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Relationship Between Oscillatory Phenomena and Templates on a Two-Template CNN

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***Abstract*— In our previous studies, we have proposed a two template CNN and have investigated some kinds of boundary conditions. As a result, it was revealed that oscillatory phenomena are not observed in the case of periodic boundary conditions.**

In this study, we investigate the relationship between oscillatory phenomena and templates on a two-template CNN whose output function is modified. By using modified output function, oscillatory phenomena are observed in the case of periodic boundary conditions. Additionally, the synchronization phenomena are observed by changing templates parameters. We consider that investigating this system is important to understanding coupled oscillatory systems. Because this system is a new class of coupled oscillatory systems.

I. Introduction

Cellular Neural Networks (CNNs) [1]-[3] is one kind of mutually coupled neural networks. The main characteristics are the local connection and the parallel signal processing. There have been many studies on CNNs and many kinds of CNNs have been proposed. One of them is two-layer CNN [4]. Two-layer CNN can generate many interesting phenomena. For instance, self-organizing pattern [4], active wave propagation [5] and so on are observed. Like this, some kinds of CNNs generate interesting phenomena. Investigating these system contributes to understand complex systems.

In our previous studies [6], we have proposed a two-template CNN and have investigated some observed phenomena. Additionally, we investigated some kinds of boundary conditions in this system. As a result, oscillatory phenomena, clustering phenomena, pattern formation, active wave propagation and so on are observed. We consider that this system is a novel coupled oscillatory system. The reason of why we consider is that the cell connects with four the other cells and the other cell also connects with four neighbor cells. Namely, a pair of cells which have two kinds of templates are needed for a simple oscillation. Like this, these cells are sharing a factor of the oscillation. This type of connection may be difficult to realize by coupling normal oscillators. However, these phenomena are not observed in the case of periodic boundary conditions.

In this study, we investigate the relationship between oscillatory phenomena and templates on a two-template CNN. By using the modified output function. We consider that investigating this system is important to understanding coupled oscillatory systems.

II. CNN using two kinds of template

Figure 1 shows a system model of two-template CNN. We assume that the system has a two-dimensional M by N array structure. Each cell in the array is denoted as $c(i, j)$, where (i, j) is the position of the cell, $1 \le i \le M$ and $1 \le j \le N$. The coupling radius is assumed to be one. Cells having one template are called as cell α and the other are called as cell β. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

1: The case that $i + j$ is an even number.

$$\begin{aligned} \frac{dx_{ij}}{dt} =& -x_{ij} + I_\alpha \\ &+ \sum_{c(k,l)} A_\alpha(i,j;k,l)y_{kl} \\ &+ \sum_{c(k,l)} B_\alpha(i,j;k,l)u_{kl} \end{aligned} \tag{1}$$

2: The case that $i + j$ is an odd number.

$$\begin{aligned} \frac{dx_{ij}}{dt} =& -x_{ij} + I_\beta \\ &+ \sum_{c(k,l)} A_\beta(i,j;k,l)y_{kl} \\ &+ \sum_{c(k,l)} B_\beta(i,j;k,l)u_{kl} \end{aligned} \tag{2}$$

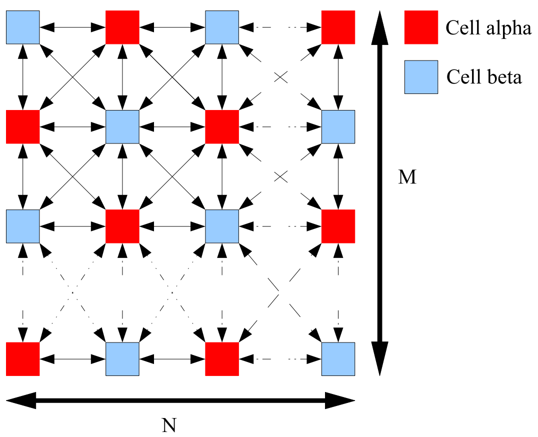

Fig. 1. Structure of two-template CNN.

