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A Spice-Oriented Frequency Domain Analysis of Electromagnetic Fields of PCBs

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Abstract

For designing PCBs (printed circuit boards), it is very important to find out the locations gradating strong EMF (electromagnetic fields), where the voltages will have large peak value. In the peak search of the frequency characteristics, it is difficult to find the peak for large scale circuits. If we carry out frequency analysis as changing the frequency, we have to increase the frequency gradually. However, if the changing step is large, we may miss the peaks. Also there is no guarantee that the peak values exist at the frequencies. We need to find out both the exact peak voltages and locations on the PCB.

In this article, we propose an algorithm to find the peak of the frequency characteristics by using the Sine-Cosine circuit [1]-[3] and nonlinear limiter to control the step size of SPICE.

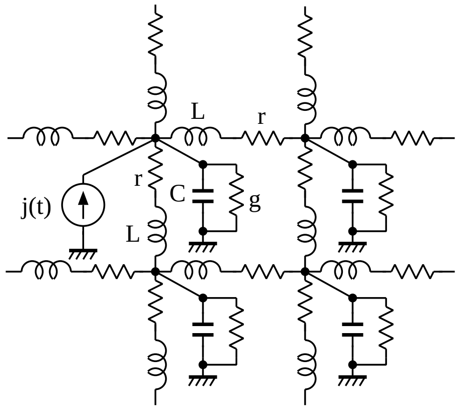


Figure 1: LRCG plane model of PCB.

1. Introduction

Nowadays, sizes of LSIs and PCBs become smaller and smaller, and the operating frequency higher and higher. In these cases, PCBs are modeled by linear LRCG large scale plane circuits as shown in Fig. 1. They can be solved by time-domain and/or frequency domain [4][5] and we concentrate on the latter technique in this study. In this case, the frequency response curve may have many sharp resonant points for high Q circuits. It is difficult to find the exact peak points.

The resonant and anti-resonant points appear at poles and zeros of driving and transfer functions, respectively. We can not find out them with analytical method when the circuit size becomes larger. They are also found by tracing the frequency response curve $|V(\omega)|$, where the resonant and anti-resonant points correspond to the highest and lowest values. In this case, when we use AC analysis of Spice simulator, we may pass over to find out all the resonant points especially for the high Q circuits. Furthermore, it is impossible to find out the exact resonant voltages. Therefore, we need to propose a new algorithm based on the HB method where the determining equation is formulated by the coupled Sine-Cosine circuits [1]-[3]. Both the resonant and anti-resonant points correspond to the solutions satisfying $f'(\omega) = 0$ on the frequency response curve, so that we use a differentiator and a nonlinear limiter to find the $f'(\omega) = 0$ points. The circuit is solved by the variable step size transient analysis of Spice. Thus, we can develop an efficient simulator to trace the frequency response curves, and to find the exact peak voltages.

2. Reactance circuit

Now, we review the circuit properties of a general linear LRC circuit as shown Fig. 1. The circuit equation is given by

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} J \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{1}$$

where Vs are nodal voltages for the input J, and the mutual admittance is given by

$$Y_{ij} = j\omega C_{ij} + G_{ij} + \frac{1}{j\omega L_{ij}} \tag{2}$$

Then, we have

$$V_k = \frac{\Delta_{1k} J}{\Delta} \tag{3}$$

where

$$\Delta = \begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{vmatrix} \tag{4}$$

Now, we consider the case $r = g = 0$ in Fig. 1 which corresponds to the reactance circuit. It is known that the voltage (3) can be written by

$$V_k(\omega) = \left[\frac{a_{0k}}{j\omega} + \sum_{i=1}^{n} \frac{ja_{2i,k}\omega}{\omega_{2i}^2 - \omega^2} + ja_{\infty,k}\omega \right] J \tag{5}$$

Observe that the resonant points correspond to the poles of both the driving and the transfer functions. They arise at the same frequencies. They also satisfy

$$0 < \omega_1 < \omega_2 < \omega_3 < \dots < \infty \tag{6}$$

Note that r and g in the high frequency domain are small compared to the reactance values, so that the frequency response behaves like as that of reactance circuits, and it has sharp resonant points.

