

# Solving Ability of Coupled Map Lattice with 2-Opt Algorithm for TSPs

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## **Abstract**

In this study, we propose a new method using the coupled map lattice with the 2-opt algorithm for solving traveling salesman problems. In this method, we utilize the 2-opt algorithm for updating solutions of traveling salesman problems, and we use sequences generated by the coupled map lattice as a adding noise. By carrying out computer simulations, we confirm that the chaotic noise generated by the coupled map lattice has a good effect to escape local minima and achieves a good performance to find near optimal solutions.

## 1. Introduction

Combinatorial optimization problems can be used for an arrangement and wiring of LSI in an engineering field. When the number of element increases, a calculation time becomes longer and a calculation becomes almost impossible in real time. Therefore, it is important to develop heuristic algorithms for finding good solutions in a short time. A technique using the Hopfield neural network [1] is one of the powerful tool to find good solutions within limited time. Recently many researchers suggested that the chaotic noise is more effective than the stochastic noise with the Hopfield neural network for solving traveling salesman problems (TSPs) [2]-[4]. Moreover, the technique is extended to the neural network with chaotic dynamics to avoid the local minimum problems [5][6].

In this study, we propose a new method using the coupled map lattice (CML) with the 2-opt algorithm for solving TSPs. We utilize the 2-opt algorithm for updating solutions of TSPs. The 2-opt algorithm is one of the simplest local search to find good solutions. The 2-opt Algorithm exchanges two baths, i-a(i) and j-a(j), with another two paths, i-j and a(i)-a(j) as shown in Fig. 1. However, a tour obtained by the 2-opt algorithm is often trapped into local minima. In order to escape from local minima, we applied the chaotic noise generated by the CML to the 2-opt algorithm. The 2-opt algorithm can find new candidates for 2-opt exchange by adding the chaotic noise. And we investigate solving abilities of the noise generated by the CML for TSPs.

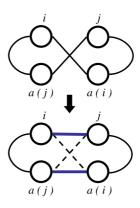


Figure 1: Example of the 2-opt algorithm.

## 2. Coupled Map Lattice

A CML is known as good models to describe spatiotemporal chaos and pattern dynamics [7][8]. In the CML, the each map connects to only adjacent maps. The CML is shown in Fig. 2. Also the CML is able to generate complex pattern [9] and phenomena [10]. And it applied to the pseudo-random-bit generator in the engineering field [11].

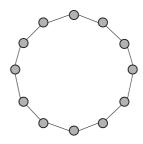


Figure 2: Coupled map lattice.

A CML is consisted as following equation:

$$x^{i}(n+1) = (1-\epsilon)f(x^{i}(n)) + \frac{\epsilon}{2} \left[ f(x^{i+1}(n)) + f(x^{i-1}(n)) \right] \ (1)$$

where  $x^i(n)$  represents the state variable for the site i at time n,  $\epsilon$  is a coupling strength and  $f(\cdot)$  is a site map. The periodic boundary conditions are  $x^0(n) = x^L(n)$  and  $x^{L+1}(n) = x^1(n)$ .

In this study, we use a skew tent map and a logistic map as the chaotic site map f. A skew tent map is expressed as following equation:

$$f(x) = \begin{cases} \frac{X}{p} & (0 \le X \le p) \\ \frac{X-1}{p-1} & (p < X \le 1) \end{cases}$$
 (2)

This map is shown in Fig. 3.

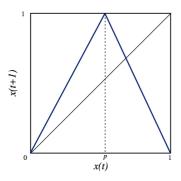


Figure 3: Skew tent map.

A logistic map is expressed as following equation:

$$f(x) = \alpha x(1 - x) \tag{3}$$

where  $\alpha$  is a bifurcation parameter. A logistic map for  $\alpha = 4.0$  is shown in Fig. 4.

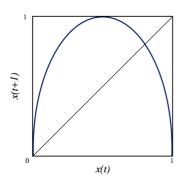


Figure 4: Logistic map.

## 3. Proposed Method

We propose a new method using the CML with the 2-opt algorithm for solving TSPs. In the proposed method, internal states of cities are defined as following equation:

$$city_i(t+1) = \max_{j} \{D_0(t) - D_{ij}(t) + \beta x^i(t)\}$$
 (4)

where  $x^i$  is sequence generated by a CML,  $D_0(t)$  is the length of the tour,  $D_{ij}(t)$  is the length of the tour generated by applying the 2-opt exchange between cities i and j,  $\beta$  is the scaling parameter of the noise effect.

