2009 International Workshop on Nonlinear Circuits and Signal Processing
NCSP'09, Waikiki, Hawaii, March 1-3, 2009

Synchronous Oscillation of Resonator-Coupling Oscillators

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Abstract

Synchronization is common phenomenon in the field of natural science. It should be noted that mutual synchronization phenomenon of oscillators gives various phase states and there have been many investigations on these phenomenon. In previous study, we confirmed the basic synchronization of three oscillators with the same natural frequencies coupled by a resonator. In this study, we investigate oscillation frequency of resonator-coupling three oscillators to carried out the sychronization phenomenon in detail.

1. Introduction

In our surroundings, there are a lot of synchronous phenomena. It is not unusual at all. For example, synchronus luminescence of firefly group is widely known for synchronous phenomena. When you observe the firefly group, the interval of firefly luminescence is gradually becomes the equal interval. However, this mechanism has not been clarified in detail. In a familiar point, the activity of our brain is a synchronous phenomenon. We feel, think about something, at the moment, the hundreds million of neuron of the brain synchronizes and exchanges pulses. In the others, cell of heart producing pulses at equal intervals and revolution of the moon etc. are good examples in which a synchronous phenomenon is comprehensible. Similarly, synchronization is common phenomenon in the field of natural science. However, it is defficult to clarified the mechanism of synchronous phenomena in natural science. We believe that investigating simpler synchronous phenomena (ex.synchronous oscillators) is the key that arrives at synchronous of firefly group and more complex synchronous phenomenon.

There have been many investigations of the mutual synchronization of oscillators ([1]-[6] and therein). Moro and one of the authors have confirmed that N oscillators with same natural frequencies mutually coupled by one resistor give N-phase oscillations. Their system can take $(N-1)!$ phase states, because of their system tends to minimize the current through the coupling resistor [7][8]. They thought that these coupling structure and huge number of steady states (for example, when their system take 479,001,600 steady states when $N = 13$.), would be structural element of cellular neural network or may be used as an extremely large memory.

In our previous stady, we observed synchronization of three oscillators coupled by a resonator. Resonator was consisted of parallel circuit of a capacitor and an inductor. We observed in-phase oscillation and two types of three-phase oscillations from the coupled oscillators by using the Runge-Kutta method [9].

In this study, we investigate frequency characteristics of the synchronized oscillations and distorted waveforms by using SPICE in detail.

2. Circuit Model

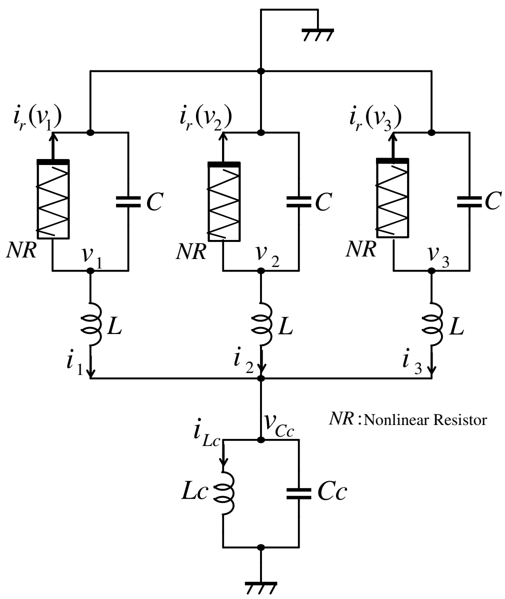

Figure 1: Circuit model.

The circuit model is shown in Fig. 1. Three oscillators with the same natural frequencies are mutually coupled by a resonator ($L_C C_C$ circuit). The circuit equations are described as Eq. (1).

$$\begin{cases} C\dfrac{dv_k}{dt} &= -i_k - i_r(v_k) \\ L\dfrac{di_k}{dt} &= v_k - v_{Cc} \qquad (k = 1, 2, 3) \\ Cc\dfrac{dv_{Cc}}{dt} &= \sum_{j=1}^{3} i_j - i_{Lc} \\ Lc\dfrac{di_{Lc}}{dt} &= v_{Cc} \end{cases} \tag{1}$$

where $i_r(v_k)$ indicates the $v-i$ characteristics of the nonlinear

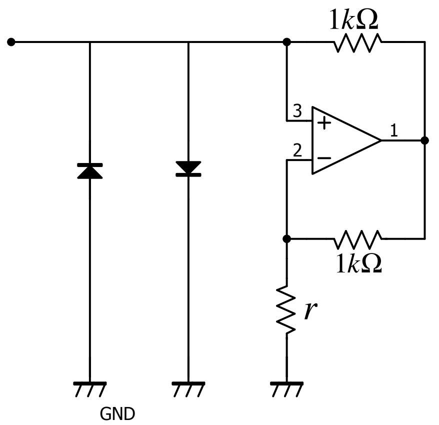

Figure 2: Nonlinear resistor.

resistor, which is approximated by Eq. (2).

