2009 International Workshop on Nonlinear Circuits and Signal Processing
NCSP'09, Waikiki, Hawaii, March 1-3, 2009

Simple Chaotic Oscillator Using Two RC Circuits

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Abstract

In this study, a simple chaotic circuit using two coupled RC circuits is proposed. We confirm that the circuit generates chaotic oscillation by both computer calculations and circuit experiments.

1. Introduction

There have been many simple circuits exhibiting chaotic oscillations. In particular, simple oscillators excited by periodic signals can be a good circuit model to study the mechanism of chaos generation, because some of them can be analyzed in a rigorous way. In [1], Tang et al. investigated a simple multivibrator with periodic pulse and explained the generation of chaos using one-dimensional map. Torikai et al. proposed a chaotic pulse generator realized by an oscillator and a periodic signal and also controlled the generated chaos [2].

In this study, we propose a simple chaotic oscillator using two RC circuits. In the circuit model, the two RC circuits are coupled via comparators whose inputs are controlled by external periodic signal. Computer simulations and circuit experiments confirm the generation of chaos.

2. Circuit model

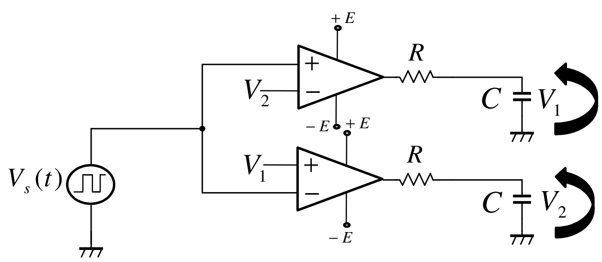


Figure 1: Circuit model.

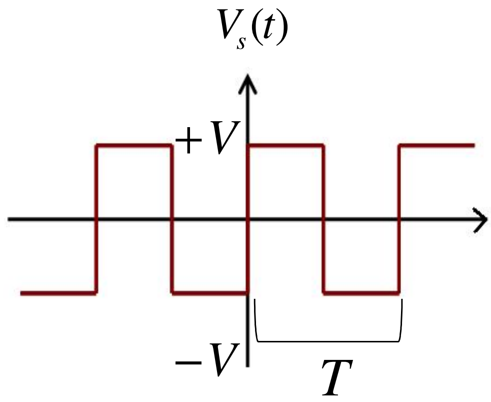


Figure 2: Rectangular voltage wave.

Figure 1 shows the circuit model. In the circuit, two RC circuits are coupled via simple comparators of operational amplifiers. The rectangular voltage wave is inputted to the other input terminals of the comparators and the comparators produce the output voltage $\pm E$ which is their power supply voltage according to the input signals.

Figure 2 shows the rectangular voltage wave inputted to the comparators. V is the amplitude of the rectangular voltage. T is the period of the rectangular voltage. The circuit equations are described as follows.

When $V_1 > V_s(t)$ and $V_2 < V_s(t)$

$$\begin{cases} RC\dfrac{dV_1}{dt} = -V_1 + E, \\ RC\dfrac{dV_2}{dt} = -V_2 + E, \end{cases} \tag{1}$$

When $V_1 < V_s(t)$ and $V_2 < V_s(t)$

$$\begin{cases} RC\dfrac{dV_1}{dt} = -V_1 + E, \\ RC\dfrac{dV_2}{dt} = -V_2 - E, \end{cases} \tag{2}$$

When $V_1 > V_s$ and $V_2 > V_s$

$$\begin{cases} RC\dfrac{dV_1}{dt} = -V_1 - E, \\ RC\dfrac{dV_2}{dt} = -V_2 + E, \end{cases} \tag{3}$$

When $V_1 < V_s(t)$ and $V_2 > V_s(t)$

$$\begin{cases} RC\dfrac{dV_1}{dt} = -V_1 - E, \\ RC\dfrac{dV_2}{dt} = -V_2 - E, \end{cases} \tag{4}$$

By using the following variables and the parameters,

$$\begin{array}{l} v_1 = E\,x_1,\;\; v_2 = E\,x_2,\;\; t = RC\,\tau, \\ V = E\alpha,\;\; T = RC\beta, \end{array} \tag{5}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dfrac{dx_1}{dt} = -x_1 \pm 1, \\ \dfrac{dx_2}{dt} = -x_2 \pm 1. \end{cases} \tag{6}$$

Because the circuit equations are linear in each region, the exact solutions can be derived as follows.

