Synchronization Phenomena in Three Oscillators Coupled by a Resonator

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Abstract

Synchronization is common phenomenon in the field of natural science. It should be noted that mutual synchronization phenomenon of oscillators gives various phase states and there have been many investigations on these phenomenon. In this study, we investigate synchronization phenomena observed from three oscillators with the same natural frequencies coupled by a resonator.

1. Introduction

In our surroundings, there are a lot of synchronous phenomena. The synchronous luminescence of firefly group, cell of heart producing pulses at equal intervals and revolution of the moon etc. are examples in which a synchronous phenomenon is comprehensible. Similarly, synchronization is common phenomenon in the field of natural science.

There have been many investigations of the mutual synchronization of oscillators ([1]-[6] and therein). Moro and one of the authors have confirmed that N oscillators with same natural frequencies mutually coupled by one resistor give N-phase oscillations. Their system can take $(N-1)!$ phase states, because of their system tends to minimize the current through the coupling resister [7][8]. They thought that these coupling structure and huge number of steady states (for example, when their system take 479,001,600 steady states when $N = 13$.), would be structural element of cellular neural network or may be used as an extremely large memory.

In this study, we observe synchronization of three oscillators coupled by a resonator. Resonator is consisted of parallel circuit of a capacitor and an inductor. Computer simulations and circuit experiments are carried out to investigate the phenomenon in detail.

2. Circuit Model

The circuit model is shown in Fig. 1. Three oscillators with the same natural frequencies are mutually coupled by a resonator ($L_C C_C$ circuit). The circuit equations are de-

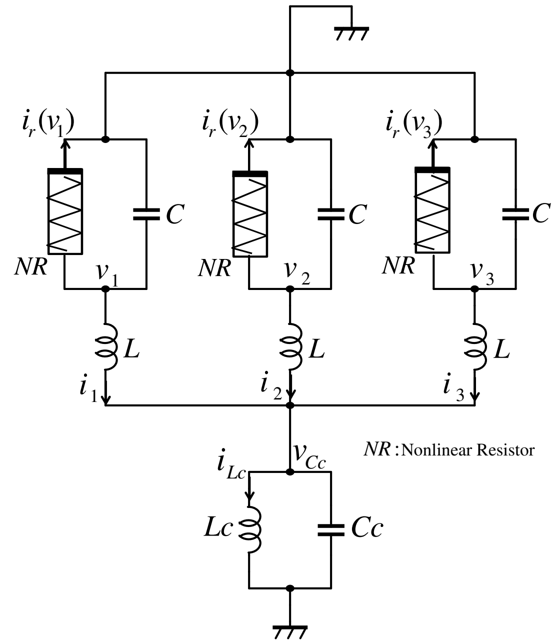

Figure 1: Circuit model.

scribed as Eq. (1).

$$\begin{cases} C\dfrac{dv_k}{dt} = -i_k - i_r(v_k) \\ L\dfrac{di_k}{dt} = v_k - v_{Cc} \qquad (k = 1, 2, 3) \\ Cc\dfrac{dv_{Cc}}{dt} = \sum_{j=1}^{3} i_j - i_{Lc} \\ Lc\dfrac{di_{Lc}}{dt} = v_{Cc} \end{cases} \tag{1}$$

where $i_r(v_k)$ indicates the $v - i$ characteristics of the nonlinear resistor, which is approximated by Eq. (2).

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3. \tag{2}$$

For circuit experiments, the nonlinear resister is realized as shown in Fig. 2. Note that when r is small, the nonlinearity

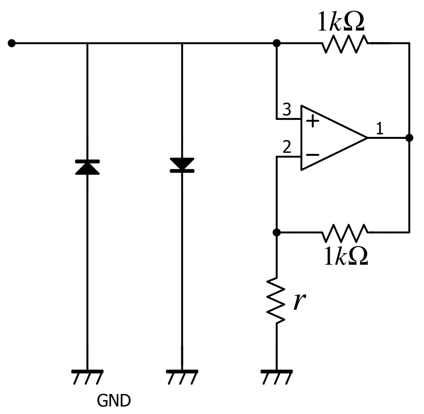

Figure 2: Nonlinear resister.

is strong. By using the following variables and parameters,

$$\begin{cases} v_k = \sqrt{\dfrac{g_1}{g_3}}\, x_k, \quad i_k = \sqrt{\dfrac{Cg_1}{Lg_3}}\, y_k, \\ v_{Cc} = \sqrt{\dfrac{g_1}{g_3}}\, X, \quad i_{Lc} = \sqrt{\dfrac{Cg_1}{Lg_3}}\, Y, \\ t = \sqrt{LC}\,\tau, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau}, \\ \epsilon = \sqrt{\dfrac{L}{C}}\, g_1, \quad \beta = \dfrac{C}{Cc}, \quad \gamma = \dfrac{L}{Lc}, \end{cases} \tag{3}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k = -y_k + \epsilon(x_k - x_k^3) \\ \dot{y}_k = x_k - X \qquad (k = 1, 2, 3) \\ \dot{X} = \beta\left(\sum_{j=1}^{3} y_j - Y\right) \\ \dot{Y} = \gamma X \end{cases} \tag{4}$$

