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Performance of Quadratic Assignment Problem by Hopfield NN with Recalling the Best Memory

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Abstract

In our past study, the solving ability of the Hopfield Neural Network with noise for quadratic assignment problem is investigated. However, even if we injected the noise to the network, the optimal solution can not occasionally be found. In this study, we propose the method adding noise with recalling the best memory to the Hopfield Neural Network for achieve better performance. We investigate effective search with proposal technique for quadratic assignment problem.

1. Introduction

Hopfield Neural Network (abbr. NN) [1] is one of the tools to solve combinatorial optimization problems. To solve the problems, the global minimum can be searched by energy decent principle of the Hopfield NN. However, the network is a model aiming at the energy of the local minimum rather than the global minimum. So, in a lot of cases, the network finds local minimums, and can not escape from there. In order to avoid this critical problem, many researchers proposed the method adding some kinds of noises to the Hopfield NN. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance as noise [2].

Recalling the best memory is one of routine of the psyshological human thinking, and the human find a solution by rethinking from the best memory.

In this study, we propose a method adding noise with recalling the best memory. Adding the noise is effective for the solution to escape from the local minimum, but at the same time it may obstruct the energy decent principle. We investigate solving ability of adding the noise with recalling the best memory to the Hopfield NN for QAP. We confirm that the method is effective to solve QAP by computer simulations.

2. Solving QAP with hopfield NN

Various methods are proposed for solving QAP which is one of the NP-hard combinatorial optimization problems. We explain QAP with a factory arrangement problem. The problem is given by two matrices, distance matrix C denoting the distances between the factories and flow matrix D denoting the flow of the products between the factories, and is to find the permutation P which corresponds to the minimum value of the objective function $f(P)$ in Eq. (1).

$$f(P) = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \quad (1)$$

where C_{ij} and D_{ij} are the (i,j)-th elements of C and D,respectively, $p(i)$ is the i-th element of vector P, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). For solving $N-$element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i,j)-th neuron at the optimal position:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{i,m:j,n} x_{i,m} x_{j,n} + \sum_{i,m=1}^{N} \theta_{i,m} x_{i,m}. \quad (2)$$

The neurons are coupled each other with weight between (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron are described by:

$$w_{i,m:j,n} = -2\left\{A(1-\delta_{mn})\delta_{ij} + B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q}\right\}. \quad (3)$$

$$\theta_{im} = A + B \quad (4)$$

where A and B are positive constant, and δ_{ij} is Kroneker's delta. The state of $N \times N$ neurons are asynchronously updated due to the following difference equation:

$$x_{im}(t+1) = f\left(\sum_{j,n=1}^{N} w_{i,m:j,n} x_{i,m}(t) x_{j,n}(t) - \theta_{im} + \beta z_{im}(t)\right) \quad (5)$$

where f is sigmoidal function defined as follows:

$$f(x) = \frac{1}{1 + exp(-\frac{x}{\varepsilon})} \quad (6)$$

z_{im} is additional noise injected to the network, and β limits amplitude of the noise. Also, we use the method suggested by Sato et al. (Ref. [3]) to decide firing of neurons.

3. Chaos noise

In this section, we describe chaos noise injected to the Hopfield NN. The logistic map is used to generate the chaos noise:

$$\hat{l}_{im}(t+1) = \alpha \hat{l}_{im}(t)(1 - \hat{l}_{im}(t)). \quad (7)$$

Varying parameter α, Eq. (7) behaves chaotically via a periodic-doubling cascade. Further, it is well known that the map produces intermittent bursts just before periodic-windows appear. Figure 1 shows an example of the intermittency chaos near the three-periodic window obtained from Eq. (7) for $\alpha = 3.8274$. As we can see from the figure, the chaotic time series could be divided into two phases; laminar parts of periodic behavior with period three and burst parts of spread points over the invariant interval. As increasing α, the ratio of the laminar parts becomes larger and finally the three-periodic window appears. We use the intermittency chaos time series after the following normalization.

$$l_{im}(t+1) = \frac{\hat{l}_{im}(t) - \overline{l}}{\sigma_l} \quad (8)$$

where $\overline{l}$ and σ_l are the average and the standard deviation of $\hat{l}(t)$, respectively.

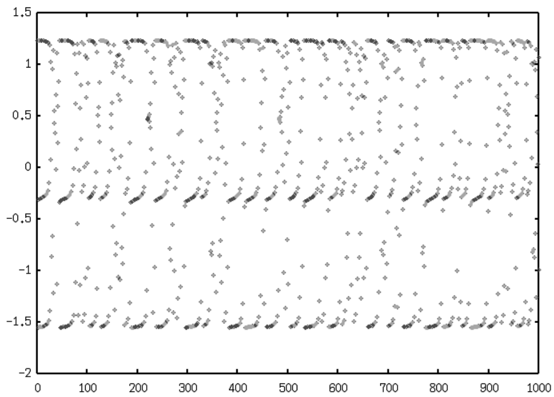

Figure 1: Time series obtained from logistic map. $\alpha_l = 3.8274$.

4. Recalling the best memory

we consider that a energy state of some of the neurons in a local minimum correspond to the energy state in a global minimum. For this reason, to add noise to the state which is near the global minimum is more efficient than to add noise immoderately. In this study, we propose a method to add noise with recalling the best memory. In the method, noises are added every update. In the update, the state of each neuron which has the lowest energy is memorized. After the lowest energy update, if the memorized energy is not updated by the dicided update number (named $LIMIT$), the state of all neurons is returned to the memorized lowest energy state. Then, repeat the update. Figure 2 shows the flow chart of recalling the best memory.

