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Wave Propagation in a Ladder of Cross-Coupled Chaotic Circuits

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Abstract

A large number of coupled oscillators are useful as models for a wide variety of systems in natural fields. In this study, wave propagation generated in a ladder of cross-coupled chaotic circuits are investigated. For the case of 20 circuits, interesting wave propagation phenomena of phase states are found. Computer simulations show that this coupled system produces several wave propagation.

1. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos. Endo et al. have reported a details of theoretical analysis and circuit experiments about some coupled oscillators as a ladder [2]. Also, wave propagation phenomenon observed from coupled chaotic circuits is also reported [3]. Yamauchi et al. have discovered interesting wave propagation phenomena of phase states in a ladder of many simple oscillators coupled by inductors [4].

In our past studies, two simple chaotic circuits cross-coupled by inductors are investigated. As a result, we confirmed that the circuits generated many different synchronization states.

In this study, we investigate wave propagation phenomena observed in a ladder of cross-coupled chaotic circuits by inductors. By computer simulations for the case of 20 circuits, we can find interesting wave propagation phenomena of phase states.

2. Basic Circuit [5][6]

In this section, we review the phenomena observed from simple two cross-coupled chaotic circuits. Figure 1 shows the basic circuit model. In this model, two simple autonomous chaotic circuits [7][8] are cross-coupled via inductors L_2.

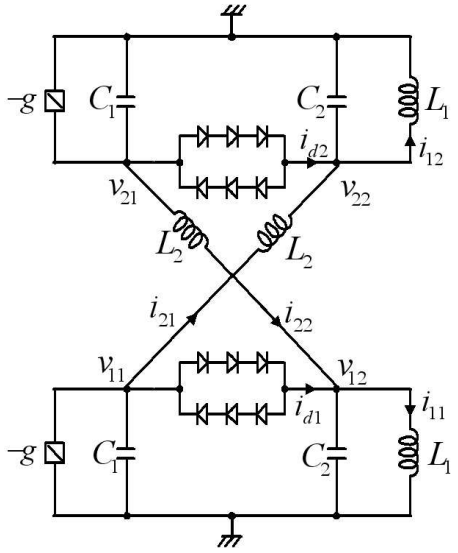


Figure 1: Basic circuit model.

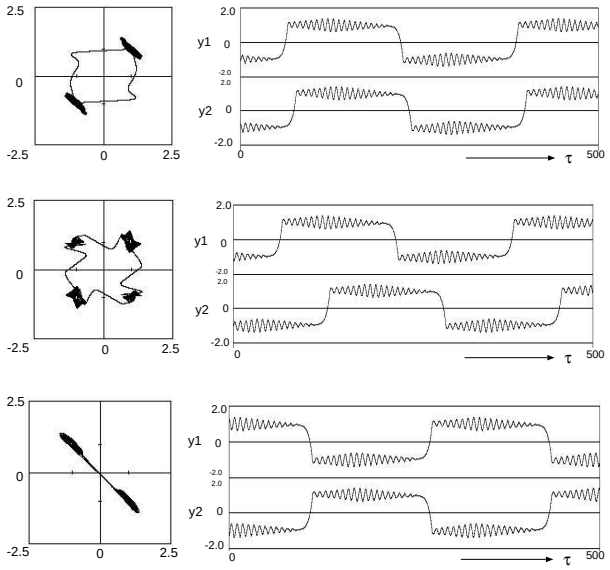
Typical examples of the observed phenomena are shown in Fig. 2 and Fig. 3. By computer simulations and circuit experiments, we confirmed that the circuits generated many different synchronization states, by changing initial condition.

3. Ladder of Cross-Coupled Chaotic Circuits

Figure 4 shows the circuit model investigated in this study. In the circuit, n chaotic circuits are cross-coupled via inductors L_2 as a ladder. The total number of the connections of n coupled system is expressed as $n \times 2 - 2 = 2(n-1)$. The normalized circuit equations are given as follows.

[First Oscillator]

$$\begin{cases} \dot{x}_1 = z_1 \\ \dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f\,(y_1 - z_1)\} \\ \dot{z}_1 = \beta\, f(y_1 - z_1) + u_2 - x_1 \\ \dot{w}_1 = \delta(y_1 - z_2 - \varepsilon w_1) \end{cases} \tag{1}$$



(a) (b)

Figure 2: Some examples of different synchronization states. (computer simulation results). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$. (a) Attractor on $y_1 - y_2$ plane. (b) Timewaveform.