3. Peak points tracing algorithm

Although, for relative low Q circuits, the AC analysis of Spice and a cubic spline combining Newton method can be usefully applied to find out the peak points, it may pass over the points for high Q circuits. Thus, we propose a new algorithm based on the HB method such that the determining equation is solved by transient analysis of Spice, where ω is a function of time t as follows:

$$\omega = Kt \tag{7}$$

Now, let us discuss the Sine-Cosine circuit corresponding to the determining equation of the HB method.

Let the current through an inductance L be

$$i_L = I_{LS} \sin \omega t + I_{LC} \cos \omega t \tag{8.1}$$

Then, the voltage v_L is given by

$$v_L = L\frac{di_L}{dt} = -\omega L I_{LC} \sin \omega t + \omega L I_{LS} \cos \omega t \tag{8.2}$$

Thus, the coefficients of $\sin \omega t$, $\cos \omega t$ are described by

$$V_{LS} = -\omega L I_{LC}, \quad V_{LC} = \omega L I_{LS} \qquad (8,3)$$

Namely, the inductance is replaced by coupled current-controlled voltage sources in the Sine-Cosine transformation of the HB method. In the same way, let the voltage across a capacitor C be

$$v_C = V_{CS} \sin \omega t + V_{CC} \cos \omega t \tag{9.1}$$

Then, the current i_C is given by

$$i_C = C\frac{dv_C}{dt} = -\omega C V_{LC} \sin \omega t + \omega C V_{LS} \cos \omega t \tag{9.2}$$

Thus, the coefficients of $\sin \omega t$, $\cos \omega t$ are described by

$$I_{LS} = -\omega C V_{LC}, \quad I_{LC} = \omega C V_{LS} \qquad (9,3)$$

Namely, the capacitor is replaced by the coupled voltage-controlled current sources in the Sine-Cosine transformation. The circuit corresponding to the determining equation of the HB method is driven by a constant voltage $E = J$ and ω is given by (7).

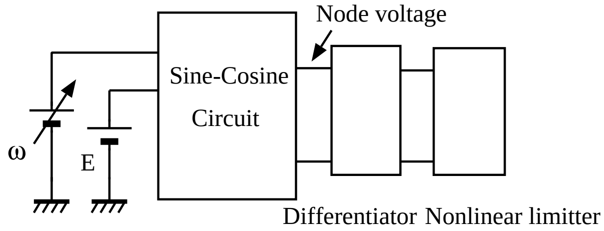


Figure 2: Peak detector of frequency response curve.

Since ωs at the peak voltages satisfy

$$\frac{d|V_k(\omega)|}{d\omega} = 0, \quad k = 1, 2, \dots n, \tag{10}$$

on the response curve, we need to find the zero points satisfying (10) Hence, $|V_k(\omega)|$ need to be firstly differentiated by a differentiator. In order to detect the exact peak points, the output is limited and expanded with a nonlinear limiter as shown in Fig. 2, which consists of a limiter and nonlinear diodes. The output of the limiter is given by

$$v_L = \begin{cases} -V_{max} & : \quad \text{for } v_{in} < -V_L \\ kv_{in} & : \quad \text{for } -V_L \geq v_{in} \geq V_L \\ V_{max} & : \quad \text{for } v_{in} > V_L \end{cases} \tag{11.1}$$

where

$$kV_L = V_{max} \tag{11.2}$$

The outputs of diodes are given by

$$i_o = \begin{cases} I_s \exp(\lambda v_o) & : \quad \text{for } v_o > 0 \\ -I_s \exp(-\lambda v_o) & : \quad \text{for } v_o < 0 \end{cases} \tag{11.3}$$

This means that the regions around $d|V_k(\omega)|/d\omega = 0$ are largely expanded. Furthermore, the characteristic has large nonlinearity around the zeros. Thus, the transient analysis around the zero points is executed with a very small step size, and we can find out precise peak points.