$A_{\{\alpha\beta\}}(i,j;k,l)y_{kl}$, $B_{\{\alpha\beta\}}(i,j;k,l)u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively. The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}). \tag{3}$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \tag{4}$$

The variables u and y are the input and output variables of the cell, respectively. $\boldsymbol{A}_\alpha$, $\boldsymbol{B}_\alpha$, $\boldsymbol{A}_\beta$ and $\boldsymbol{B}_\beta$ are 3 times 3 matrices, which can be described to have a similar form to Eq. (5).

$$\begin{pmatrix} A_\alpha(i,j;i-1,j-1) & A_\alpha(i,j;i-1,j) & A_\alpha(i,j;i-1,j+1) \\ A_\alpha(i,j;i,j-1) & A_\alpha(i,j;i,j) & A_\alpha(i,j;i,j+1) \\ A_\alpha(i,j;i+1,j-1) & A_\alpha(i,j;i+1,j) & A_\alpha(i,j;i+1,j+1) \end{pmatrix} \tag{5}$$

In previous study, a piecewise-linear function shown in Eq. 4 was applied as the output function. We could not observe oscillatory phenomena in the case of periodic boundary condition. However, the oscillatory phenomena are observed by modifying a output function. The modified output function is given as a following function.

$$f(x) = -x + |x+1| - |x-1|. \tag{6}$$

This proposed system is more complex than the normal CNNs. Although this system has a unique characteristic. Namely, a pair of cell α and cell β are needed for a simple oscillation. Additionally, one cell α connects with four neighbor cells β and one cell β also connects with four neighbor cells α. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators. Hence, we consider that this system is a new class of coupled oscillatory systems.

III. Relationship between Oscillatory Phenomena and Parameters

In our previous study, some kinds of boundary conditions have been investigated. Then, initial state values set as random values. The parameter of templates are given as follows:

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} 1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \tag{7}$$
$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0.$$

As a result, it was revealed that oscillatory phenomena are affected by virtual cells. Furthermore, it was also revealed that oscillatory phenomena are not observed in the case of periodic boundary conditions which has no virtual cells.

In this study, we investigate in the case of modified output function shown in Eq. 6. We guess that coupling coefficients and oscillation factors are corresponding to values of A_α and A_β. Thus, the templates are set as follows.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -u & v & -u \\ v & w & v \\ -u & v & -u \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} u & -v & u \\ -v & -w & -v \\ u & -v & u \end{pmatrix},$$
$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0. \tag{8}$$

Namely, we guess that coupling coefficients and oscillation factors are corresponding to u and v, respectively. The number of cells is fixed as 8×8. Because that in the case of lower than 8×8, only one kind of synchronization phenomena is observed and in the case of over 8×8, various kinds of oscillatory phenomena are observed.

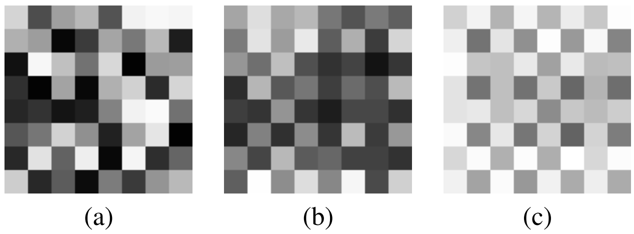

Fig. 2. Simulation results in the case of periodic boundary condition. The number of cells is 8×8. $u = 1$, $v = 1$ and $w = 1$. (a) Initial state. (b) Transient state. (c) Stable state.

Figure 2 shows a computer simulation result in the case of $u = 1$, $v = 1$ and $w = 1$. Each square shows a output of cells. Oscillatory phenomena are observed in Fig. 2 (b) and (c). In Fig 2 (b), each cell oscillates independently. This phenomenon is a transient state. After that, synchronization-like phenomena are observed like as Fig. 2 (c). In this result, we seem to be able to classify synchronization-like phenomena into four groups as shown in Fig. 3. In order to investigate the phenomena in detail, some computer simulations are carried out. The template parameters set as Eq. 8.

Figure 4 shows waveforms of state values. Vertical axes are state variables. Horizontal axes are time. The first row shows state variables of α_1 group. In the same way, the second, third and fourth row shows state variables of β_1, β_2 and α_2, respectively. Each row includes sixteen waveforms. This result is corresponding to Fig. 2. This result shows that these are not synchronized. However, some waveforms of them are similar forms. Like this method, we investigate waveforms in some parameter and initial values. Figures 5–8 show some typical results.

Figure 5 shows waveforms of state values in the case of $u = 1.2$, $v = 1$ and $w = 1$. By changing initial values, two kinds of stable states are observed like as Fig. 5 (a) and (b). All cells of each row are synchronized. Thus, we can see one waveform per one row. A difference of Fig. 5 (a) and (b) is that first row and fourth row are replaced each other. The reason of why is symmetrical property of the system. Namely, by rotating the system 90 degree, Fig. 5 (a) becomes Fig. 5 (b).