[Middle Oscillators]

$$\begin{cases} \dot{x}_k = z_k \\ \dot{y}_k = \alpha\{\gamma y_k - w_k - u_k - \beta f\,(y_k - z_k)\} \\ \dot{z}_k = \beta\, f(y_k - z_k) + w_{k-1} + u_{k+1} - x_k \\ \dot{w}_k = \delta(y_k - z_{k+1} - \varepsilon w_k) \\ \dot{u}_k = \delta(y_k - z_{k-1} - \varepsilon u_k) \\ (k = 2, 3, 4..., n-1) \end{cases} \tag{2}$$

[Last Oscillator]

$$\begin{cases} \dot{x}_n = z_n \\ \dot{y}_n = \alpha\{\gamma y_n - u_n - \beta f\,(y_n - z_n)\} \\ \dot{z}_n = \beta\, f(y_n - z_n) + w_{n-1} - x_n \\ \dot{u}_n = \delta(y_n - z_{n-1} - \varepsilon u_n) \end{cases} \tag{3}$$

where

$$\begin{aligned} & x_k = \sqrt{\frac{L_1}{C_2}}\,\frac{i_{1k}}{V},\ \ w_k = \sqrt{\frac{L_1}{C_2}}\,\frac{i_{2k}}{V},\ \ u_k = \sqrt{\frac{L_1}{C_2}}\,\frac{i_{3k}}{V}, \\ & y_k = \frac{v_{1k}}{V},\ \ z_k = \frac{v_{2k}}{V},\ \ t = \sqrt{L_1 C_2}\,\tau, \\ & \alpha = \frac{C_2}{C_1},\ \ \beta = \sqrt{\frac{L_1}{C_2}}\,G,\ \ \gamma = \sqrt{\frac{L_1}{C_2}}\,g, \\ & \delta = \frac{L_1}{L_2},\ \ \varepsilon = \sqrt{\frac{C_2}{L_1}}\,R,\ \ \text{“}\cdot\text{”} = \frac{d}{d\tau} \\ & (k = 1, 2, 3, ..., n). \end{aligned} \tag{4}$$

The function f is nonlinear function corresponding to the $v - i$ characteristics of the nonlinear resistors of the diodes and are described as follows.

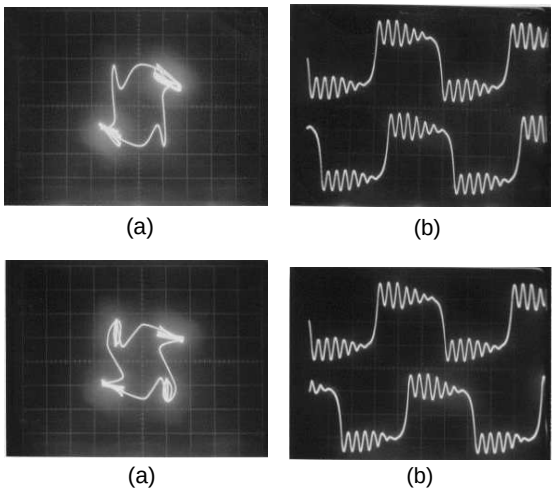


Figure 3: Two examples of different synchronization states. (circuit experimental results). L_1 = 10.56mH, L_2 = 1.28H, C_1=33.3nF, C_2=49.5nF and g=515mS. (a) Attractor on $v_{11}-v_{12}$ plane. Horizontal and vertical: 5 V/div. (b) Time waveform v_{11} and v_{21}. Horizontal 0.5 ms/div and vertical: 5 V/div.

$$f(y_k - z_k) = \begin{cases} y_k - z_k - 1 & (y_k - z_k > 1) \\ 0 & (|y_k - z_k| \le 1) \\ y_k - z_k + 1 & (y_k - z_k < -1) \\ (k = 1, 2, 3, ..., n). \end{cases} \tag{5}$$

4. Wave propagation phenomenon

In this study, we observe wave propagation from the 20 circuits. Figure 5 shows examples of wave propagation phenomena change as the parameter γ corresponding to the negative resistor. By changing γ, we can see the various wave propagation patterns. We cannot understand why the wave propagation happens. Investigating the mechanism of the wave propagation is our future research.

5. Conclusions

In this study, we investigated wave propagation phenomena observed in a ladder of chaotic circuits. By computer simulations for the case of 20 circuits, we could find interesting wave propagation phenomena of phase states. We confirmed that several wave propagation patterns could be observed by changing parameter γ. The investigation of the various size circuits and the their experiments are our future research.

