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Self-Switching Phenomenon of Synchronization States in Coupled Parametrically Excited LC Oscillators

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Abstract

In this study, we investigate synchronization phenomena in parametrically excited simple LC oscillators are coupled by a capacitor. We use parametorons that transformers have nonlinear characteristics for the parametrically excited simple LC oscillators. In this system, we observe two different types of synchronization phenomena; coexistence phenomenon of in-phase synchronization and opposite-phase synchronization and self-switching of in and opposite-phases synchronizations. Additionally, we also confirm that a sojourn time of the self-switching is changed by changing the amplitude of the time-varying inductor.

1. Introduction

Synchronization is one of the fundamental phenomena in nature, and one of typical nonlinear phenomena. Therefore studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. In particular, in the field of electrical engineering various kinds of models can be implemented easily. The phenomena generated in the models can be confirmed easily. Recently, studies on coupled nonlinear circuits have come into important things as active studies on neural networks. For this reason we consider that it is important to investigate the synchronization phenomena of coupled oscillators for future engineering applications.

In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Ref. [4]. A method of analysis for the bifurcation in the system is proposed in Ref. [5]. In our past studies we investigated synchronization phenomena of parametrically excited van der Pol oscillators coupled by a resister [6]. We confirmed that various kinds of interesting synchronization phenomena can be generated in the system. Therefore, we consider that there are special effects of parametric excitation for synchronization phenomenon.

In this study, we investigate synchronization phenomena in parametrically excited simple LC oscillators coupled by a capacitor. We use parametrons for the parametrically excited simple LC oscillators. In general, oscillators coupled by a capacitor generate coexistence of synchronization states that are in-phase synchronization and opposite-phase synchronization. However, in this system we confirm the above synchronization phenomenon and self-switching phenomenon of synchronization state. Additionally, we also confirm that a sojourn time of the self-switching is changed by changing the amplitude of the time-varying inductor.

2. Parametron

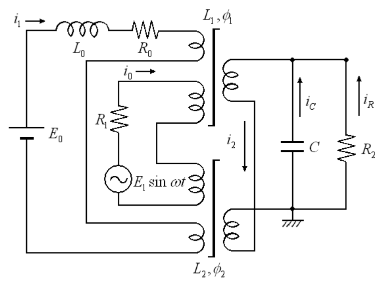

Figure 1: Parametron.

A resonance circuit is made of two transformers that the two magnetic cores has similar magnetic characteristics and a capacitor in secondary side. A source is used for give a change of parametric excitation. If the two magnetic cores have linear magnetic characteristics, a direct-current power source does not affect the resonance circuit. The excitation source also does not affect the resonance circuit because connections of the coils are inverse. However, if magnetic cores have nonlinear characteristics, inductance viewed of resonance circuit is changed with time. So-called parametric excitation occurs (Fig. 2). Additionally, periodic attractors, quasi-periodic attractors and chaotic attractors are confirmed in this system. Figure 3 shows examples of the attractors and the Poincaré maps.

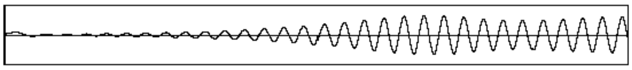

Figure 2: Parametric excitation

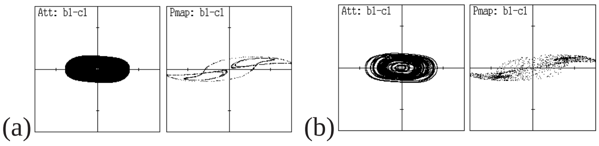

Figure 3: Attractors and Poincaré maps. $k_1 = 0.37$, $k_2 = 0.06$ and $u_0 = 0.07$. (a) Quasi-periodic attractor at $B = 0.26$. (b) Chaotic attractor at $B = 0.29$.

3. Circuit model

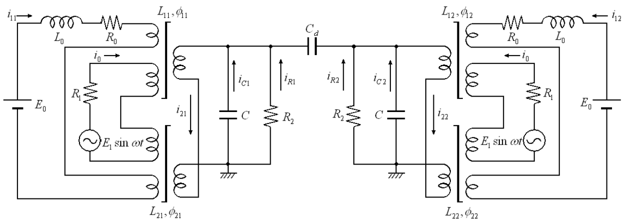

Figure 4: Circuit model.