Figure 6 shows the case that some waves of each group synchronize and over three stable states exist. Different initial values are applied to Fig. 6 (a), (b) and (c). In this case, waves of each row are classified some groups. Waves of each group are synchronized. Frequencies of these waves are same. Some of waves are same waveforms and different phases. Anti-phase synchronization phenomena are also observed. The first row of Fig. 6 (b) is similar to the inverted fourth row of Fig. 6 (c). Likewise, the second, third and fourth rows of Fig. 6 (b) is similar to the inverted third, second and first rows of Fig. 6 (c), respectively. Like this, many stable states are observed by changing initial values.

Figure 7 shows the case that switching phenomena are observed. In the left side of figure, all waves exist negative sides of vertical axes and these are not synchronized. In the center of figure, almost waves switch to the positive values in same time. Like this switching phenomena are observed randomly. Sojourn time until switching is corresponding to parameter v.

Figure 8 shows the case of increasing a parameter v from Fig. 6. Different initial values are applied to Fig. 8 (a) and (b). Same synchronization phenomena as shown in Fig. 6 are observed in Fig. 8 (a). On the other hand, similar phenomena as shown in Fig. 4 are observed in Fig. 8 (b). Like this, two kinds of phenomena are observed by initial values. By increasing or decreasing a parameter v from Fig. 6, a decrease of observation rates of synchronization phenomena is caused.

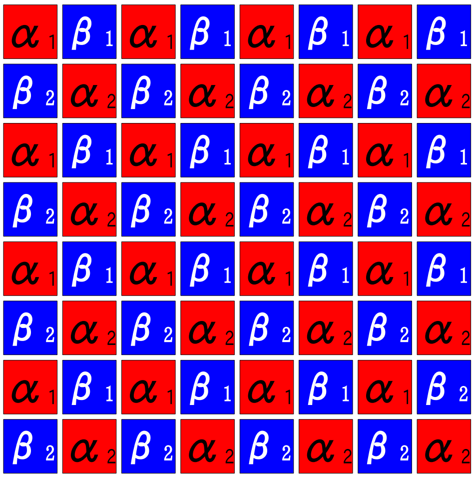

Fig. 3. The CNN array consists of 8 × 8.

IV. Conclusions

In this study, we have investigated the relationship between oscillatory phenomena and templates on a two-template CNN whose output function is modified. As results, follows phenomena are confirmed.

- Coexistence of some kinds of stable states
- Various kinds of synchronization phenomena
- Switching phenomena
- Existence of a relationship between synchronization phenomena and parameters

As future works, we will investigate these phenomena in detail.

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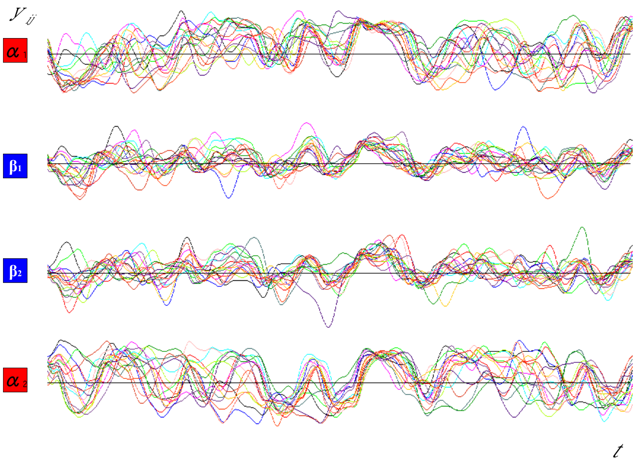

Fig. 4. Waveforms of state values. $u = 1.0$, $w = 1.0$ and $v = 1.0$. This result is corresponding to Fig. 2.

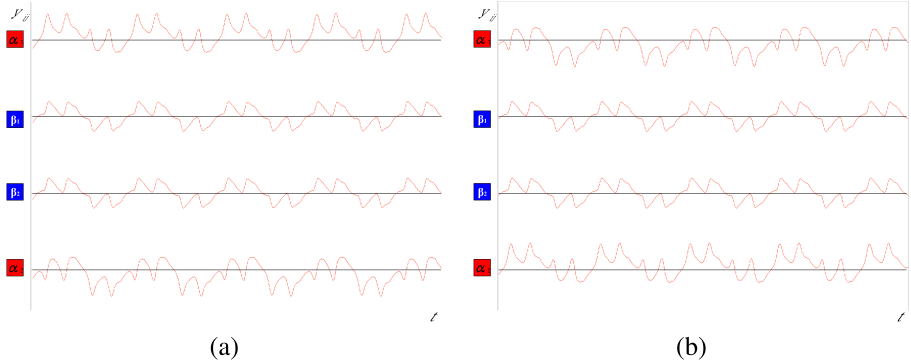

Fig. 5. Waveforms of state values in the case that some waves of each group synchronize and two stable states exist. $u = 1.2$, $w = 1.0$ and $v = 1.0$. Different initial values are applied to (a) and (b).