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3. \tag{2}$$

For circuit experiments, the nonlinear resistor is realized as shown in Fig. 2. Note that when r is small, the nonlinearity is strong. By using the following variables and parameters,

$$\begin{cases} v_k = \sqrt{\dfrac{g_1}{g_3}}\, x_k, \quad i_k = \sqrt{\dfrac{Cg_1}{Lg_3}}\, y_k, \\ v_{Cc} = \sqrt{\dfrac{g_1}{g_3}}\, X, \quad i_{Lc} = \sqrt{\dfrac{Cg_1}{Lg_3}}\, Y, \\ t = \sqrt{LC}\,\tau, \quad \text{“}\,\cdot\,\text{”} = \dfrac{d}{d\tau}, \\ \varepsilon = \sqrt{\dfrac{L}{C}}\, g_1, \quad \beta = \dfrac{C}{Cc}, \quad \gamma = \dfrac{L}{Lc}, \end{cases} \tag{3}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k &= -y_k + \varepsilon(x_k - x_k^3) \\ \dot{y}_k &= x_k - X \qquad (k = 1, 2, 3) \\ \dot{X} &= \beta\left(\sum_{j=1}^{3} y_j - Y\right) \\ \dot{Y} &= \gamma X \end{cases} \tag{4}$$

3. Basic Synchronization Phenomena

Basic synchronization phenomena are shown in Fig. 3. We can observe three patterns of oscillations for the same parameter; in-phase synchronization and two types of three-phase synchronizations. The observed synchronous patterns depend on the initial states. Also the circuit experimental results show similar phenomena to the numerical results.

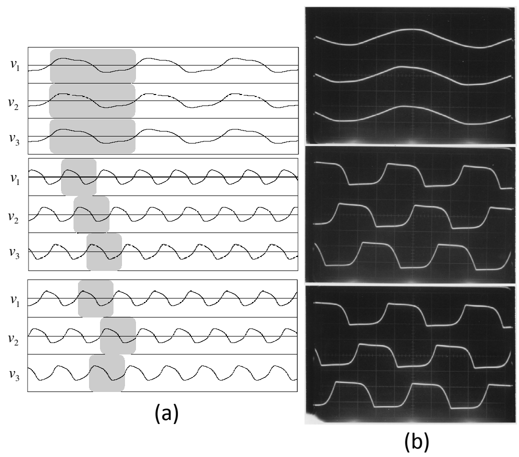

Figure 3: (a)Time waveform of in-phase oscillation and two types of three-phase oscillations (numerical results). ε=β=γ=1.0. (b)Time waveform of in-phase oscillation and two types of three-phase oscillations (experimental results). L=L_c=10mH, C=C_c=68nF and r=250Ω. Horizontal scale: 50μs/div. and Vertical scale: 1.0V/div.

To investigate the synchronization phenomenon when the parameters of the coupling resonator are changed, one of the parameters are fixed to 1.0, and the other parameter (β or γ) is changed. First, the parameter β is fixed to 1, and the parameter γ is changed from 0.3 to 2.7. For any values of γ, the three patterns of synchronization; in-phase oscillation and two types of three-phase oscillations, are able to be confirmed. We should note that the oscillation frequency of the three-phase oscillations is almost the same for different γ. On the other hand, the frequency of the in-phase oscillation increases as γ increases.

Secondly, the parameter γ is fixed to 1, and the parameter β is changed from 0.3 to 2.7. We can observe the three patterns of synchronization as well as the previous case. However, in this case, when β was changed, the oscillation frequency of either in-phase oscillation or three-phase oscillation does not change.

