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Comparison of Chaos Synchronization and Noise Effect on Simple Oscillators

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Abstract

In this study, the breakdown of synchronization observed from four coupled chaotic oscillators is investigated. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with chaos noise is considered. And the logistic map is used to generate chaos noise. The comparison of the coupled chaotic oscillators with the coupled modified van der Pol oscillators with chaos noise gives us some interesting results.

1. Introduction

Synchronization is one of typical nonlinear phenomena observed in the field of natural science. There have been many investigations on mutual synchronization of oscillators. Breakdown of synchronization is a kind of cooperative phenomenon for dissipated assembly oscillators and is important to clarify its mechanism for better understanding of higher-dimensional complicated phenomena [1]-[6]. In the case of coupled chaotic oscillators, chaotic fluctuations of their waveforms are supposed to play a role to break the synchronization and the breakdown sometimes causes chaotic wandering phenomenon.

On the other hand, there is certainly noise in actual physical systems. In these years, investigations on systems including noise attract many researchers' attentions. We are interested in the difference between chaotic fluctuation and noise, because sometimes chaos exhibits better performance than random noise in information processing tasks [7].

In our previous research, we have investigated the breakdown of synchronization observed from four coupled chaotic oscillators. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with the additive white Gaussian noise (AWGN) was proposed. By computer simulations, we have confirmed that chaotic systems were synchronized more stably than modified van der Pol oscillators with AWGN [8].

In this study, we propose chaos noise for adding the van der Pol oscillators to confirm the different between chaotic systems and van der Pol systems with AWGN.

2. Circuit Model

2.1. Chaotic Oscillators

Figure 1 shows the circuit model.

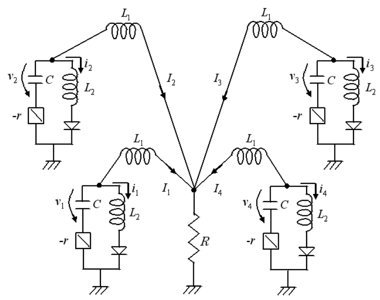

Figure 1: Coupled chaotic oscillators.

In this circuit, four identical chaotic oscillators are coupled by one resistor. First the $i-v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = \frac{1}{2}(r_d i_k + E - |r_d i_k - E|). \tag{1}$$

By changing the variables and the parameters,

$$\begin{aligned} &I_k = \sqrt{\frac{C}{L_1}}Ex_k, \quad i_k = \sqrt{\frac{C}{L_1}}Ey_k, \quad v_k = Ez_k, \\ &t = \sqrt{L_1 C}\tau, \quad \alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \\ &\gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = r_d\sqrt{\frac{C}{L_1}}, \end{aligned} \tag{2}$$

the normalized circuit equations of the circuit are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k - \gamma\displaystyle\sum_{j=1}^{4} x_j \\ \dfrac{dy_k}{d\tau} = \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dfrac{dz_k}{d\tau} = x_k + y_k \qquad\qquad (k = 1, 2, 3, 4) \end{cases} \tag{3}$$

where

$$f(y_k) = 0.5(\delta y_k + 1 - |\delta y_k - 1|). \tag{4}$$

Figure 2 shows an example of the observed four-phase quasi-synchronization of chaos. In the figures the phase differences of x_2, x_3 and x_4 with respect to x_1 are almost 90°, 180° and 270°, respectively.

Because of symmetry of the coupling structure, six different combinations of phase states coexist.

As the coupling resistance R (γ) decreases, we can observe that the synchronization states become unstable and that the self-switching phenomenon of the states occurs [6]. The breakdown of the synchronization means the start of the self-switching, hence it is important to investigate how the synchronization breaks down.