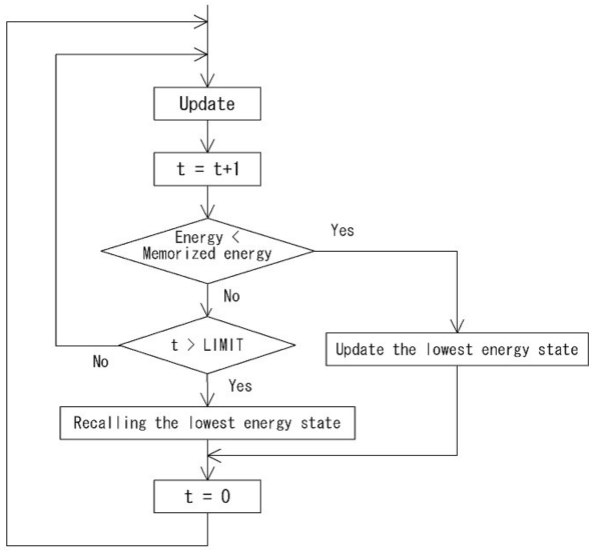

Figure 2: Flow chart of recalling the best memory.

5. Simulation results

In this section, the simulation results of Hopfield NN with recalling the best memory for 12-element QAP are shown. The problem used here was chosen from the site QAPLIB named "Nug12". The global minimum is known as 578. The parameters of Hopfield NN are fixed as $A = 0.9$, $B = 0.9$, $q = 140$, $\varepsilon = 0.02$ and $\beta = 0.64$. We carried out 1000 trails of 10000 iterations. The control parameter of the logistic map is fixed as $\alpha_l = 3.8274$. The limit of update number of the recalling the best memory is fixed as $LIMIT = 50$.

Table 1: Solving abilities for Nug12.

Iteration	Conventional	Recalling
1000	632.96	606.802
2000	623.30	598.450
3000	619.82	594.482
4000	616.18	592.116
5000	613.56	590.588
6000	612.68	589.466
7000	611.74	588.562
8000	610.48	587.952
9000	610.30	587.448
10000	609.96	586.998

Next, we tried another problem, whose name is "Tai12a". The global minimum is 224416. The parameters of the Hopfield NN are fixed as $A = 0.9$, $B = 0.9$, $q = 16000$, and $\varepsilon = 0.02$. We carried out 1000 trails of 40000 iterations. Othe parameters are $\beta_0 = 0.64$, $\alpha_l = 3.8274$ and $LIMIT = 40$.

Table 2: Solving abilities for Tai12.

Iteration	Conventional	Recalling
4000	252624.44	245091.840
8000	251337.90	242225.154
12000	250291.38	240731.618
16000	249638.16	239919.308
20000	249520.72	239180.534
24000	249495.62	238717.338
28000	249458.04	238176.598
32000	249335.58	237803.944
36000	249228.40	237485.112
40000	249225.84	237098.142

Tables 1 and 2 show the mean values of the best solutions obtained during each iteration numbers for Nug12 and Tai12a, respectively. From these results, the proposed method gains better performance than the conventional method.

Finally, we investigate the performance against changing the limit of the update number of recalling the best memory. From the results of changing $LIMIT$ (Fig. 3 and Fig. 4), by change $LIMIT$ the performance of the proposed method is perturbed, however the proposed method gains better performance than the conventional method.

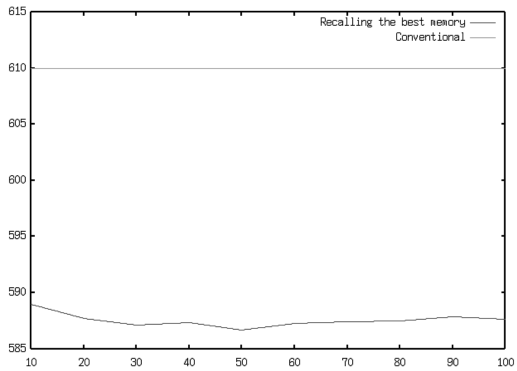

Figure 3: Simulation result of Nug12. Horizontal axis: LIMIT. Vertical axis: Average of solutions.

6. Conclusions

In this study, we proposed the method adding noise with recalling the best memory to the Hopfield Neural Network. By computer simulations, we investigated the solving ability of the Hopfield NN with the recalling the best memory for QAP. We confirmed that the proposed method is effective to solve QAP.

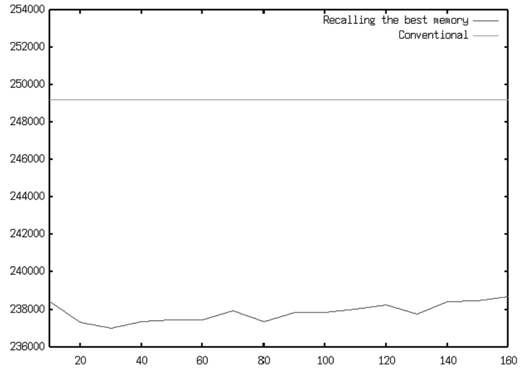

Figure 4: Simulation result of Tai12. Horizontal axis: LIMIT. Vertical axis: Average of solutions.

References

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