When $x_1 > \alpha_s(\tau)$ and $x_2 < \alpha_s(\tau)$ (Dpp),

$$\begin{cases} x_1 = k_1 e^{-\tau} + 1, \\ x_2 = k_2 e^{-\tau} + 1, \end{cases} \tag{7}$$

When $x_1 < \alpha_s(\tau)$ and $x_2 < \alpha_s(\tau)$ (Dpm),

$$\begin{cases} x_1 = k_1 e^{-\tau} + 1, \\ x_2 = k_2 e^{-\tau} - 1, \end{cases} \tag{8}$$

When $x_1 > \alpha_s(\tau)$ and $x_2 > \alpha_s(\tau)$ (Dmp),

$$\begin{cases} x_1 = k_1 e^{-\tau} - 1, \\ x_2 = k_2 e^{-\tau} + 1, \end{cases} \tag{9}$$

When $x_1 < \alpha_s(\tau)$ and $x_2 > \alpha_s(\tau)$ (Dmm),

$$\begin{cases} x_1 = k_1 e^{-\tau} - 1, \\ x_2 = k_2 e^{-\tau} - 1, \end{cases} \tag{10}$$

where $\alpha(\tau)$ is a function corresponding to $V_s(t)$, k_1 and k_2 are the arbitrary constants determined by initial values. We named four linear regions as Dpp , Dpm , Dmp and Dmm.

3. Generation of chaos

Figures 3 (a) and (b) show computer calculated results for α = 0.075 and β = 3.0. We can confirm that the circuit exhibits chaos.

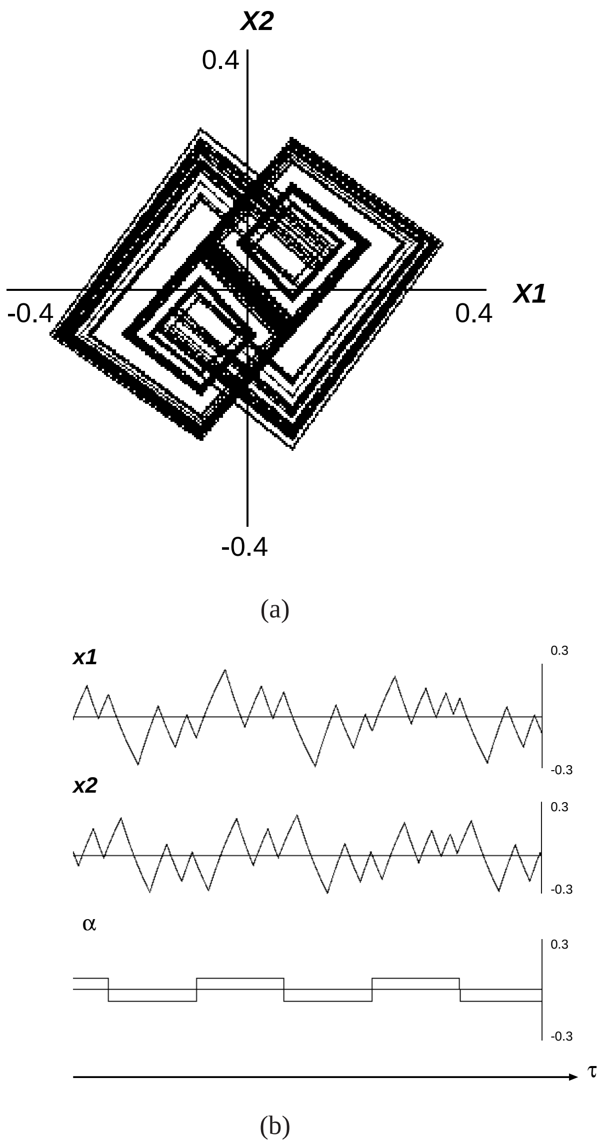


Figure 3: Computer simulation results. (a) Attractor on x_1–x_2 plane. (b) Time waveform.

Figures 4 (a) and (b) show circuit experimental results for C = 47μF , R = 1.2Ω and $f = 6.75kHz$. These results agree well to the computer calculated results.

