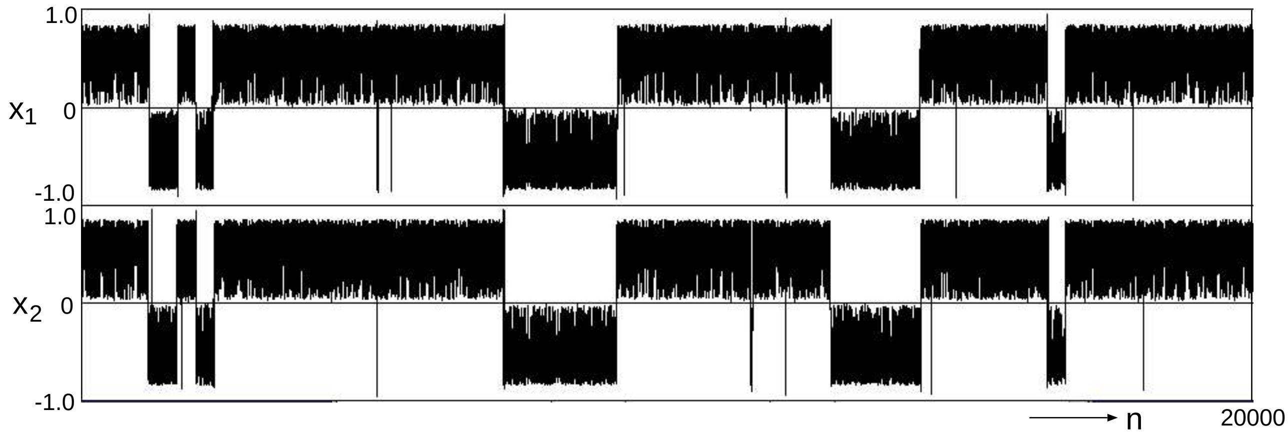
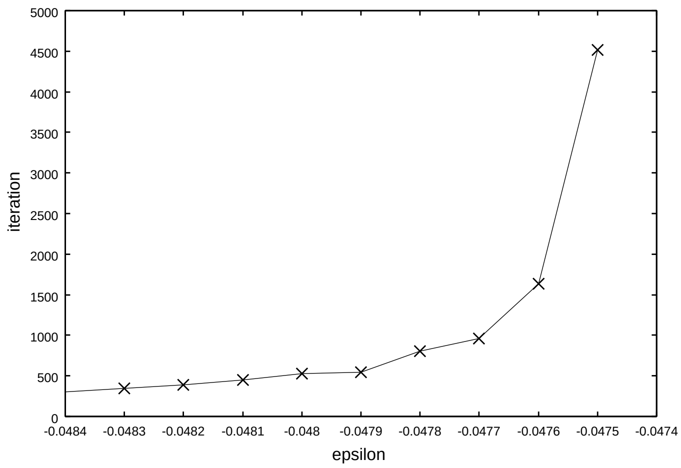


(a)

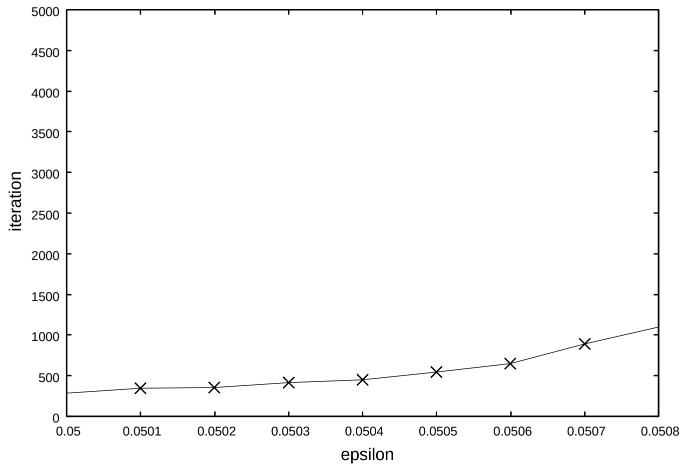


(b)

Fig. 6. Timewaveform of coupled cubic maps. (a) $\alpha = -3.451$, and $\epsilon = -0.0477$. (b) $\alpha = -3.70$, and $\epsilon = 0.0508$.



(a)



(b)

Fig. 7. Sojourn time between state transitions versus coupling parameter ϵ. (a) $a = -3.451$. (b) $a = -3.70$.