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Classification of phenomena on coupled oscillators system as a lattice

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Abstract

In our previous study, we observed that phenomena of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase and continuously exist. We call the phenomenon “phase-inversion waves.” The phase-inversion waves were observed on a ladder of coupled oscillators. In this study, the phase-inversion waves are observed on a lattice of coupled oscillators. The various phenomena of propagating phase differences, which include phase-inversion waves, are observed by changing initial values of each oscillator, coupling parameter and a nonlinearity of the circuit. Then, we clarify regions of the observed the phase-inversion waves.

1. Introduction

The studies of synchronization phenomena are researched in physics, biology, and various fields[1]-[2]. For example, the phase synchronization phenomena of the nervous system are researched. The coupled oscillators system is known that oscillators synchronize[3]. We investigate the synchronization phenomena by coupled oscillators system.

The phase-inversion waves, which are propagate in in-and-anti-phase synchronization and in in-phase synchronization, are observed on a ladder of coupled oscillators system[4].

In our previous study, we made a prediction system of time-series data[5]. The system is that some van der Pol oscillators are coupled by inductors as a lattice. We made clear that the system can predict time-series data, but the system does not predict automatically. Therefore, phenomena on the system shall be analyzed for developing automatic prediction system.

In this study, we investigate the synchronization phenomena on a lattice-shaped coupled van der Pol oscillators system. The phase-inversion waves of vertical and horizontal direction are observed when initial values of voltages and currents of oscillators are set as a plus value or a minus value alternately. State of phase-inversion waves are able to be observed three types when we change pattern of initial values. The observation regions of each kind of phase-inversion waves are made clear.

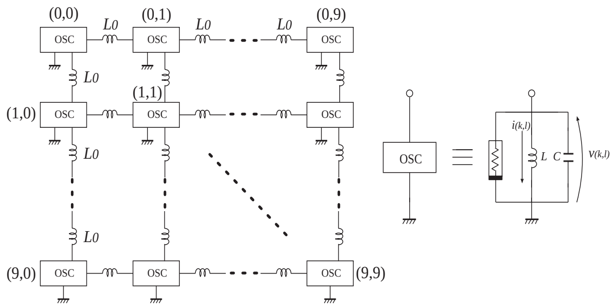


Figure 1: Circuit model.

2. Circuit model

The van der Pol oscillators are coupled by inductors L_0 as a lattice(see Fig.1). The number of rows of the system is 10. The number of columns of the system is 10. Each oscillator is named as OSC(k,l), a voltage of each oscillator is called $v_{(k,l)}$. $i_{(k,l)}$ is the current flowing through the inductor of each oscillator(see Fig.1). An equation of the nonlinear negative resistor is shown as Eq. (1). The circuit equations are normalized by Eq. (2). The normalized circuit equations are shown as Eqs. (3)–(7).

$$f_{(k,l)} = -g_1 v_{(k,l)} + g_3 v_{(k,l)}^3. \tag{1}$$

$$\begin{aligned} &i_{(k,l)} = \sqrt{\frac{Cg_1}{3Lg_3}} x_{(k,l)}, \quad v_{(k,l)} = \sqrt{\frac{g_1}{3g_3}} y_{(k,l)}, \\ &t = \sqrt{LC}\tau, \quad \frac{d}{d\tau} = \text{“ } \cdot \text{ ”}, \alpha = \frac{L}{L_0}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}}. \end{aligned} \tag{2}$$

[Corner–top: left and right]

$$\begin{aligned} \frac{dx_{(0,a)}}{d\tau} &= y_{(0,a)}, \\ \frac{dy_{(0,a)}}{d\tau} &= -x_{(0,a)} + \alpha(x_{(0,b)} + x_{(1,a)} - 2x_{(0,a)}) \\ &\quad + \varepsilon(y_{(0,a)} - \frac{1}{3}y_{(0,a)}^3). \end{aligned} \tag{3}$$

left:$a = 0$ and $b = 1$. right:$a = 9$ and $b = 8$.