The circuit model used in this study is shown in Fig 4. In our system two identical parametrically LC oscillators are coupled by one capacitor C_d. The circuit equations are given by the following equations.

$$
\begin{cases}
n\dfrac{d}{dt}(\phi_{11}+\phi_{21}) + R_1 i_{11} = E_1 \sin(\omega t) \\
n\dfrac{d}{dt}(\phi_{11}-\phi_{21}) = -\dfrac{1}{C}\displaystyle\int i_{C1}dt = -R_2 i_{R1} \\
i_{21} = i_{C1} + i_{R1} + i_d \\
i_d = \dfrac{C_d}{C}(i_{C2} - i_{C1}) \\
n\dfrac{d}{dt}(\phi_{12}+\phi_{22}) + R_1 i_{12} = E_1 \sin(\omega t) \\
n\dfrac{d}{dt}(\phi_{12}-\phi_{22}) = -\dfrac{1}{C}\displaystyle\int i_{C2}dt = -R_2 i_{R2} \\
i_{22} + i_d = i_{C2} + i_{R2},
\end{cases}
\tag{1}
$$

where ϕ_{11}, ϕ_{21}, ϕ_{12} and ϕ_{22} are respectively fluxes of the iron cores of inductors L_{11}, L_{21}, L_{12} and L_{22}, and n is each coil turn. By changing the variables and the parameters,

$$
\begin{array}{l}
\phi_{11} = \Phi_n v_{11},\ \phi_{21} = \Phi_n v_{21}, \\
\phi_{12} = \Phi_n v_{12},\ \phi_{22} = \Phi_n v_{22} \\
i_{11} = I_n u_{11},\ i_{21} = I_n u_{21}, \\
i_{12} = I_n u_{12},\ i_{22} = I_n u_{22} \\
\tau = \omega t,
\end{array}
\tag{2}
$$

the first and fifth equations in Eq. (1) are transformed into

$$
\begin{cases}
n\omega\Phi_n \dfrac{d}{d\tau}(v_{11}+v_{21}) + R_1 I_n u_{11} = E_1 \sin\tau \\
n\omega\Phi_n \dfrac{d}{d\tau}(v_{12}+v_{22}) + R_1 I_n u_{12} = E_1 \sin\tau,
\end{cases}
\tag{3}
$$

where v_{11}, v_{21}, v_{12}, v_{22}, u_{11}, u_{21}, u_{12} and u_{22} are nondimensional variables. In a similar way, the remaining equations in Eq. (1) are described by

$$
\begin{cases}
n\omega^2 C\Phi_n \dfrac{d^2}{d\tau^2}(v_{11}-v_{21}) \\
\quad + \dfrac{1-\frac{C_d}{C}}{1-2\frac{C_d}{C}} n\omega\dfrac{1}{R_2}\Phi_n\dfrac{d}{d\tau}(v_{11}-v_{21}) + \dfrac{1-\frac{C_d}{C}}{1-2\frac{C_d}{C}} I_n u_{21} \\
\quad = \dfrac{\frac{C_d}{C}}{1-2\frac{C_d}{C}}\left\{ I_n u_{22} + n\omega\dfrac{1}{R_2}\Phi_n\dfrac{d}{d\tau}(v_{12}-v_{22})\right\} \\
n\omega^2 C\Phi_n \dfrac{d^2}{d\tau^2}(v_{12}-v_{22}) \\
\quad + \dfrac{1-\frac{C_d}{C}}{1-2\frac{C_d}{C}+2\frac{C_d^2}{C^2}} n\omega\dfrac{1}{R_2}\Phi_n\dfrac{d}{d\tau}(v_{12}-v_{22}) \\
\quad + \dfrac{1-\frac{C_d}{C}}{1-2\frac{C_d}{C}+2\frac{C_d^2}{C^2}} I_n u_{21} \\
\quad = -\dfrac{\frac{C_d}{C}}{1-2\frac{C_d}{C}+2\frac{C_d^2}{C^2}}\left\{ I_n u_{21} + n\omega\dfrac{1}{R_2}\Phi_n\dfrac{d}{d\tau}(v_{11}-v_{21})\right\}.
\end{cases}
\tag{4}
$$

By taking following parameters,

$$
\begin{array}{l}
k_1 = \omega C R_1,\ k_2 = \dfrac{1}{\omega C R_2} \\
B = \dfrac{E_1}{n\omega\Phi_n},\ I_n = n\omega^2 C\Phi_n,\ \gamma = \dfrac{C_d}{C},
\end{array}
\tag{5}
$$

