IWVCC'08
@ Xian
Nov. 1-3, 2008

Spatio-Temporal Phenomena of Coupled Multi-State Chaotic Circuits

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Abstract

An oscillator including a chaotic system is an important device and one of essential component in the natural world to solve a mechanism of a nonlinear dynamics of network. In this study, a simple chaotic circuit with three states of both chaotic oscillation and limit cycles which is named multi-state chaotic circuit is proposed. Synchronization phenomena and complex behavior on a network system of the multi-state chaotic circuits coupled by inductors are investigated. Several interesting chaotic phenomena of spatio-temporal behavior have been observed in the coupled network system.

1. Introduction

In recent years, dynamical network system such as small-world, scale-free and other types is attracted many researchers' attention from the view point of complex systems and several engineering applications. It is also important for solving the process of mechanisms in social and biological systems. We consider that it is a good model for realization of the real systems to make a coupled network with nonlinear oscillators. Therefore, an oscillator including a chaotic system is an important device and one of essential component in the natural world. Nonlinear dynamics on coupled oscillators is considerable interesting for a wide variety of systems in several scientific fields and some engineering applications, especially in stochastic models. Thought, many types of coupled circuit systems have been widely studied in order to clarify inherent features and many researchers have already proposed and investigated mechanism of them. The dynamics of spatio-temporal phenomena or phase synchronization on several coupled systems is still considerable interest from the viewpoint of both natural scientific fields and several applications. They have been confirmed in several coupled chaotic systems [1]–[3]. Phase synchronization and pattern dynamics are also interesting for several engineering applications. On the other hand, many types of chaotic systems and circuits have already been proposed and investigated in detail. As interesting phenomena, there are famous chaotic attractors such a double-scroll family [4], *n*-double scroll [5]–[7] and scroll grid attractors [8]. If the active elements including in the systems have complexity constructed by compound some nonlinear elements, it can be easily considered that they yield several interesting features. In our previous studies, the circuit which can individually behave both chaotic or periodic oscillations in the same parameters had been investigated [9]-[13]. This type of circuit was called a multi-state chaotic circuit (abbr. MSCC). Multimode oscillations in coupled two or more multi-state chaotic circuits had been shown [10] on real circuit. A complicated and interesting phenomena of phase synchronization had also been investigated [11]-[13]. It is known that complex behavior can be confirmed such chaotic itinerancy and spatio-temporal chaos on the large scale coupled networks.

In this study, several synchronization phenomena and complex behavior on coupled network system of multi-state chaotic circuits are investigated. This paper shows several phase synchronization phenomena of multi-state chaotic oscillators coupled by some inductors as a simple network. Each oscillator circuit can individually behave both limit cycles and chaos in the same parameters. This proposed circuit can behave three steady states of both chaotic and two different size of limit cycles when an appropriate initial conditions are supplied. We consider a coupled chaotic network system which each chaotic oscillator is connected to neighbors by inductors. In numerical simulation, many types of phase synchronization modes are asynchronously confirmed in the proposed systems, further all parameters are the same. This means several phase synchronization modes are coexisting in the same parameters. Several interesting chaotic phenomena of spatio-temporal behavior have been observed in the coupled network system.

2. Model description of an MSCC

An MSCC is a basic circuit as a subcomponent of a coupled network model. The designed circuit is shown in Fig. 1. In this study, we substitute a symmetrical continuous piecewise linear resistor for the negative active resistor including in the original chaotic circuit. The piecewise linear resistor can be easily constructed by combining some components in parallel [9][10]. By changing the following variables and parameters as follows

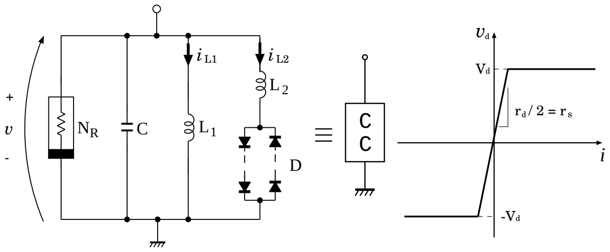

Figure 1: A multi-state chaotic circuit with piecewise linear resistors N_R and $v - i$ characteristic of the diode D.