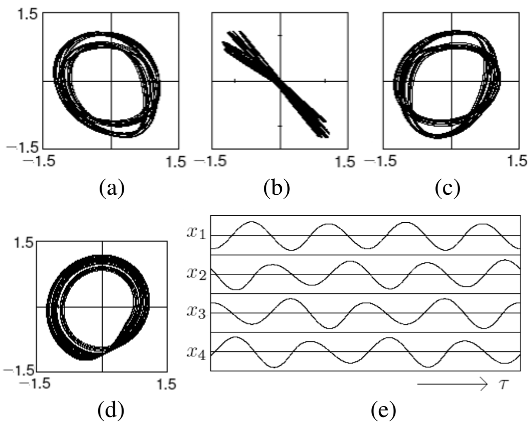

Figure 2: Four-phase quasi-synchronization of chaos (computer calculated result). $\alpha = 7.0$, $\beta = 0.10$, $\gamma = 0.10$ and $\delta = 100.0$. (a) x_1 vs. x_2. (b) x_1 vs. x_3. (c) x_1 vs. x_4. (d) x_1 vs. z_1. (e) Time waveforms.

2.2. Modified van der Pol Oscillators

Next, we consider four coupled van der Pol oscillators. In

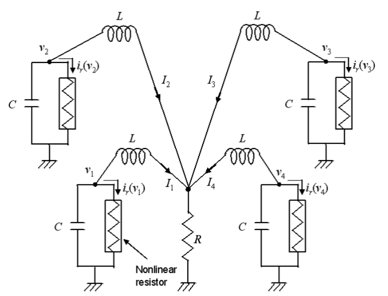

Figure 3: Coupled van der Pol oscillators.

order to obtain the waveforms similar to those of the chaotic oscillator, we modify the van der Pol oscillator with the nonlinear resistor whose $v - i$ characteristics are described by the following asymmetric function

$$i_r(v_k) = -g_1 v_k + g_2 v_k^2 + g_3 v_k^3 \ \ (g_1, g_2, g_3 > 0). \tag{5}$$

By changing the variables and the parameters,

$$\begin{aligned} v_k &= \sqrt{\frac{g_1}{g_3}} x_k, \ \ i_k = \sqrt{\frac{C g_1}{L g_3}} y_k, \ \ t = \sqrt{LC}\tau, \\ \nu &= \frac{g_2}{\sqrt{g_1 g_3}}, \ \ \gamma = R\sqrt{\frac{C}{L}}, \ \ \varepsilon = g_1\sqrt{\frac{L}{C}}, \end{aligned} \tag{6}$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \xi\left\{-y_k + \varepsilon(x_k - \nu x_k^2 - x_k^3)\right\} \\ \dfrac{dy_k}{d\tau} = x_k - \gamma\displaystyle\sum_{j=1}^{4} y_j \\ (k = 1, 2, 3, 4) \end{cases} \tag{7}$$

where ξ is the parameter added to tune the period of the waveform.

Figure 4 shows an example of the observed four-phase synchronization of the modified van der Pol oscillators.

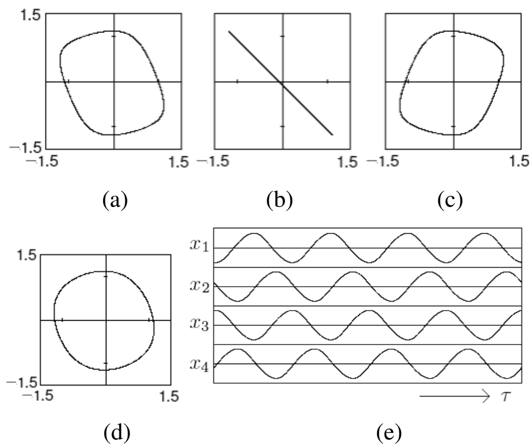

Figure 4: Four-phase synchronization of modified van der Pol oscillators (computer calculated result). $\varepsilon = 0.50$, $\gamma = 0.60$, $\xi = 1.07$ and $\nu = 0.1035$. (a) x_1 vs. x_2. (b) x_1 vs. x_3. (c) x_1 vs. x_4. (d) x_1 vs. y_1. (e) Time waveforms.

In this study, we consider the case that the noise is added to the modified van der Pol oscillator in order to simulate the breakdown of the synchronization in the coupled chaotic oscillators caused by chaotic fluctuations of their waveforms.