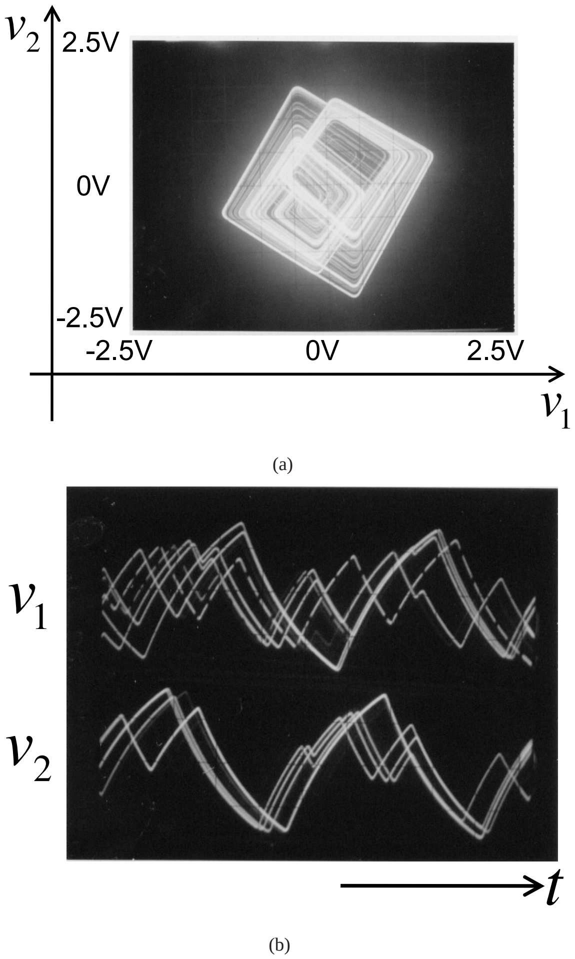


Figure 4: Circuit experimental results. (a) Attractor on v_1–v_2 plane. (b) Time waveform.

4. Analysis of chaotic attractor

Figure 5 shows bifurcation of the attractor when β is held at 3.0 and α changes from 0.65 to 0.075. We can confirm changing process of attractors. At first, the attractor shows that two waves x_1 and x_2 are synchronized at anti-phase. However, as α is smaller, synchronization begins to break. And finally, the attractor shows chaotic behavior.

Figure 6 shows only the part in each region of the chaotic attractor.

We can know convergence points in four linear regions from equations $(7) - (10)$. Figure 7 shows convergence points and formulas of the most outward line in four linear regions ($\alpha = 0.075$ and $\beta = 3.0$). From this result, we can confirm the most outward lines in Dpp and in Dmm are parallel, also in Dpm and in Dmp are parallel.

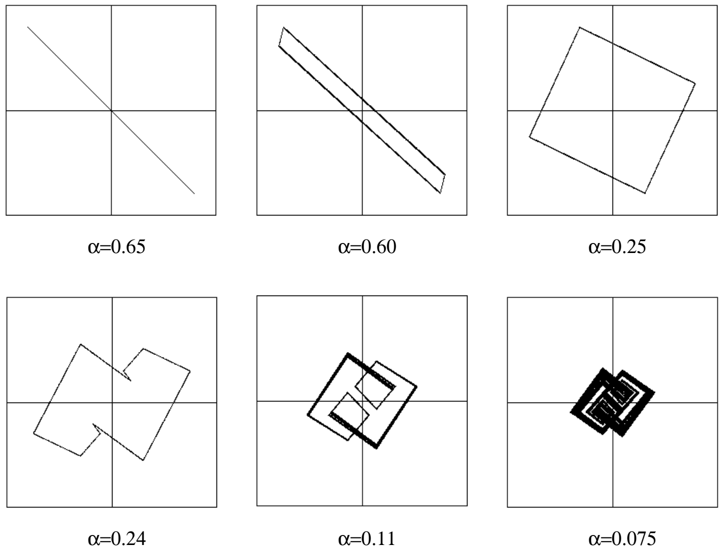


Figure 5: Bifurcation of chaos attractor.

Figure 8 shows the switching points from other three linear regions to one region ($\alpha = 0.075$ and $\beta = 3.0$). The points are dotted only when the region switches to one region from other three regions. We can see that (a) and (d) are origin symmetry, (b) and (d) are too. The distance between two lines shown in four regions is always 2α.

5. Conclusions

In this study, we have proposed a simple chaotic oscillator using two RC circuits. Computer simulations and circuit experiments confirmed the generation of chaos. In the future work, we will advance theoretical analysis of this circuit and try to consider engineering application.