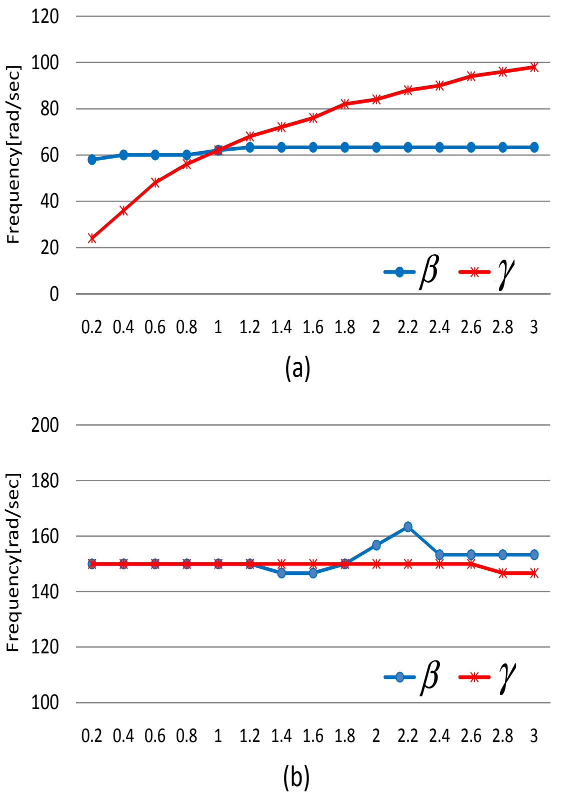

Figure 4: The change of the frequency for changing the parameters. (a)The frequency of the in-phase oscillation. (b)The frequency of the three-phase oscillation. Horizontal axis is the change β or γ.

4. Frequency Analysis by SPICE

Because of the detailed oscillation frequency is not obtained by the Runge-Kutta method, we analyzed the oscillation frequency by using SPICE. We investigated the change in the oscillation frequency as changing β or γ. The change in oscillation frequency of the in-phase oscillation and the three-phase oscillation are shown in Fig. 4. Horizontal axis is the changing β or γ. When β is changed, γ is fixed to 1.0. Also, When γ is expanded, β is fixed to 1.0. The oscillation frequency of the in-phase oscillation increases as γ increases. On the other hands, there are no remarkable change in other oscillation frequencies. The value of L_c is more strongly influenced to the oscillation frequencies than the value of C_c. We think the reason of this result is that L of the oscillator is connected to the resonator.

5. Analysis of distorted waveforms

Also, we observed distorted waveforms of synchronizations for some parameter values in SPICE. The distorted oscillations obtained by using the Runge-Kutta method are shown in Fig. 5 for the reference. The distorted oscillations obtained by using SPICE are shown in Figs. 6 and 7. The frequency characteristics of the distorted oscillations obtained by using FFT of SPICE are shown in Figs. 8 and 9. We confirmed that the distortion comes from the third-order harmonics of the oscillations.

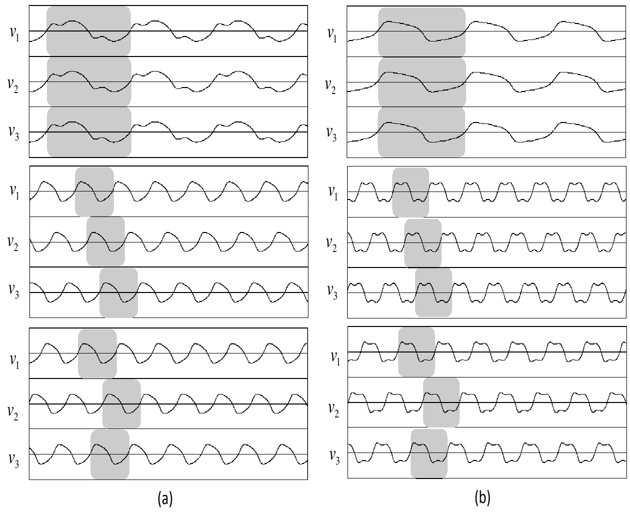

Figure 5: Distorted oscillation by Runge-Kutta method for ε=γ=1.0. (a) β=0.3. (b) β=2.1.

6. Conclusions

In this study, we have investigated frequency characteristics of the synchronized oscillations in three oscillators coupled by a resonator by using SPICE. Further, detailed analysis of the distorted waveforms was carried out.

Our future work is to investigate synchronous oscillators with various circuits in more detail.

Acknowledgments

This work was partly supported by Yazaki Memorial Foundation for Science and Technology.

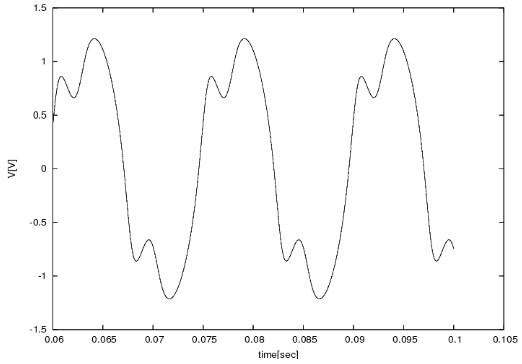

Figure 6: The distorted in-phase oscillation by SPICE for ε=γ=1.0, β=2.1.