When we add the noise to the voltage amplitude of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \xi[-y_k + \varepsilon\{(1 + \rho_k n_k(\tau))x_k \\ \qquad - \nu((1 + \rho_k n_k(\tau))x_k)^2 - ((1 + \rho_k n_k(\tau))x_k)^3\}] \\ \dfrac{dy_k}{d\tau} = (1 + \rho_k n_k(\tau))x_k - \gamma\displaystyle\sum_{j=1}^{4} y_j \\ (k = 1, 2, 3, 4) \end{cases} \tag{8}$$

where $n_k(\tau)$ is the added noise and ρ_k is constant to tune the amplitude of the noise.

While when we add the noise to the voltage period of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = (1 + \rho_k n_k(\tau))\xi\{-y_k + \varepsilon(x_k - \nu x_k^2 - x_k^3)\} \\ \dfrac{dy_k}{d\tau} = x_k - \gamma \displaystyle\sum_{j=1}^{4} y_j \qquad (k = 1, 2, 3, 4). \end{cases} \tag{9}$$

The noise $n_k(\tau)$ is the additive white Gaussian noise (AWGN) with the average 0 and the variance σ^2.

In the next section, the computer calculated results are described. Moreover, for all of the computer calculations, the fourth-order Runge-Kutta method is used with step size $h = 0.005$.

3. Breakdown of Synchronization

When the coupling parameter γ is relatively large, both the coupled chaotic oscillators and the modified van der Pol oscillators with noise exhibit four phase synchronizations. While for relatively smaller γ, the synchronizations break down and we observe the switchings of phase states. This means that a critical value of the coupling parameter exists which divides the parameter space into the synchronization region and the self-switching region. We define this critical coupling parameter as γ_c and investigate how γ_c changes when the strength of chaos or noise increases.

First, we define the breakdown of synchronization as at least one switching during 50,000 periods [8]. Next, in order to compare the coupled chaotic oscillators with the modified van der Pol oscillators, we set the parameters as follows. We investigate the critical coupling parameter γ_c for the coupled chaotic oscillators when the chaotic oscillator exhibits the first period-doubling bifurcation (β=0.0425). This value γ_c=1.189 is set as the standard value. We tune the parameter of the modified van der Pol oscillators without noise in order to the synchronization breaks down at this standard value of γ_c. By this setting of the parameter, the chaotic oscillators without chaotic fluctuation and the modified van der Pol oscillators without noise are equalized in the sense of the stability of the synchronization.

Figures 5 and 6 show one-parameter bifurcation diagram of the chaotic oscillator and the critical coupling parameter γ_c, respectively. In Fig. 6, the region over the curve corresponds to the synchronization region and the region under the curve corresponds to the self-switching region. We can observe that γ_c increases as β increases. This means that the synchronization becomes easier to be broken down for larger β, namely larger chaotic fluctuation.

The breakdown of the synchronization between chaotic oscillators and modified van der Pol oscillators with AWGN was rather different. Therefore we propose two types of noise for adding to modified van der Pol oscillators.

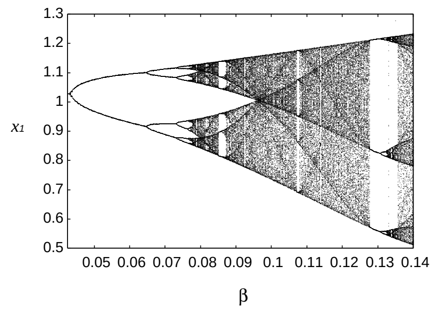

Figure 5: One-parameter bifurcation diagram of chaotic oscillator on the Poincaré section as $z_1 = 0$ and $x_1 < 0$ ($\alpha = 7.0$, $\gamma = 0.0$ and $\delta = 100.0$).

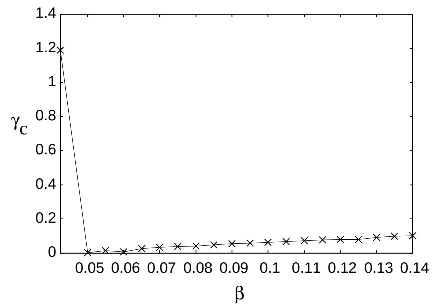

Figure 6: Breakdown of synchronization of coupled chaotic oscillators $\alpha = 7.0$ and $\delta = 100.0$).