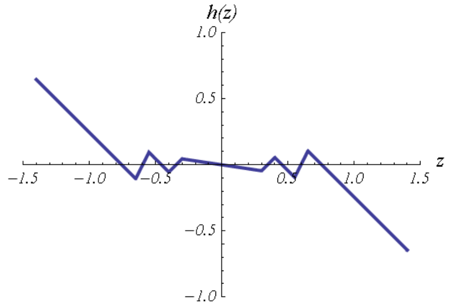

Figure 2: Design for sawtooth nonlinear resistor N_R in the circuit.

$$
\begin{aligned}
& i_{L1} = \sqrt{\frac{C}{L_1}} V_d\, x\,, \quad i_{L2} = \sqrt{\frac{C}{L_1}} V_d\, y\,, \\
& v = V_d\, z\,, \quad t = \sqrt{L_1 C}\, \tau\,, \quad \text{“}\cdot\text{”} = \frac{d}{d\tau}\,, \\
& \beta = \frac{L_1}{L_2}\,, \quad \gamma = g\sqrt{\frac{L_1}{C}}\,, \quad \delta = r_s\sqrt{\frac{C}{L_1}}
\end{aligned}
\tag{1}
$$

where g is a linear negative conductance value of N_R if we consider the negative resistor as an ideal element. Consider that the part of negative resistance N_R in Fig. 1 replaces to the function $h(z)$ represented by a voltage source z as canonical form with 9-segments as shown in Fig. 2. When we chose the threshold voltage V_d for a normalized parameter, then the circuit equations can be normalized and rewritten as follows.

$$
\left\{
\begin{array}{lcl}
\dot{x} & = & z \\
\dot{y} & = & \beta\left(z - f(y)\right) \\
\dot{z} & = & -(x+y) - h(z)
\end{array}
\right.
\tag{2}
$$

$$
f(y) = \frac{1}{2}\Big\{|\delta y + 1| - |\delta y - 1|\Big\} \tag{3}
$$

$$
h(z) = m_0\gamma^* z + \frac{\gamma^*}{2}\left\{\sum_{k=0}^{K}(m_k - m_{k+1})\left\{|z - p_{k+1}| - |z + p_{k+1}|\right\}\right\} \tag{4}
$$

where $f(y)$ is a function of the current y and $h(z)$ is a function of the voltage z, respectively. The function $h(z)$ which is designed for several segment piecewise linear as symmetric with respect to the origin. The parameter γ^* is used for a basic common value, hence the values $m_k (k = 0, 1, 2, \cdots, K)$ mean magnitude of the slope to the ratio for γ^*.

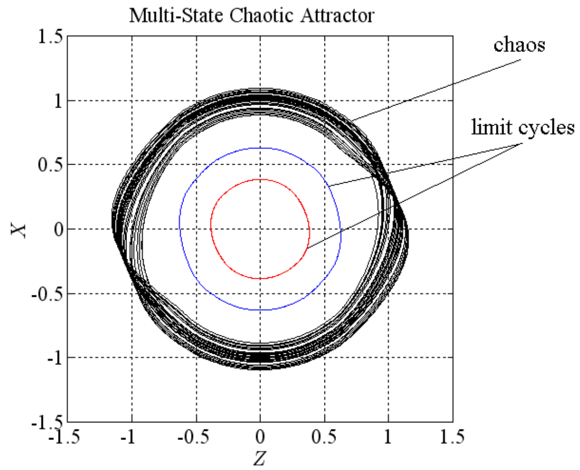

Figure 3: Attractor drawing onto the $z - x$ plane for the parameters $\beta = 10.0$, $\gamma^* = 0.78$ and $\delta = 100$. $h(z)$: $\{p_1, p_2, p_3, p_4\}$=$\{0.65, 0.55, 0.40, 0.30\}$, $\{m_0, m_1, m_2, m_3, m_4\}$ = $\{-1.0, 2.0, -1.0, 1.0, -0.15\}$.