4. Illustrative example

4.1. Theoretical analysis of transmission line

We consider a single transmission line of the length $d[mm]$, whose parameters are

$$r[\Omega/mm],\ \ L[H/mm],\ \ g[S/mm],\ \ C[F/mm].$$

The circuit equation with a complex frequency s at $x[mm]$ from the near end is given by

$$\frac{dV(x,s)}{dx} = -(r+sL)I(x,s) \tag{12.1}$$

$$\frac{dI(x,s)}{dx} = -(g+sC)V(x,s) \tag{12.2}$$

Thus, we have

$$\frac{d^2V(x,s)}{dx^2} = (r+sL)(g+sC)V(x,s) \tag{13.1}$$

$$\frac{d^2I(x,s)}{dx^2} = (r+sL)(g+sC)I(x,s) \tag{13.2}$$

Set the boundary conditions $V(0,s),\ I(0,s)$ at the near end and $V(d,s),\ I(d,s)$ at the far end. Then, we have

$$\begin{bmatrix} I(0,s) \\ I(d,s) \end{bmatrix} = \frac{1}{Z_0(s)} \begin{bmatrix} \coth\gamma(s)d & -\frac{1}{\sinh\gamma(s)d} \\ -\frac{1}{\sinh\gamma(s)d} & \coth\gamma(s)d \end{bmatrix} \begin{bmatrix} V(0,s) \\ V(d,s) \end{bmatrix} \tag{14}$$

where

$$Z_0(s) = \sqrt{(r+sL)/(g+sC)}$$

$$\gamma(s) = \sqrt{(r+sL)(g+sC)}$$

Thus, the poles are located at the frequencies satisfying $\sinh\gamma(s)d = 0$, namely

$$(r+sL)(g+sC) = (jn\pi/d)^2,\ \ n = 1,2,\ldots \tag{15}$$

Hence,

$$p_0 = -\frac{r}{L},\ \ p_n = u_n \pm jv_n,\ \ n = 1,2,\ldots \tag{16}$$

where

$$u_n = -\frac{Lg+rC}{2LC},$$

$$v_n = \frac{\sqrt{4LC(gr+(n\pi/d)^2)-(Lg+rC)^2}}{2LC}.$$

Observe that the transmission line has an infinite number of poles, where the frequency response curve has peak points.

4.2. Discrete model of transmission line

Transmission line is usually modeled by discrete RLCG ladder circuit, where we neglected G. This time, we analyze the two-dimensional circuit in the case of 5×5 as shown in Fig.1, where $C = 1[nF]$, $L = 1[mH]$, $e(t) = E\cos\omega t$ and $E = 100[\mu V]$. In order to obtain the frequency response curve, the circuit is transformed by HB method into the Sine-Cosine circuit. Note that the detector can be attached to an arbitrary node. We use a simple RC differentiator with $C_d = 1[pF]$ and $R_d = 1[\Omega]$ in the detector. Now, we show the simulation results using the transient analysis of Spice. Figures 3(a), (b) and (c) show the results for the cases of $R = 1[\Omega], 10[\Omega]$ and $100[\Omega]$, respectively, where we set the peaks 1-12 from the left hand side. The sharpnesses depend on the quality factors so that we have changed them by Rs. We found that although the resonant frequencies are almost same for all Rs as shown in Table 1, the peak values are largely different.

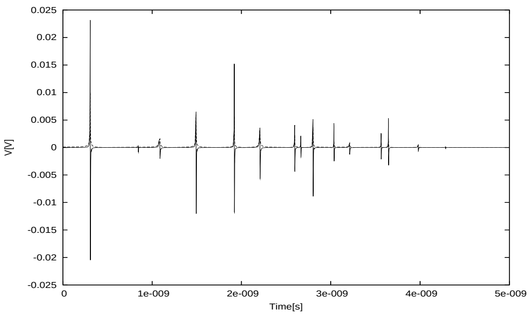
Table 1 Calculated peak values.