Eq. (3) and Eq. (4) are transformed into

$$
\begin{cases}
\frac{d}{d\tau}(v_{11}+v_{21})+k_1u_{11}=B\sin\tau \\
\frac{d}{d\tau}(v_{12}+v_{22})+k_1u_{12}=B\sin\tau \\
\frac{d^2}{d\tau^2}(v_{11}-v_{21}) \\
\quad+\frac{1-\gamma}{1-2\gamma}k_2\frac{d}{d\tau}(v_{11}-v_{21})+\frac{1-\gamma}{1-2\gamma}u_{21} \\
\quad=\frac{\gamma}{1-2\gamma}\left\{k_2\frac{d}{d\tau}(v_{12}-v_{22})+u_{22}\right\} \\
\frac{d^2}{d\tau^2}(v_{12}-v_{22}) \\
\quad+\frac{1-\gamma}{1-2\gamma+2\gamma^2}k_2\frac{d}{d\tau}(v_{12}-v_{22}) \\
\quad+\frac{1-\gamma}{1-2\gamma+2\gamma^2}u_{21} \\
\quad=-\frac{\gamma}{1-2\gamma+2\gamma^2}\left\{k_2\frac{d}{d\tau}(v_{11}-v_{21})+u_{21}\right\}.
\end{cases} \tag{6}
$$

The saturation characteristics of the iron core are approximated by the following third order polynomial functions;

$$
\begin{cases}
v_{11}^3=u_0+u_{11}+u_{21} \\
v_{21}^3=u_0+u_{11}-u_{21} \\
v_{12}^3=u_0+u_{12}+u_{22} \\
v_{22}^3=u_0+u_{12}-u_{22},
\end{cases} \tag{7}
$$

where $i_0=I_n u_0$. Here, the new variables a_1, a_2, $b_1$and b_2 are defined as

$$
\begin{array}{l}
a_1=v_{11}+v_{21},\ b_1=v_{11}-v_{21} \\
a_2=v_{12}+v_{22},\ b_1=v_{12}-v_{22}.
\end{array} \tag{8}
$$

Then, by taking above variables Eq. (7) is transformed into

$$
\begin{cases}
u_{11}=\frac{1}{8}(a_1^3+3a_1b_1^2)-u_0 \\
u_{21}=\frac{1}{8}(b_1^3+3a_1^2b_1) \\
u_{12}=\frac{1}{8}(a_2^3+3a_2b_2^2)-u_0 \\
u_{22}=\frac{1}{8}(b_2^3+3a_2^2b_2).
\end{cases} \tag{9}
$$

By taking Eq. (7) and Eq. (9), the normalized circuit equations are given by the following equations.

$$
\begin{cases}
\frac{da_1}{d\tau}+\frac{1}{8}k_1\left\{(a_1^2+3b_1^2)a_1-8u_0\right\}=B\sin\tau \\
\frac{da_2}{d\tau}+\frac{1}{8}k_1\left\{(a_2^2+3b_2^2)a_2-8u_0\right\}=B\sin\tau \\
\frac{d^2b_1}{d\tau^2}+\frac{1-\gamma}{1-2\gamma}k_2\frac{db_1}{d\tau}+\frac{1-\gamma}{1-2\gamma}\left\{\frac{1}{8}(b_1^3+3a_1^2b_1)\right\} \\
\quad=\frac{\gamma}{1-2\gamma}\left\{k_2\frac{db_2}{d\tau}+\frac{1}{8}(b_2^3+3a_2^2b_2)\right\} \\
\frac{d^2b_2}{d\tau^2}+\frac{1-\gamma}{1-2\gamma+2\gamma^2}k_2\frac{db_2}{d\tau} \\
\quad+\frac{1-\gamma}{1-2\gamma+2\gamma^2}\left\{\frac{1}{8}(b_2^3+3a_2^2b_2)\right\} \\
\quad=-\frac{\gamma}{1-2\gamma+2\gamma^2}\left\{k_2\frac{db_1}{d\tau}+\frac{1}{8}(b_1^3+3a_1^2b_1)\right\}.
\end{cases} \tag{10}
$$

4. Simulation results

We calculated Eq. (10) by using the Runge-Kutta method. To simulate the synchronization pattern, we fix parameters as $k_1 = 0.37$, $k_2 = 0.06$, $u = 0.07$ and $\gamma = 0.05$. In this system, we observe two different types of synchronization phenomena; coexistence phenomenon of in-phase synchronization and opposite-phase synchronization and self-switching of these in-phase and opposite-phase synchronizations. Figure 5 shows examples of attractors, Poincaré maps, phase differences and time series. When the parameter $B < 0.19$, the coupled oscillators are synchronized at the in-phase or the opposite-phase for the same parameter. In this situation the in-phase synchronization and the opposite-phase synchronization coexist by different initial values as Figs. 5 (a) and (a'). However, the self-switching phenomenon of synchronizations is observed for the parameter $0.19 < B < 0.42$. In this phenomenon the synchronization states switch from the in-phase to the opposite-phase or vice versa with time as Figs. 5 (b) and (c). With further increase of parameter B, the coexistence of the in-phase synchronization and the opposite-phase synchronization is observed again.