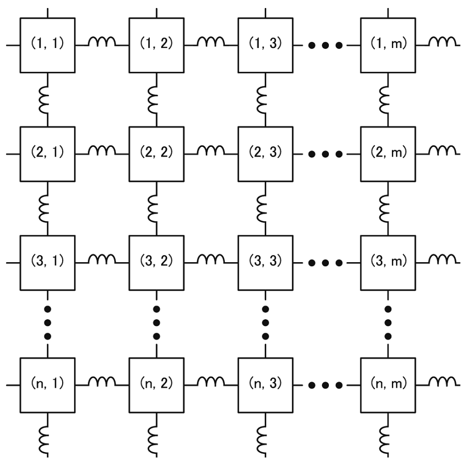

Figure 4: A coupled network model which each circuit is connected to four neighbors.

Figure 3 also shows a typical chaotic attractor obtained for the parameters $\beta = 10.0$, $\gamma^* = 0.78$, $\delta = 100$, with piecewise linear characteristics realized by breakpoints $p_1 = 0.65$, $p_2 = 0.55$, $p_3 = 0.40$, $p_4 = 0.30$, slopes $m_0 = -1.0$, $m_1 = 2.0$, $m_2 = -1.0$, $m_3 = 1.0$ and $m_4 = -0.15$. We can confirm that chaotic and two periodic attractors coexist in the circuit. This means coexistence of both chaos and two different size of limit cycles in the same parameters.

3. Simulation for coupled MSCCs on a network model

In this section, the model of coupled MSCCs by inductors are investigated. For example of a coupled network, now let us consider the coupled MSCCs model which combined number of $N \times M$ chaotic circuits are connected by inductors L_0 to neighbors' circuit as a network structure shown in Fig. 4. The circuit index is defined as (i, j). It is note that every chaotic circuit is composed by all the same parameters and connected to four neighbors circuits. The circuit on the edge of this coupled network is connected to an opposite side circuit, it seems like a distribution on surface of the torus structure. By the similar way described before, the circuit equations of coupled MSCCs can be normalized by the variables (1) with a new parameter $\alpha = L_1/L_0$. Therefore, the whole circuit equations can be rewritten as follows.

$$\left\{\begin{array}{rcl} \dot{x}_{(i,j)} & = & z_{(i,j)} \\ \dot{y}_{(i,j)} & = & \beta\Big(z_{(i,j)} - f\big(y_{(i,j)}\big)\Big) \\ \dot{z}_{(i,j)} & = & \alpha\left(\sum_{\Phi} x_{\Phi} - 4x_{(i,j)}\right) \\ & & \qquad -(x_{(i,j)} + y_{(i,j)}) - h(z_{(i,j)}) \end{array}\right. \tag{5}$$

where functions $f(y)$ and $h(z)$ are similar to (3) and (4), respectively. Φ means a set of four neighbors to $x_{(i,j)}$.

We show some computer calculation results by using 4-th order Runge–Kutta method with time step size $\Delta t = 0.001$ for the circuit equations in some cases of (N, M) as follows. The parameters of each circuit are the same in the Sec. 2. The initial conditions of each circuit are supplied at random. If the case $(N, M) = (2, 2)$, this coupled system is corresponding to the case of a ring structure in [12]. We show some computer simulation results for some cases. Figure 5 shows some typical results obtained from computer simulation in some cases of coupled number (N, M). The left part of the figure shows attractors of the circuit, and the right part indicates synchronization state between two circuits. We can confirm complex and chaotic synchronization phenomena on the coupled system. Further we can several types of complex and interesting synchronization phenomena in the same parameters, i.e., in-phase synchronization, anti-phase synchronization, clustering of phase synchronization, phase locking and other types. Several phase synchronization modes are coexisting in spite of the same parameters.