If the internal state  $city_i(t)$  is the largest in all cities, the path between cities i and j is connected by the 2-opt exchange.

#### 4. Simulated Results

In this research, we use a problem "lin105" from TSPLIB [12]. A known optimal tour length of "lin105" is 14372. The simulated results are the average values of 10 trials with different initial values. And the number of iterations is 1000 times. The simulated result by using the CML consisting of the logistic maps is shown in Fig. 5. We can see that the proposed method with  $\alpha$  close to 4.0 becomes better solutions.

Next, we investigate the relation between each parameters and the tour length in detail. Figure 6 shows results of the tour length versus the amplitude of added noise. The horizontal axis shows the value of  $\beta$ , and the vertical axis shows the tour length. Here,  $\beta=0$  means that the chaotic noise does not be added to the 2-opt algorithm. From this figure, we can see that the solutions becomes close to the optimal solution by adding the chaotic noise.

The result of the tour length versus the coupling strength is shown in Fig. 7. The horizontal axis shows the value of  $\epsilon$ , and the vertical axis shows the tour length. In the case of  $\epsilon=0$ , the CML is not coupled. We confirm that the 2-opt algorithm with chaotic noise consisting of the CML exhibits much better performance than the 2-opt algorithm. However, as the coupling strength becomes large, the solutions cannot find the solutions closer to the optimal solution than the 2-opt algorithm with simple chaotic map. Moreover, we can see that the proposed method has a very sensitive to the coupling strength.

Finally, we investigate the  $\alpha$  sensibility.  $\alpha$  corresponds to the bifurcation parameter of the logistic map. From Fig. 8, we can obtain the good solution around  $\alpha = 3.8$ .

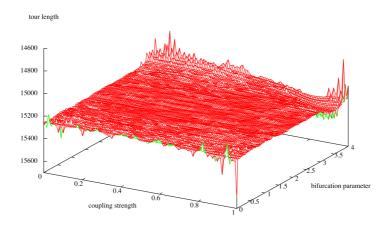


Figure 5: Simulated result

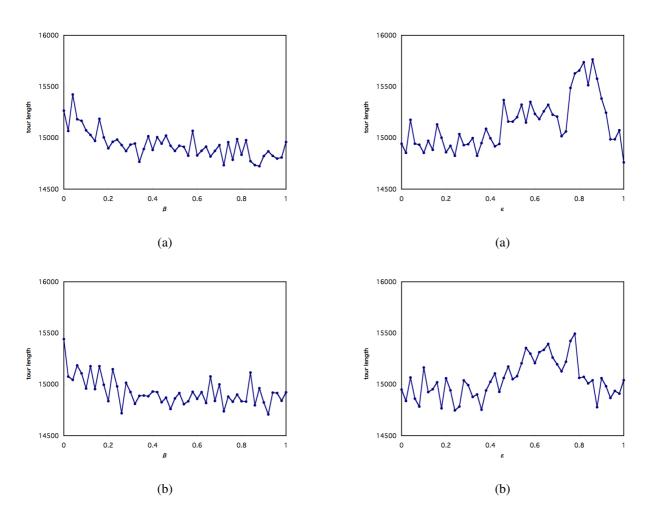


Figure 6:  $\beta$  vs. tour length (a) logistic map, (b) skew tent map ( $\alpha=4.0,\epsilon=0.1,p=5.1$ )

Figure 7:  $\epsilon$  vs. tour length (a) logistic map, (b) skew tent map  $(\beta = 0.2, \alpha = 4.0, p = 5.1)$ 

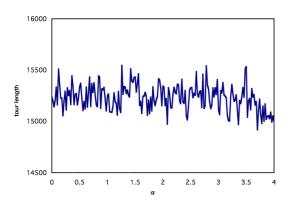


Figure 8:  $\alpha$  vs. tour length ( $\beta = 0.2$ ,  $\epsilon = 0.1$ )

## 5. Conclusions

In this study, we have proposed a method using the CML with the 2-opt algorithm for solving TSPs. And we have investigated solving abilities of the noise generated by the CML for TSPs. By carrying out computer simulations, we have confirmed that the chaotic noise generated by the CML has a good effect to escape local minima and achieves a good performance to find near the optimal solution for TSPs.

As the future subject, we will investigate the potentially the tabu effect by coupling chaotic maps.

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