3. Synchronization Phenomena for $L_C = L$ and $C_C = C$

As shown in Fig. 3, we can observe three patterns of oscillations for the same parameter; in-phase oscillation and two types of three-phase oscillations. Which synchronous pattern was seen depends on the initial states. It is interesting that three patterns can be observed for the same parameter and the oscillation frequency depends on the synchronization patterns. Also, as Fig. 4, the circuit experimental results show similar phenomena to the numerical results.

4. Synchronization Phenomena for $L_C \neq L$ or $C_C \neq C$

To investigate the synchronization phenomenon when the parameters of the coupling resonator are changed, one of the parameters are fixed to 1.0, and the other parameter (β or γ) is changed.

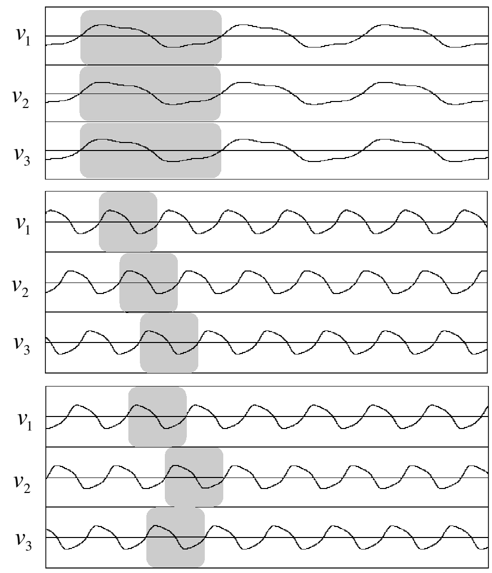

Figure 3: Time waveform of in-phase oscillation and two types of three-phase oscillations (numerical results). ϵ=β=γ=1.0.

4.1. Synchronization when γ is changed

First, the parameter β is fixed to 1, and the parameter γ is changed from 0.3 to 2.7. Three typical examples are shown in Figs. 5(a), (b) and (c). For any values of γ, the three patterns of synchronization; in-phase oscillation and two types of three-phase oscillations, are able to be confirmed. We should note that the oscillation frequency of the three-phase oscillations is almost the same for different γ. On the other hand, the frequency of the in-phase oscillation decreases as γ decreases.

4.2. Synchronization when β is changed

Secondly, the parameter γ is fixed to 1, and the parameter β is changed from 0.3 to 2.7. Two typical examples are shown in Figs. 6(a) and (b). We can observe the three patterns of synchronization as well as the previous case. However, in this case, when β was changed, the oscillation frequency of either in-phase oscillation or three-phase oscillation does not change.

Further, as shown in Figs. 7(a) and (b), we observed some strange waveforms for some parameter values. For Fig. 7(a), the waveform of the three-phase oscillations becomes distorted, while for Fig. 7(b) the waveform of the in-phase oscillation becomes distorted.

It is very interesting that a variety of synchronization phenomena can be seen for different coupling parameter values.

5. Conclusion

In this study, we have investigated the synchronization phenomena in three oscillators coupled by a resonator. From the coupled oscillators, we observed the in-phase oscillation and two types of three-phase oscillations. Moreover, we investigated the dependences of the oscillation frequencies on the coupling parameter values. Our future work is to investigate these synchronization phenomena in more detail.