[Corner–bottom: left and right]

$$\frac{dx_{(9,a)}}{d\tau} = y_{(9,a)}, \tag{4}$$

$$\frac{dy_{(9,a)}}{d\tau} = -x_{(9,a)} + \alpha(x_{(8,a)} + x_{(9,b)} - 2x_{(9,a)}) + \varepsilon(y_{(9,a)} - \frac{1}{3}y^3_{(9,a)}).$$

left:$a = 0$ and $b = 1$. right:$a = 9$ and $b = 8$.
[Center]

$$\frac{dx_{(k,l)}}{d\tau} = y_{(k,l)}, \tag{5}$$

$$\frac{dy_{(k,l)}}{d\tau} = -x_{(k,l)} + \alpha(x_{(k+1,l)} + x_{(k-1,l)} + x_{(k,l+1)} + x_{(k,l-1)} - 4x_{(k,l)}) + \varepsilon(y_{(k,l)} - \frac{1}{3}y^3_{(k,l)}).$$

where $0 < k < 9$, $0 < l < 9$.
[Edge–top and bottom]

$$\frac{dx_{(a,l)}}{d\tau} = y_{(a,l)}, \tag{6}$$

$$\frac{dy_{(a,l)}}{d\tau} = -x_{(a,l)} + \alpha(x_{(a,l-1)} + x_{(a,l+1)} + x_{(b,l)} - 3x_{(a,l)}) + \varepsilon(y_{(a,l)} - \frac{1}{3}y^3_{(a,l)}).$$

top:$a = 0$ and $b = 1$.
bottom:$a = 9$ and $b = 8$. both:$0 < l < 9$.
[Edge–left and right]

$$\frac{dx_{(k,a)}}{d\tau} = y_{(k,a)}, \tag{7}$$

$$\frac{dy_{(k,a)}}{d\tau} = -x_{(k,a)} + \alpha(x_{(k-1,a)} + x_{(k+1,a)} + x_{(k,b)} - 3x_{(k,a)}) + \varepsilon(y_{(k,a)} - \frac{1}{3}y^3_{(k,a)}).$$

left:$a = 0$ and $b = 1$.
right:$a = 9$ and $b = 8$. both:$0 < k < 9$.
The α corresponds to the coupling of the oscillators. The ε corresponds to nonlinearity of each oscillator. These equations are calculated by the fourth order Runge-Kutta methods.

3. Classification of the phase-inversion waves

The phase-inversion waves on a lattice of coupled oscillators are observed and are classified some phenomena. The initial values are changed to two patterns(A and B in Fig. 2), and the phenomena are observe. Figure 2 shows the signs of the initial values of the voltage and current of each oscillator.

+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−
+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−
+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−
+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−
+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−

(a) Initial values of pattern A.

+	+	−	−	+	+	−	−	+	+
+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−
−	−	+	+	−	−	+	+	−	−
+	+	−	−	+	+	−	−	+	+
+	+	−	−	+	+	−	−	+	+
−	−	+	+	−	−	+	+	−	−
−	−	+	+	−	−	+	+	−	−
+	+	−	−	+	+	−	−	+	+
+	+	−	−	+	+	−	−	+	+

(b) Initial values of pattern B.

Figure 2: Initial value.

3.1. Observation conditions

Initial values are fixed to two patterns and the phase-inversion waves are observed. We observe the phase-inversion waves under the following observation conditions.
<Pattern A>
Observation conditions of Pattern A are fixed as follows.

1. Initial values are fixed as shown in Fig 2(a).
2. The coupling parameter α and nonlinearity ε are changed.

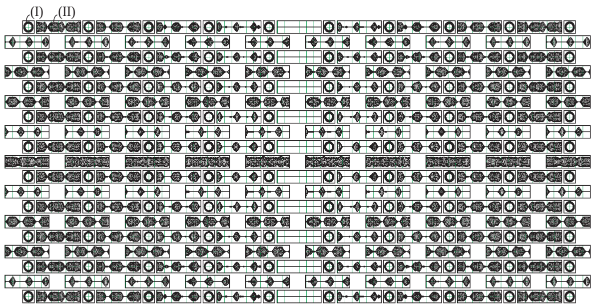
<Pattern B>
Observation conditions of Pattern B are fixed as follows.