4. Conclusions

In this study, we have investigated several synchronization modes in coupled multi-state chaotic oscillator circuits. Coexistence of several types oscillation modes have been confirmed in coupled MSCCs by inductors as a coupled network systems. Several interesting chaotic phenomena of spatio-temporal behavior have been observed in the coupled network system. On a large scale of coupled chaotic oscillators such a small-world and a scale-free network, we consider that several types of complex behavior are expected to yield novel applications and inherent emergent properties in the natural systems.

Acknowledgment

This work was partly supported by GRANT-IN-AID for ”Open Research Center” project from MEXT, Japan.

References

[1] Y. Nishio, A. Ushida, “On a Ring of Chaotic Circuits Coupled by Inductors,” *IEICE Trans.*, vol. E78-A, no. 5, pp. 608–617, 1995.

[2] Y. Nishio, A. Ushida, “Phase Synchronization in a Ring of Chaotic Circuits,” *Proc. of NOLTA’01*, pp. 163–166, 2001.

[3] Y. Nishio, A. Ushida, “Statistical Analysis of Chaotic Wandering in Simple Chaotic Circuits,” *Proc. of ITC-CSCC’01*, pp. 1034–1037, 2001.

[4] L.O. Chua, M. Komuro, T. Matsumoto, “The Double Scroll Family,” *IEEE Trans. Circuits Syst.–I*, vol. 33, no. 11, pp. 1072–1118, 1986.

[5] J.A.K. Suykens, J. Vandewalle, “Generation of *n*-Double Scrolls (*n* = 1,2,3,4,...),” *IEEE Trans. Circuits Syst.–I*, vol. 40, no. 11, pp. 861–867, 1993.

[6] M. Wada, Y. Nishio, A. Ushida, ”Chaotic Itinerancy Phenomena on Coupled n-Double Scrolls Chaotic Circuits,” *Proc. of ISCAS’99*, vol. 5, pp. 487–490, 1999.

[7] G. Zhong, K. Man, G. Chen, “A Systematic Approach to Generating *n*-Scroll Attractors,” *Int. J. of Bif. and Chaos*, vol. 12, no. 12, pp. 2907–2915, 2002.

[8] M.E. Yalçin, J.A. Suykens, J. Vandewalle, “Families of Scroll Grid Attractors,” *Int. J. of Bif. and Chaos*, vol. 12, no. 1, pp. 23–41, 2002.

[9] M. Wada, Y. Nishio, “Coexistence Phenomena of Chaotic and Non-Chaotic Oscillations in a Three-Dimensional Chaotic Circuits,” *Proc. NOLTA2002*, vol. 2, pp. 869–872, 2002.

[10] M. Wada, Y. Nishio, “Multimode Asynchronous Oscillations in Coupled Multi-State Chaotic Circuits,” *Proc. of NCSP’04*, pp. 535–538, 2004.

[11] M. Wada, Y. Nishio, “Several Phase Synchronization Modes in Coupled Multi-State Chaotic Circuits,” *Proc. of NOLTA’04*, pp. 525–528, 2004.

[12] M. Wada, Y. Nishio, “Coexistence of Several Phase Synchronization in a Ring of Coupled Multi-State Chaotic Oscillators,” *Proc. of NOLTA’07*, pp. 393–396, 2007.

[13] M. Wada, Y. Nishio, “Phase Synchronization Phenomena on Coupled Multi-State Oscillators,” *Proc. of NCSP’08*, pp. 144–147, 2008.

