Comparison between Cross-Coupled Chaotic Circuits and Direct-Coupled Chaotic Circuits

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Abstract

Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields. In this study, we compare the synchronization behaviors of cross-coupled chaotic circuits and direct-coupled chaotic circuits.

1. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1]-[9].

In our past studies [10][11], two simple chaotic circuits cross-coupled by inductors are investigated. As a result, we could observe interesting state transition phenomena.

In this study, we compare the synchronization behaviors of cross-coupled chaotic circuits and direct-coupled chaotic circuits in order to know how the structure of the coupling affects the observed phenomena.

2. Circuit model

Figure 1 shows the cross-coupled circuit model. In the circuit, simple chaotic circuits [12][13] are cross-coupled via inductors L_2.

By using the following variables and the parameters,

$$\begin{cases} x_1 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{11}}{V}, \quad y_1 = \dfrac{v_{11}}{V}, \quad z_1 = \dfrac{v_{12}}{V}, \\ x_2 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{21}}{V}, \quad y_2 = \dfrac{v_{21}}{V}, \quad z_2 = \dfrac{v_{22}}{V}, \\ w_1 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{12}}{V}, \quad w_2 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{22}}{V}, \\ \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\,G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}\,g, \\ \delta = \dfrac{L_1}{L_2}, \quad t = \sqrt{L_1 C_2}\,\tau, \end{cases} \tag{1}$$

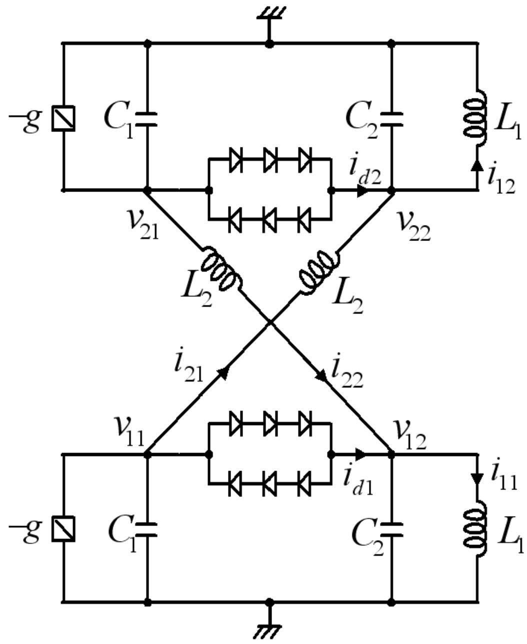

Figure 1: Cross-coupled chaotic circuits.

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_1 = z_1 \\ \dot{x}_2 = z_2 \\ \dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f\,(y_1 - z_1)\} \\ \dot{y}_2 = \alpha\{\gamma y_2 - w_2 - \beta f\,(y_2 - z_2)\} \\ \dot{z}_1 = \beta\, f(y_1 - z_1) + w_2 - x_1 \\ \dot{z}_2 = \beta\, f(y_2 - z_2) + w_1 - x_2 \\ \dot{w}_1 = \delta(y_1 - z_2) \\ \dot{w}_2 = \delta(y_2 - z_1) \end{cases} \tag{2}$$

While, Fig. 2 shows the direct-coupled circuit model. Equation (3) indicates the normalized circuit equa-