The breakdown of the synchronization between chaotic oscillators and modified van der Pol oscillators with AWGN was rather different. And the breakdown of synchronization of modified van der Pol oscillators with noise with two band characteristics almost as well as with AWGN [9]. Therefore we propose chaos noise for adding to modified van der Pol oscillators.

3.1. Chaos Noise

Chaos noise is added the coupled van der Pol oscillators. The logistic map is used to generate chaos noise:

$$\hat{z}_{im}(t+1) = \alpha \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)). \tag{10}$$

Varying parameter α, Eq. 10 behaves chaotically via a period-doubling cascade. When we added chaos noise to the modified van der Pol oscillators, we normalize $\hat{z}_{im}$ by

$$\hat{z}_{im}(t+1) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z}. \tag{11}$$

where $\bar{z}$ is average of $\hat{z}(t)$, and σ_z is standard deviation of $\hat{z}(t)$.

Figure 7 shows bifurcation diagram of the logistic map obtained by Eq. 10. And the breakdown of synchronization of

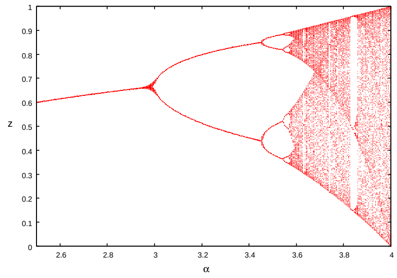

Figure 7: Bifurcation diagram of the logistic map.

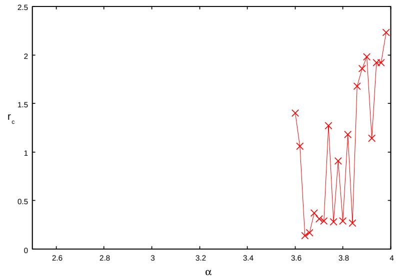

Figure 8: The breakdown of synchronization of modified van der Pol oscillators with chaos noise.

modified van der Pol oscillators with chaos noises is shown in Fig. 8. The horizontal axis is the control parameter α and the vertical axis is the coupling strength γ_c.

In order to confirm the effect of the noise we add the two types of chaos noises to the coupled modified van der Pol oscillators by setting the control parameter $\alpha = 3.64$ and $\alpha = 3.88$. Figure 9 shows the time series of the chaos noise near the two-period window. Figure 10 shows the chaotic time series without window.

By adding the chaos noise near two-periodic window (Fig 9), breakdown of synchronization characteristics of modified van der Pol oscillators becomes closer to the coupled chaotic circuit systems. However adding other chaos noise (Fig. 10), it is ineffective. From these results, we confirm that by adding chaos noise near two-periodic window, the modified van der Pol oscillators can be demonstrated similar phenomena with coupled chaotic systems.

4. Conclusions

In this study, we proposed chaos noise for adding to the van der Pol oscillators to confirm the difference between chaotic systems and van der Pol systems with AWGN. By adding chaos noise, modified van der Pol oscillators became closer to the coupled chaotic circuit systems. However adding other chaos noise, it was ineffective.

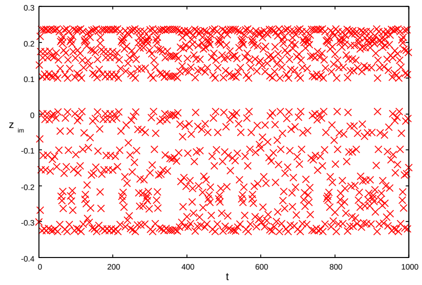

Figure 9: The time series of the chaos noise near the two-period window for $\alpha = 3.64$.

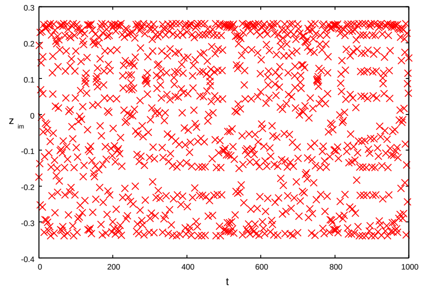

Figure 10: The time series of the chaos noise for $\alpha = 3.88$.

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