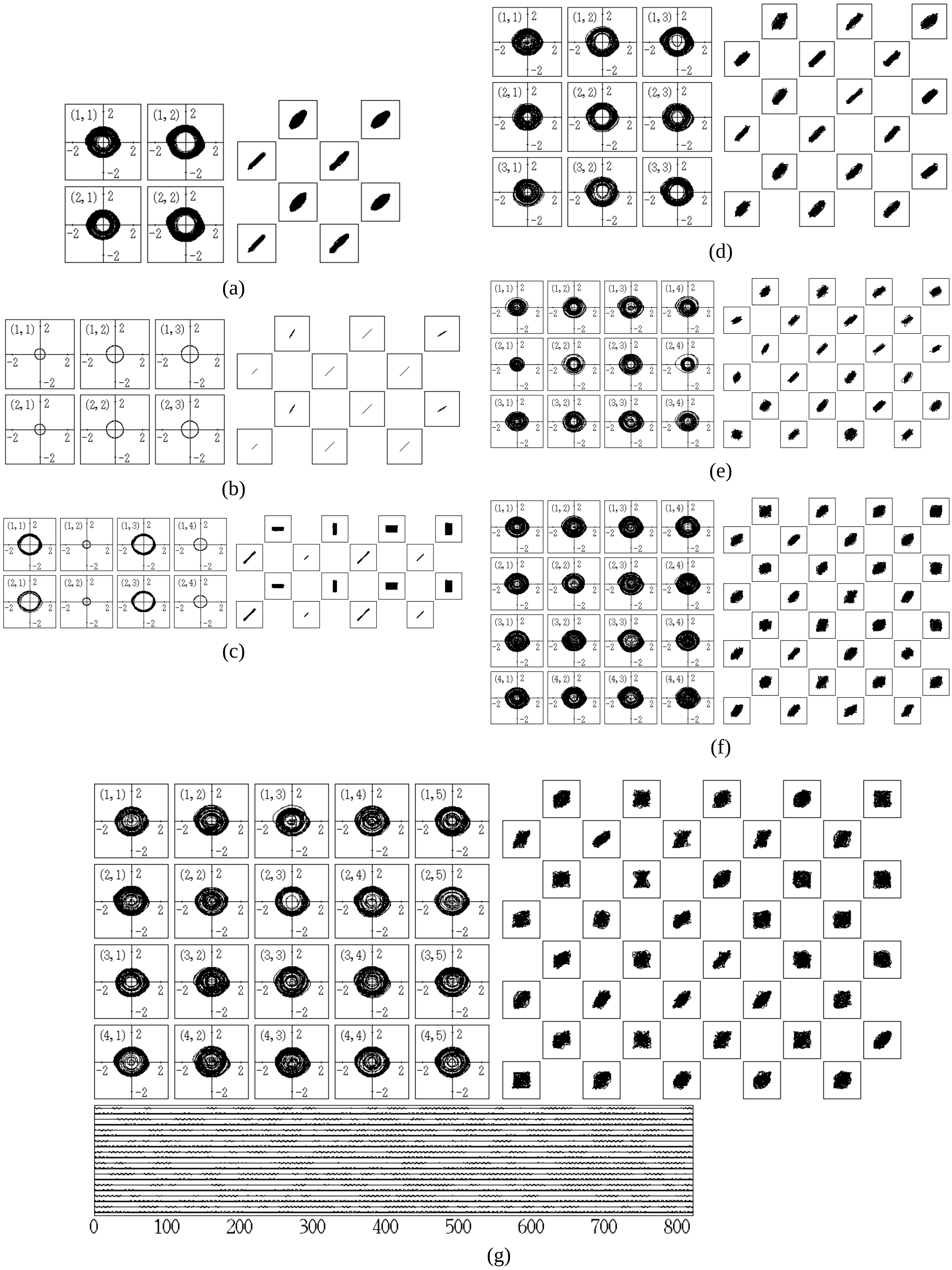

Figure 5: Simulation results obtained from coupled MSCCs for $\alpha = 0.10$, $\beta = 10.0$, $\gamma^* = 0.78$, $\delta = 100$. $h(z)$: $\{p_1, p_2, p_3, p_4\}$=$\{0.65, 0.55, 0.40, 0.30\}$, $\{m_0, m_1, m_2, m_3, m_4\} = \{-1.0, 2.0, -1.0, 1.0, -0.15\}$. Size (N, M) of the network: (a) (2,2), (b) (2,3), (c) (2,4), (d) (3,3), (e) (3,4), (f) (4,4), and (g) (4,5) with waveform of each oscillator.