Synchronization of Small Oscillations in Cross-Coupled Chaotic Circuits

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***Abstract*— In this study, we investigate synchronization states of small oscillations observed from simple two chaotic circuits cross-coupled by inductors by both computer simulations and circuit experiments. We confirm that there are many different synchronization states coexist.**

I. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1][2]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos.

In our past studies, two simple chaotic circuits cross-coupled by inductors are investigated. As a result, we could observe interesting state transition phenomena [3][4]. In particular, we noticed that small oscillations between transitions from positive region to negative region tend to be synchronized in anti-phase in spite of the synchronization of the transitions.

In this study, we investigate different synchronization states corresponding to anti-phase synchronizations of small oscillations between the transitions. We can see that the quadrature-phase synchronization in [4] is one of many different synchronization states. The computer simulation results are verified by real circuit experiments and we also carry out computer simulations for Chua's circuit in order to confirm some kinds of universality of the phenomenon.

II. Circuit Model

Figure 1 shows the circuit model [3]. In the circuit, two Shinriki-Mori chaotic circuits [5][6] are cross-coupled via inductors L_2.

By using the following variables and the parameters,

$$\begin{cases} x_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{1k}}{V}, \quad w_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{2k}}{V}, \\ y_k = \dfrac{v_{1k}}{V}, \quad z_k = \dfrac{v_{2k}}{V}, \quad t = \sqrt{L_1 C_2}\,\tau, \\ \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\,G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}\,g, \\ \delta = \dfrac{L_1}{L_2}, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau} \end{cases} \tag{1}$$

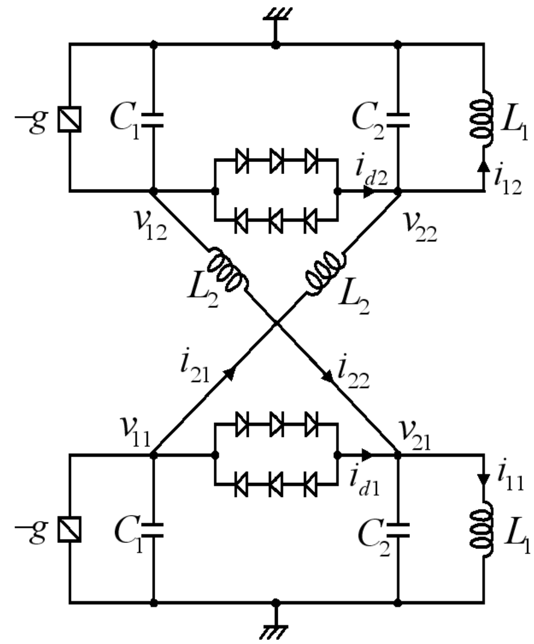

Fig. 1. Circuit model.

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_1 = z_1 \\ \dot{x}_2 = z_2 \\ \dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f\,(y_1 - z_1)\} \\ \dot{y}_2 = \alpha\{\gamma y_2 - w_2 - \beta f\,(y_2 - z_2)\} \\ \dot{z}_1 = \beta\,f(y_1 - z_1) + w_2 - x_1 \\ \dot{z}_2 = \beta\,f(y_2 - z_2) + w_1 - x_2 \\ \dot{w}_1 = \delta(y_1 - z_2) \\ \dot{w}_2 = \delta(y_2 - z_1) \end{cases} \tag{2}$$

where f are the nonlinear functions corresponding to the $v-i$ characteristics of the nonlinear resistors consisting of the diodes and are assumed to be described by the following 3-segment piecewise-linear functions:

$$f(y_k - z_k) = \begin{cases} y_k - z_k - 1 & (y_k - z_k > 1) \\ 0 & (|y_k - z_k| \le 1) \\ y_k - z_k + 1 & (y_k - z_k < -1) \end{cases} \tag{3}$$