Acknowledgments

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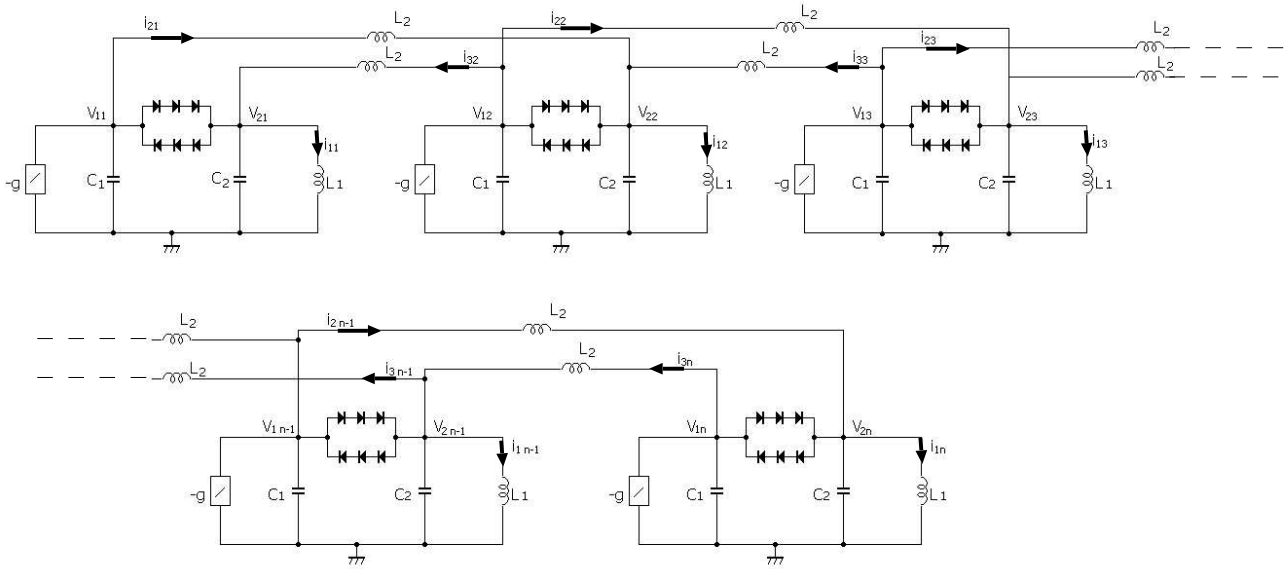

Figure 4: Ladder of cross-coupled chaotic circuits.

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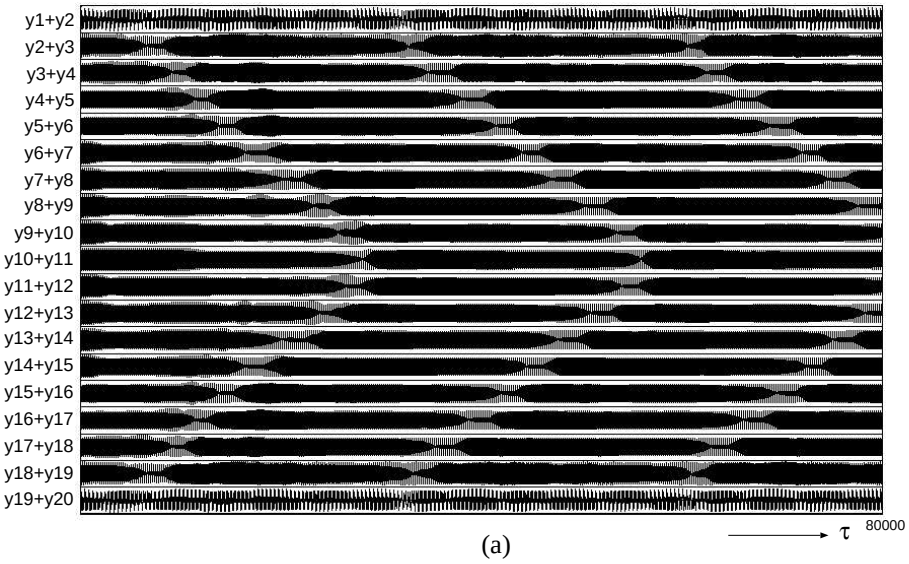
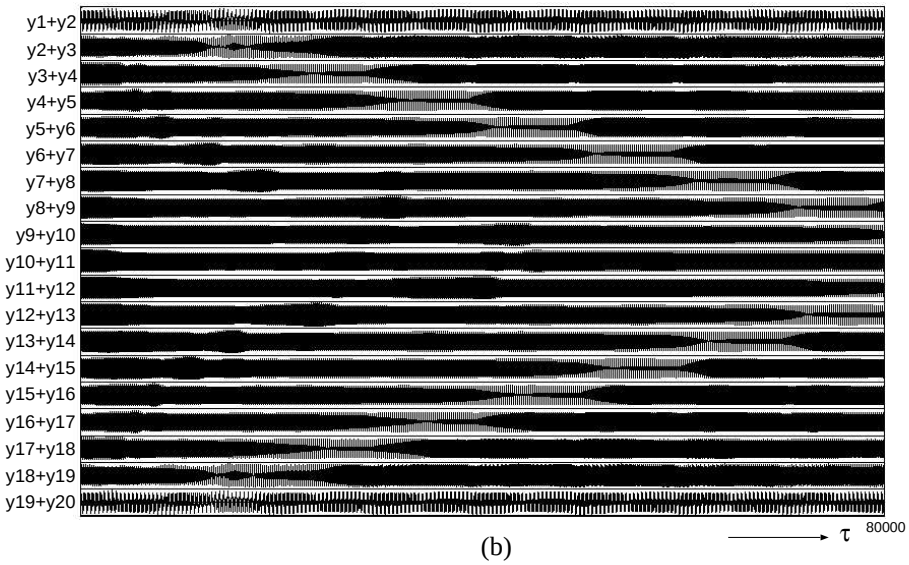


(a)



(b)

Figure 5: Wave propagation phenomena. $\alpha = 2.0$, $\beta = 4.0$, $\delta = 0.0014$, and $\varepsilon = 0.0005$. (a) $\gamma = 0.1266$. (b) $\gamma = 0.1270$.