Acknowledgments

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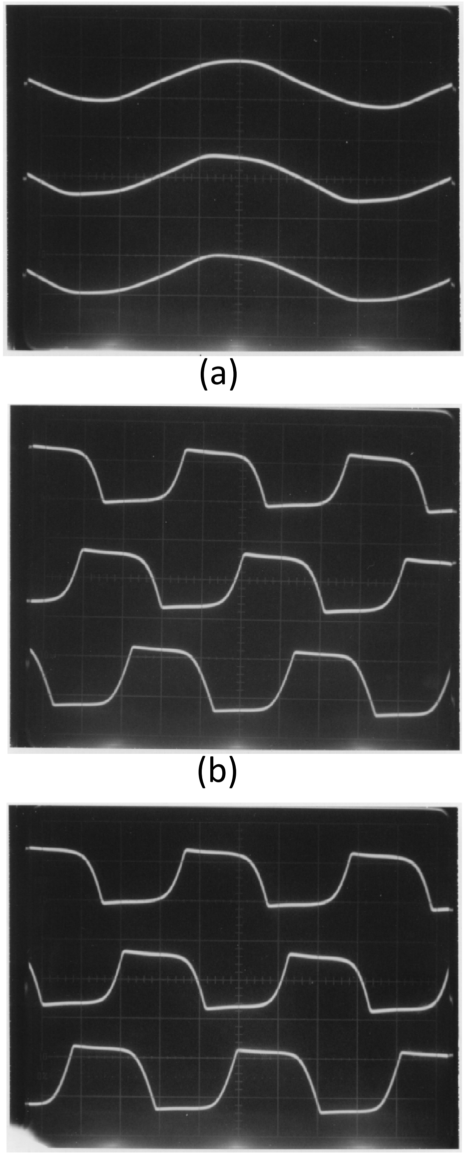

(a)

(b)

(c)

Figure 4: Time waveform of in-phase oscillation and two types of three-phase oscillations (experimental results). L=L_c=10mH, C=C_c=68nF and r=250Ω. Horizontal scale: 50μs/div. and Vertical scale: 1.0V/div.

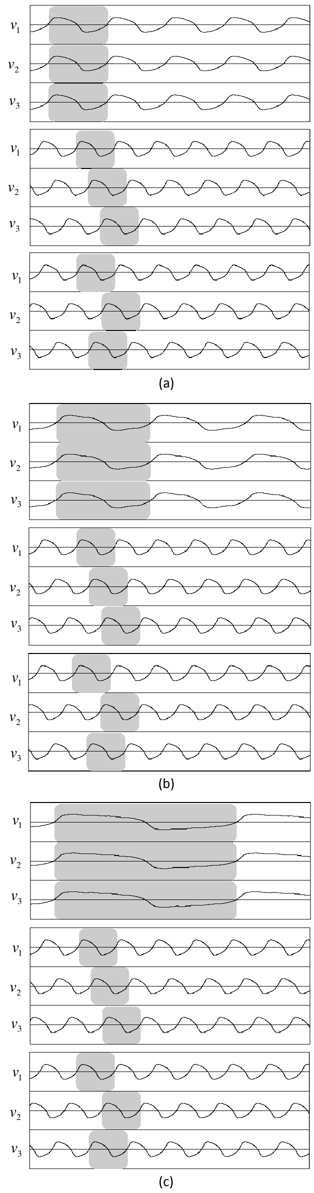

Figure 5: Numerical results for $L_C \neq L$. ϵ=β=1.0. (a) γ=2.7. (b) γ=1.2. (c) γ=0.3.

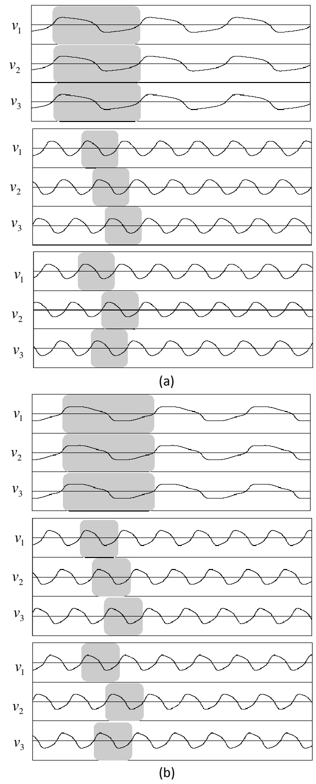

Figure 6: Numerical results for $C_C \neq C$. ϵ=γ=1.0. (a) β=2.7. (b) β=0.6.

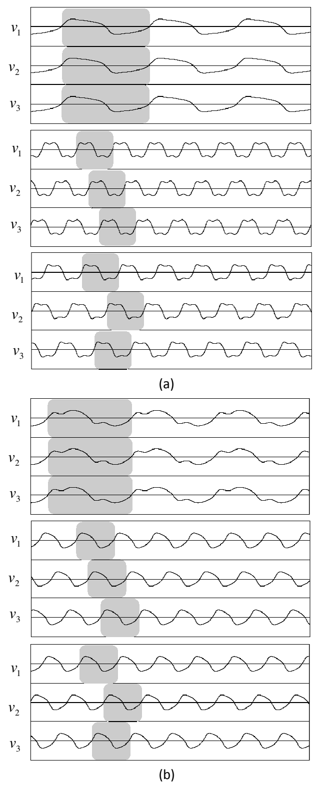

Figure 7: Distorted waveforms observed for $C_C \neq C$. ϵ=γ=1.0. (a) β=2.1. (b) β=0.3.