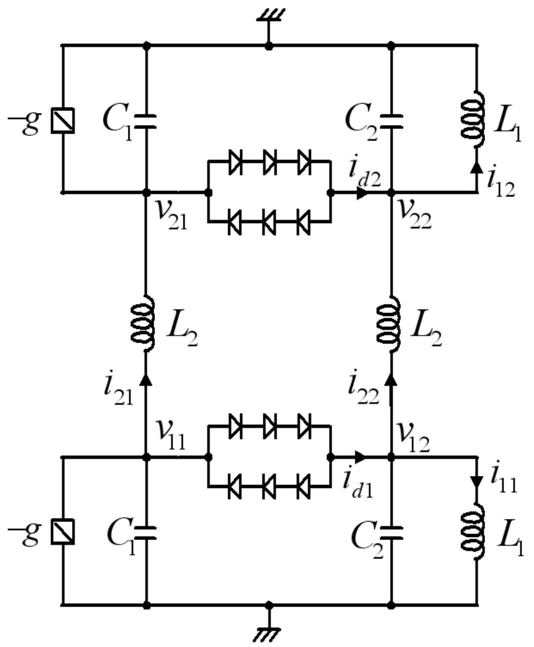

Figure 2: Direct-coupled chaotic circuits.

tions of the direct-coupled model.

$$
\begin{cases}
\dot{x}_1 = z_1 \\
\dot{x}_2 = z_2 \\
\dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f\,(y_1 - z_1)\} \\
\dot{y}_2 = \alpha\{\gamma y_2 + w_1 - \beta f\,(y_2 - z_2)\} \\
\dot{z}_1 = \beta\, f(y_1 - z_1) - w_2 - x_1 \\
\dot{z}_2 = \beta\, f(y_2 - z_2) + w_2 - x_2 \\
\dot{w}_1 = \delta(y_1 - y_2) \\
\dot{w}_2 = \delta(z_1 - z_2)
\end{cases} \tag{3}
$$

where f are the nonlinear functions corresponding to the $v-i$ characteristics of the nonlinear resistors consisting of the diodes and are assumed to be described by the following 3-segment piecewise-linear functions:

$$
f(y_1 - z_1) = \begin{cases}
y_1 - z_1 - 1 & (y_1 - z_1 > 1) \\
0 & (|y_1 - z_1| \le 1) \\
y_1 - z_1 + 1 & (y_1 - z_1 < -1)
\end{cases} \tag{4}
$$

$$
f(y_2 - z_2) = \begin{cases}
y_2 - z_2 - 1 & (y_2 - z_2 > 1) \\
0 & (|y_2 - z_2| \le 1) \\
y_2 - z_2 + 1 & (y_2 - z_2 < -1).
\end{cases} \tag{5}
$$

3. State transition phenomenon

From the cross-coupled chaotic circuits, we could observe interesting state transition phenomenon. Figure 3(1) shows computer calculated results of Eq. (2)