1. Initial values are fixed as shown in Fig 2(b).
2. The coupling parameter α and nonlinearity ε are changed.

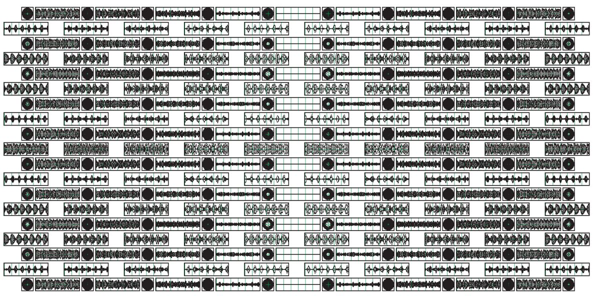
3.2. Experiment result

The Figs.3 and 6 show the observation phenomena. The Fig.3(a)-(I) shows the attractor of each circuit. The horizontal axis expresses voltage $y_{(n,m)}$, and the vertical axis expresses current $x_{(n,m)}$. The Fig.3(a)-(II) shows a transition of a phase difference between adjacent oscillators. The horizontal axis expresses time, and the vertical axis expresses a voltage which is sum of two voltages of adjacent oscillators. The vertical axis is $y_{(n,m)} + y_{(n+1,m)}$ or $y_{(n,m)} + y_{(n,m+1)}$. The amplitude doubles if in-phase synchronization is observed between adjacent oscillators. On the other hand, the amplitude becomes zero if anti-phase synchronization can be observed between adjacent oscillators. The Figs.5 and 7 show the region of the observation phenomena. The vertical axis is nonlinearity. The horizontal axis is coupling parameter.

States, that the phase-inversion wave propagate, are classified to three types. First type wave of phase-inversion waves is observed in in-and-anti-phase synchronization, and the phase-inversion waves propagate to vertical direction. Second type wave of phase-inversion waves is observed in in-phase synchronization, and the phase-inversion waves propagate to horizontal direction. Third type wave of phase-inversion waves is observed in in-and-anti-phase synchronization, and the phase-inversion waves propagate to horizontal direction.



(a) Phase-inversion waves of region-(i)(α=0.01 and ε=0.05).



(b) Phase-inversion waves of region-(ii)(α=0.02 and ε=0.02).

Figure 3: Phenomena of phase-inversion waves.

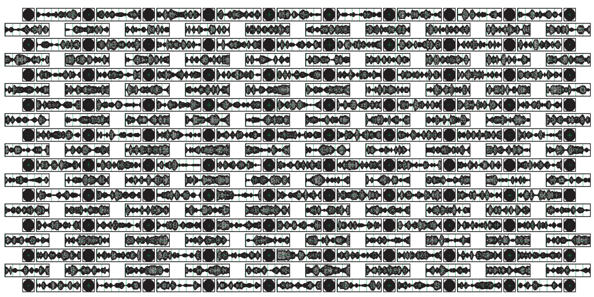


Figure 4: An example of region-(iii)(pattern A, α=0.01 and ε=0.02).

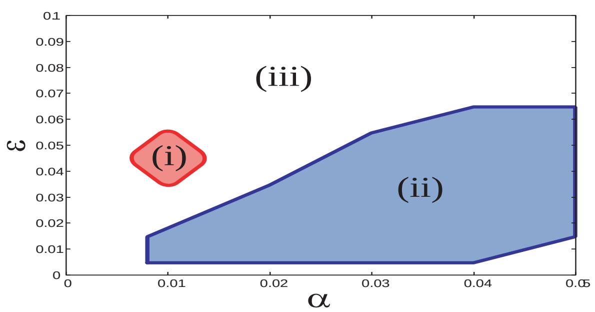


Figure 5: Region of pattern A. Region-(i): There are the first and the second types. Region-(ii): There are the first type. Region-(iii): The complex phenomena.

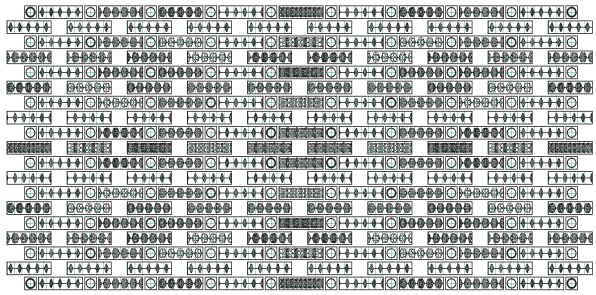


Figure 6: Phase-inversion waves of region-(iv)(α=0.02 and ε=0.15).

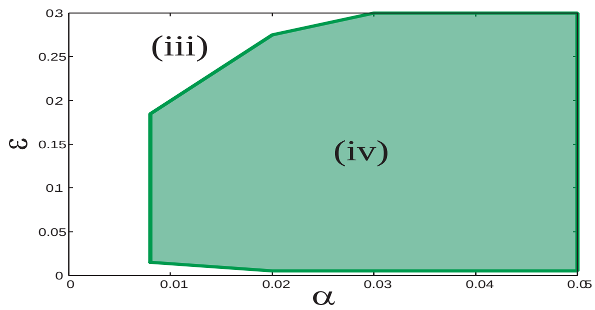


Figure 7: Region of pattern B. Region-(iii): The complex phenomena. Region-(iv): There are the first and the third types.