R=1[Ω]	peak 1	peak 2	peak 3	peak 4
f[kHz]	30.7	108.7	149.2	192.0
V[mV]	8.0	1.6	5.5	4.7
-	peak 5	peak 6	peak 7	peak 8
f[kHz]	220.7	259.7	280.2	303.6
V[mV]	3.4	1.9	3.4	1.1
-	peak 9	peak 10	peak 11	peak 12
f[kHz]	356.4	397.9	428.5	477.7
V[mV]	0.80	0.37	0.08	0.004
R=10[Ω]	peak 1	peak 2	peak 3	peak 4
f[kHz]	30.4	108.2	149.3	191.9
V[μV]	1169.7	847.5	943.3	572.8
-	peak 5	peak 6	peak 7	peak 8
f[kHz]	220.5	259.6	282.0	320.4
V[μV]	970.9	309.3	204.4	227.8
-	peak 9	peak 10	peak 11	peak 12
f[kHz]	359.5	400.9	427.9	477.3
V[μV]	19.6	18.4	14.5	3.6
R=100[Ω]	peak 1	peak 2	peak 3	peak 4
f[kHz]	25.6	104.8	147.2	190.4
V[μV]	144.5	101.5	107.7	69.3
-	peak 5	peak 6	peak 7	peak 8
f[kHz]	220.3	260.0	282.2	308.2
V[μV]	106.0	44.9	62.3	52.5
-	peak 9	peak 10	peak 11	peak 12
f[kHz]	353.1	394.0	424.1	475.5
V[μV]	29.6	12.2	4.0	3.7

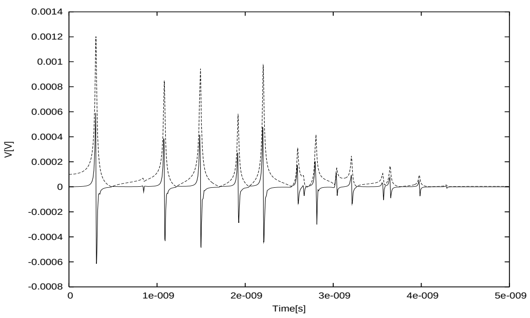
It seems that there are small differences in the calculated peak points for $R = 1$ and for $R = 100$. It is due to pass over the point because the peak is too narrow and small to detect even when the nonlinear limiter is introduced. In this case, we need to trace the resonant curve around peak points for $R = 1$ with a selection of much smaller initial step size.

5. Conclusions and remarks

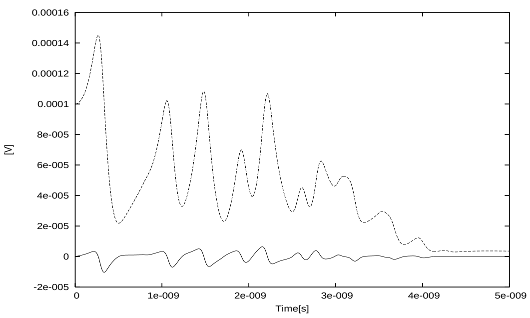
In this study, we have proposed an algorithm to find the peaks of the frequency response curve by combining the Sine-Cosine circuit based on the HB method. The circuit is traced by the transient analysis of Spice. Since the peak points correspond to the gradient being to zeros, we differentiate the curve and find its zero points. In order to find the exact zero points, we have developed the peak detector using nonlinear limiter such that the regions are largely expanded. Thus, the step size around zero values becomes small when we use variable step size transient analysis.



(a)



(b)



(c)

Figure 3: Frequency characteristics of the circuit. (a) R=1[Ω]. (b) R=10[Ω]. (c) R=100[Ω].$K = 2\pi \times 10^{14}$

Acknowledgments

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