Finally, we investigate sojourn times of the self-switching as varying B. Figure 6 shows frequencys of sojourn periods in the self-switching. In paticular, Figs. 6 (b) and (c) show examples of the frequency when $B = 0.24$ and $B = 0.30$, respectively. When $B < 0.255$, the sojourn time has two peaks and diffuse widely (Fig. 6 (b)). From this result, short sojourn period stands out as 2 period and 6 period. However, a few long sojourn periods over 30 period are confirmed. When $B = 0.255$, the subcircuits generate the quasi-periodic attractors, the sojourn time becomes decided value. With further increase of the parameter B, the chaotic attractors are generated, and the sojourn time becomes smaller (Fig. 6 (c)).

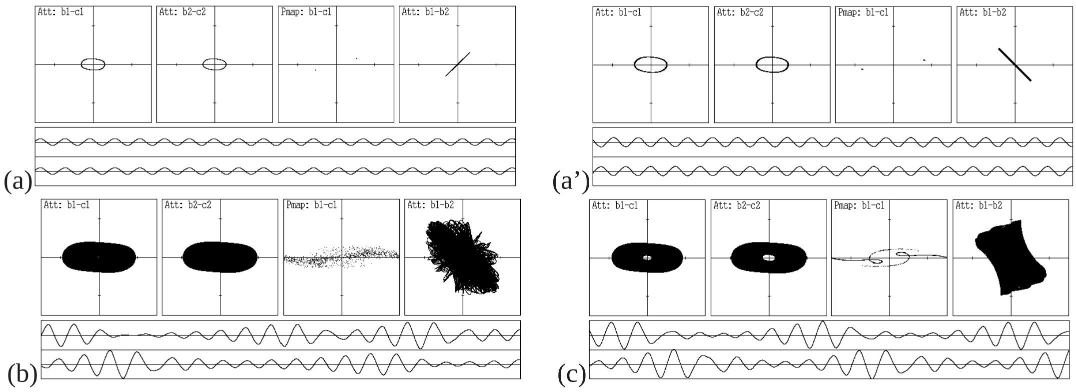

Figure 5: Attractors, Poincaré maps, phase differences and time series. $k_1 = 0.37$, $k_2 = 0.06$, $u_0 = 0.07$ and $\gamma = 0.05$. (a) and (a') $B = 0.14$. (b) $B = 0.24$. (c) $B = 0.256$.

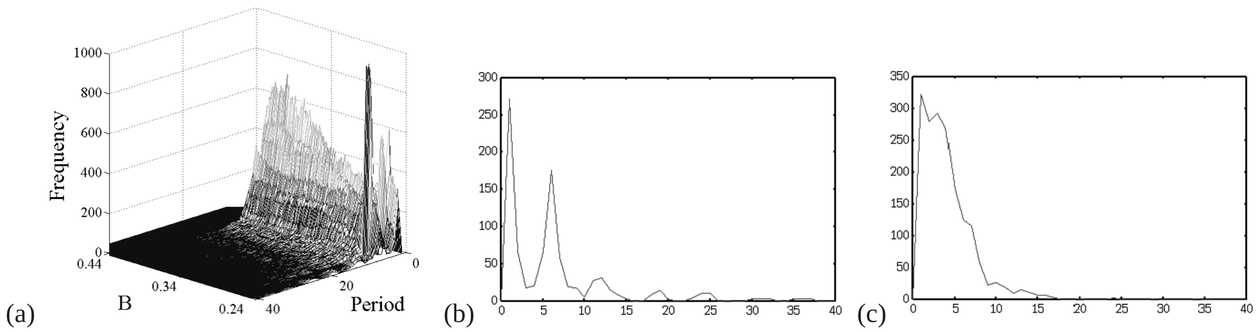

Figure 6: Sojourn periods for the self-switching. $k_1 = 0.37$, $k_2 = 0.06$, $u_0 = 0.07$ and $\gamma = 0.05$. (a) $0.24 \leq B < 0.40$. (b) $B = 0.24$. (c) $B = 0.300$.

In this region, long sojourn periods over 20 period are not confirmed. Additionally, the sojourn period becomes smaller with incleasing B. From these results, we confirm that characteristics of the sojourn time are different across the region that the quasi-periodic attractors are generated.

5. Conclusions

In this study, we investigated synchronization phenomena in parametrically excited simple LC oscillators coupled by a capacitor. In this system, we observed two different types of synchronization phenomena; coexistence phenomenon of in-phase synchronization and opposite-phase synchronization and self-switching of in and opposite-phases synchronizations. Additionally, we also confirmed that a sojourn time of the self-switching is changed by changing the amplitude of the time-varying inductor.

Acknowledgments

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