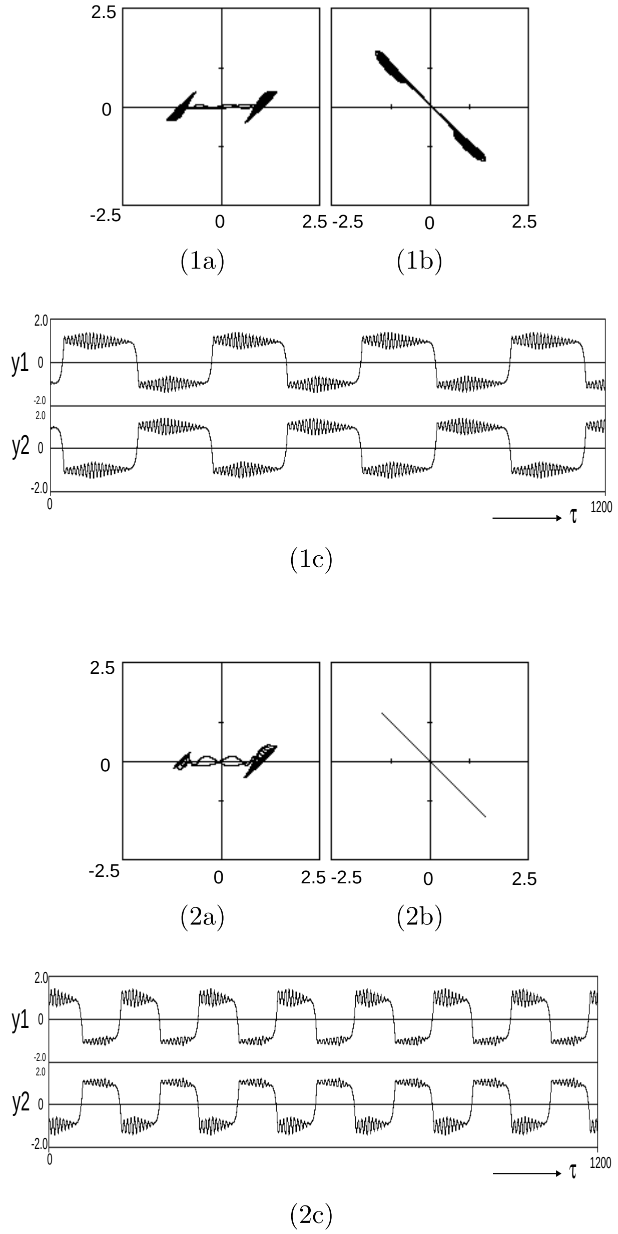

Figure 3: State transition phenomenon around anti-phase synchronization (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$. (1) Cross-coupled chaotic circuits. (2) Direct-coupled chaotic circuits. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

with the Runge-Kutta method. While, Fig. 3(2) shows the computer simulated result for the case of the direct-coupled chaotic circuits. In both cases, the two circuits exhibited chaos but almost synchronized in anti-phase in the sense that the attractors were almost in the quadrant II or IV on the $y_1 - y_2$ plane. The behaviors of the circuits are very interesting because the solutions on the $y_i - z_i$ planes seem to be attracted to the fixed points located at around $(y_i,\ z_i)=(\pm 1.2, 0)$. However, after converging to the fixed points the solution abruptly moves toward the other fixed point. When one circuit switches to/from the positive region from/to the negative region in this way, the other follows the transition after a few instants.

4. Comparison of synchronization behaviors

In this section, we compare the synchronization behaviors of the cross-coupled chaotic circuits and direct-coupled chaotic circuits.

First, for the cross-coupled chaotic circuits, we can observe similar state transition phenomenon around in-phase synchronization as Fig. 4(1) and quadrature-phase synchronization as Fig. 4(2). However, as far as we carried out computer simulations, we cannot observe the state transition phenomena around in-phase or quadrature-phase for the case of the direct-coupled chaotic circuits.

Secondly, we notice that the sojourn times between the state transitions are different from Figs. 3(1c) and 3(2c). Figure 5 shows how the sojourn times around anti-phase synchronization change as the coupling parameter changes. The horizontal axis is coupling parameter δ and the vertical axis is the average lengths of the transitions in τ. The curve of circles shows the average period of the state transitions of y_1 for the cross-coupled chaotic circuits. The curve of crosses shows the average period of the state transitions of y_1 for the direct-coupled chaotic circuits. From this figure, we can see that the sojourn time of the direct-coupled chaotic circuits is almost half of that of the cross-coupled chaotic circuits.

5. Conclusions

In this study, we have compared the synchronization behaviors of the cross-coupled chaotic circuits and the direct-coupled chaotic circuits in order to know how the structure of the coupling affected the observed phenomena. By computer simulations, we confirmed two differences. The first is that the state transition phenomenon of the cross-coupled chaotic circuits appears around anti-phase, in-phase, and quadrature-phase synchronizations, while that of the direct-coupled chaotic circuits appears only around anti-phase synchronization. The second is that the sojourn time of