3.2.1. Pattern A

Observed phenomena are classified into two patterns. Observation results are shown in Figs. 3 and 4, and are classified to Fig. 5.

Two types of states of the phase-inversion waves are observed. In the Fig. 3(a), the first and the second type wave are observed(see Fig. 5-(i)). In the Fig. 3(b), the first type wave can be observed, but the second type wave can not be observed(see Fig. 5-(ii)).

The complex phenomena are observed outside the region where the phase-inversion waves are observed(see Fig. 4 and 5-(iii)).

3.2.2. Pattern B

Observation result is shown in Fig. 6. Figure 7 shows the regions of the observation phenomena. In the Fig. 6, the first and the third type waves are observed(see Fig. 7-(iv)).

The complex phenomena are observed outside the region where the phase-inversion waves are observed(see Fig. 4 and 7-(iii)).

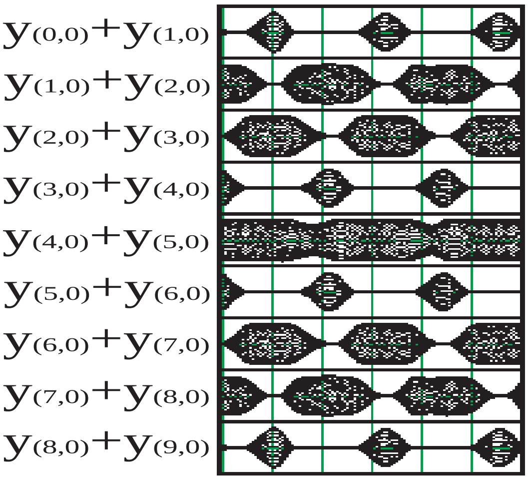


Figure 8: Extraction of the Fig. 3(a) in vertical direction.

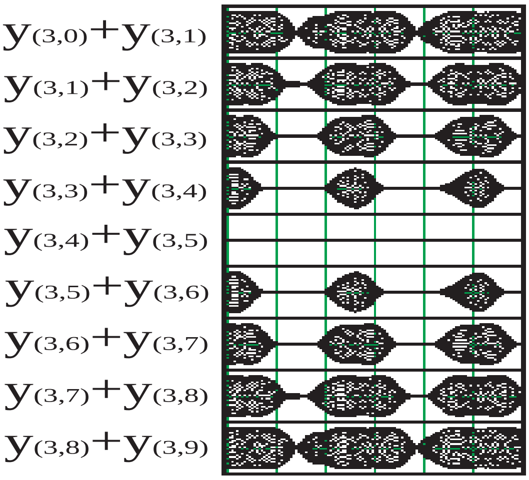


Figure 9: Extraction of the Fig. 3(a) in horizontal direction.

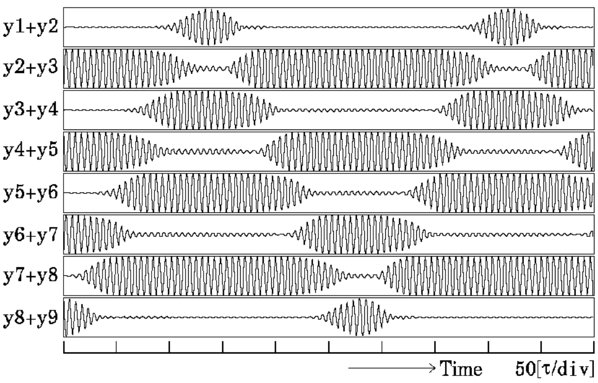


Figure 10: Phase-inversion waves in the state of in-and-anti-phase synchronization on a ladder of coupled oscillators system.

3.3. Specification of phenomena

Figures 8 and 9 are extracted from the Fig. 3(a). The phase-inversion waves propagate in in-and-anti-phase synchronization and in-phase synchronization.

Figure 10 show the phase-inversion waves in the state of in-and-anti-phase synchronization on a ladder of coupled oscillators. When we think about the vertical direction and horizontal direction separately, the same phenomena are observed on the ladder system and the lattice system.

4. Conclusion

We observed and classified the phenomena of the phase-inversion waves. Observed phenomena were classified into two patterns when initial values were fixed as Fig. 2(a). Observed phenomena were clarified when initial values were fixed as Fig. 2(b). On the ladder shape systems, we can observe phase-inversion waves which propagate in in-and-anti-phase synchronization and in in-phase synchronization. On the lattice shape systems, we can observe both types of propagation of phase-inversion waves at the same time.

Acknowledgements

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