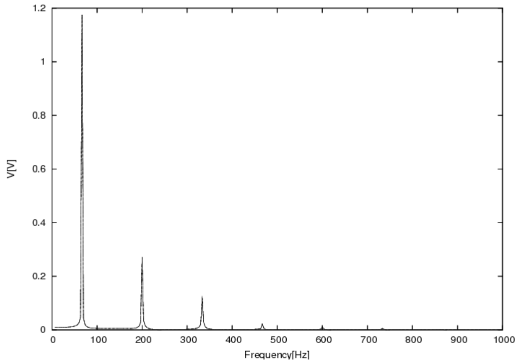

Figure 8: The frequency of the distorted in-phase oscillation by SPICE for ε=γ=1.0, β=2.1.

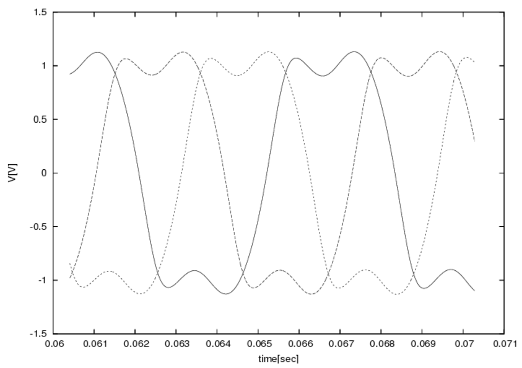

Figure 7: The distorted three-phase oscillation by SPICE for ε=γ=1.0, β=0.3.

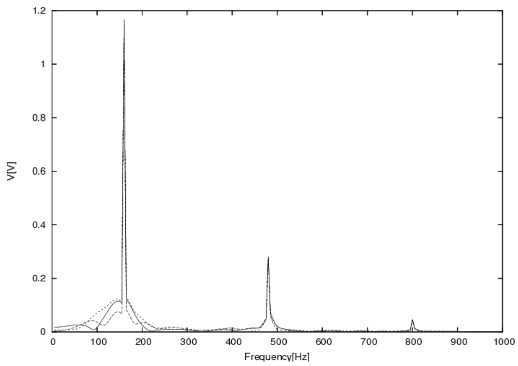

Figure 9: The frequency of the distorted three-phase oscillation by SPICE for ε=γ=1.0, β=0.3.

References

[1] L.L. Bonilla, C.J. Perez Vicente and R. Spigler, "Timeperiodic phases in populations of nonlinearly coupled oscillators with bimodal frequency distributions," Physica D: Nonlinear Phenomena, vol.113, no.1, pp.79-97, Feb. 1998.

[2] J.A. Sherratt, "Invading wave fronts and their oscillatory wakes are linked by a modulated traveling phase resetting wave," Physica D: Nonlinear Phenomena, vol.117, no.1-4, pp.145-166, June 1998.

[3] G. Abramson, V.M. Kenkre and A.R. Bishop, "Analytic solutions for nonlinear waves in coupled reacting systems," Physica A: Statistical Mechanics and its Applications, vol.305, no.3-4, pp.427-436, Mar. 2002.

[4] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, "Synchronization and graph topology," Int. J. Bifurcation and Chaos, vol.15, no.11, pp.3423-3433, Nov. 2005.

[5] C.M. Gray, "Synchronous oscillations in neural systems: mechanisms and functions," J. Computational Neuroscience, vol.1, pp.11-38, 1994.

[6] J. Cosp, J. Madrenas, E. Alarcon, E. Vidal and G. Villar, "Synchronization of nonlinear electronic oscillators for neural computation," IEEE Transactions on Neural Networks, vol.15, no.5, pp.1315-1327, Sep. 2004.

[7] Y. Nishio and S. Mori, "Mutually Coupled Oscillators with an Extremely Large Number of Steady States," Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS'92), vol.2, pp.819-822, May 1992.

[8] S. Moro, Y. Nishio and S. Mori, "Synchronization Phenomena in Oscillators Coupled by One Resistor," IEICE Transactions on Fundamentals, vol.E78-A, no.2, pp.244-253, Feb. 1995.

[9] C. Higashi, and Y. Nishio. "Synchronization Phenomena in Three Oscillators Coupled by a Resonator," Proceedings of International Workshop on Vision, Communications and Circuits (IWVCC'08), pp.35-38, Nov 2008.