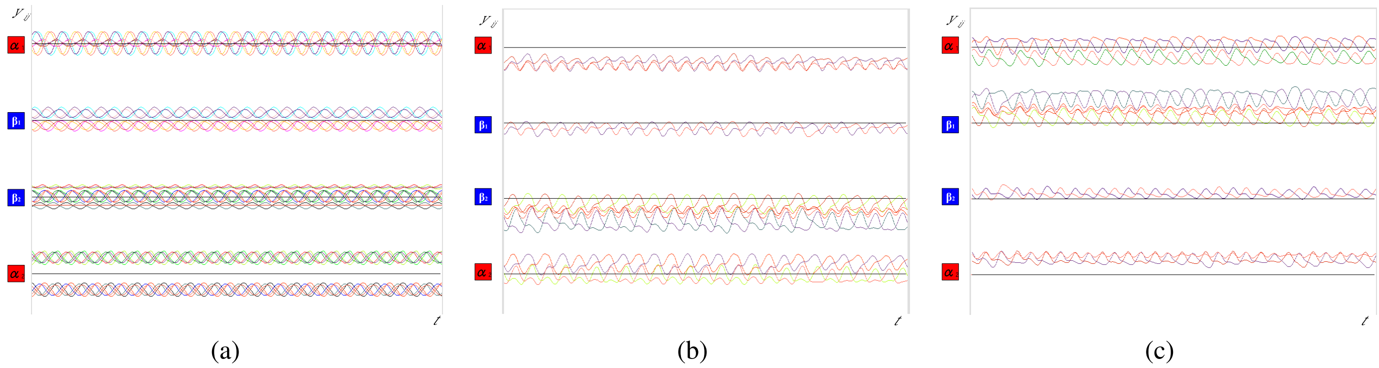

Fig. 6. Waveforms of state values in the case that waves of each group synchronize and over three stable states exist. $u = 1.0$, $w = 1.0$ and $v = 0.6$. Different initial values are applied to (a), (b) and (c).

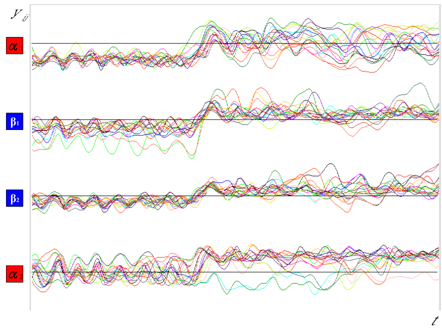

Fig. 7. Waveforms of state values in the case that switching phenomena are observed. $u = 1.0$, $w = 1.0$ and $v = 0.75$.

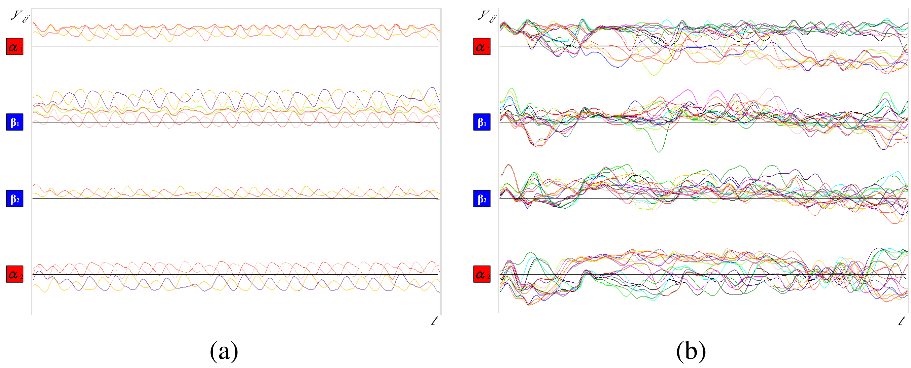

Fig. 8. Waveforms of state values in the case of changing a parameter from Fig. 6. $u = 1.0$, $w = 1.0$ and $v = 0.67$. Different initial values are applied to (a) and (b).