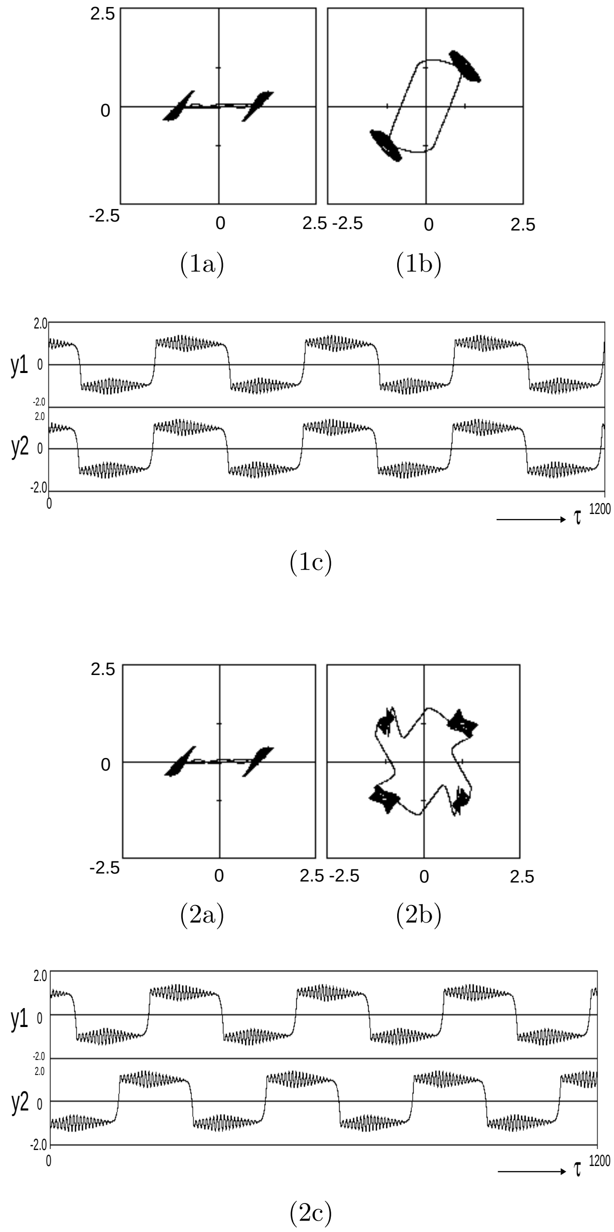

Figure 4: State transition phenomenon observed from cross-coupled chaotic circuits (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$. (1) Around in-phase synchronization. (2) Around quadrature-phase synchronization. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

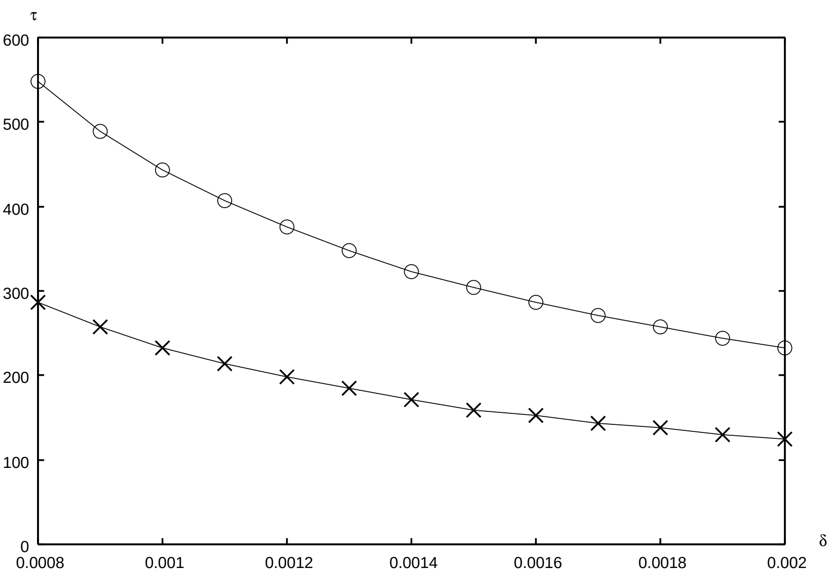

Figure 5: Sojourn time between state transitions versus coupling parameter (anti-phase synchronization). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$. Circle: Cross-coupled. Cross: Direct-coupled.

the direct-coupled chaotic circuits is almost half of that of the cross-coupled chaotic circuits. Clarifying the mechanism of these differences is our future work.

Acknowledgments

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