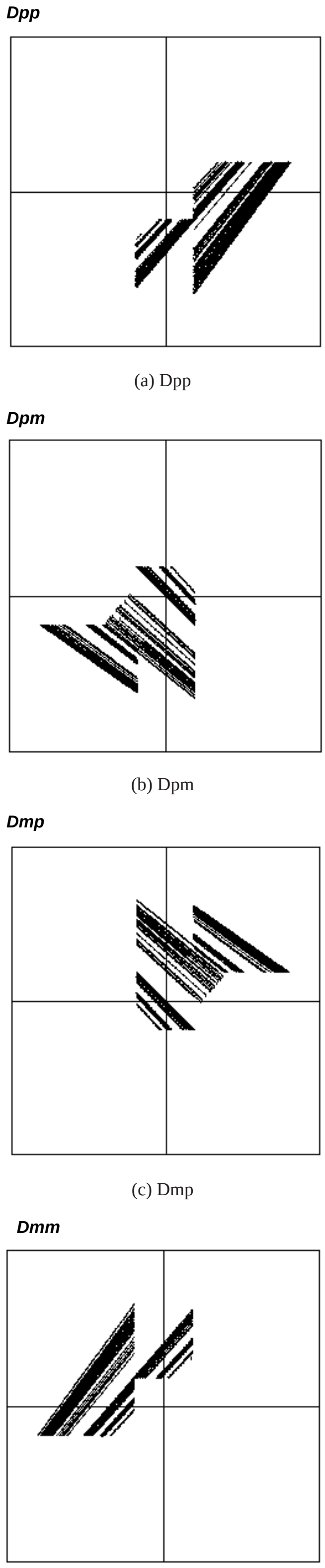
Acknowledgments

This work was partly supported by Yazaki Memorial Foundation for Science and Technology.

References

[1] Y.S. Tang, A.I. Mees and L.O. Chua, "Synchronization and Chaos," *IEEE Trans. Circuits Syst.*, vol. 30, no. 9, pp. 620-626, Sep. 1983.

[2] H. Torikai, T. Saito and W. Schwarz, "A Chaotic Pulse Generator and Sawtooth Control for Information Processing," *Proc. of ISCAS'97*, pp. 729-732, Jun. 1997.



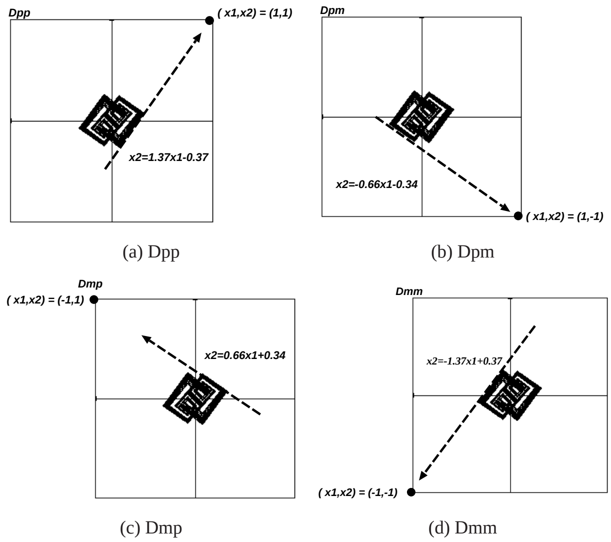
(a) Dpp

(b) Dpm

(c) Dmp

(d) Dmm

Figure 6: Four linear regions. ($\alpha = 0.075$ and $\beta = 3.0$.)



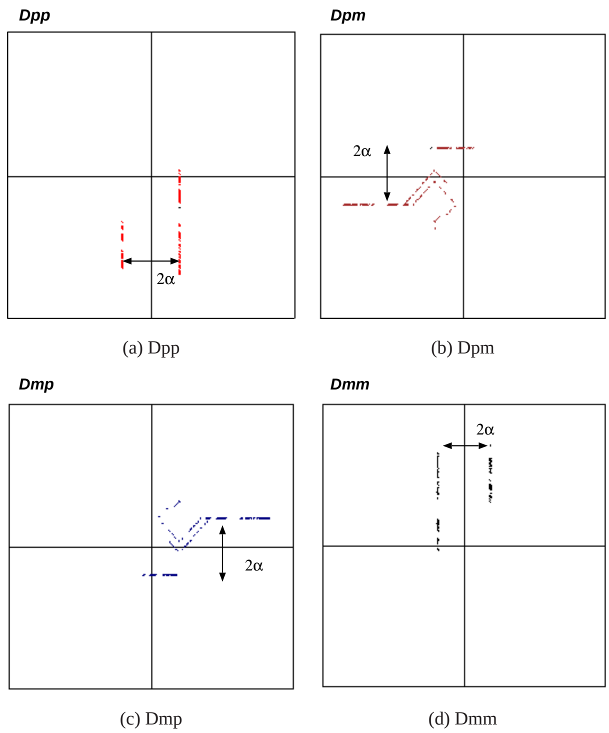
(a) Dpp

(b) Dpm

(c) Dmp

(d) Dmm

Figure 7: Convergence points and formulas of the most outward line in four linear regions. ($\alpha = 0.075$ and $\beta = 3.0$.)



(a) Dpp

(b) Dpm

(c) Dmp

(d) Dmm

Figure 8: Switching points. ($\alpha = 0.075$ and $\beta = 3.0$.)