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Chaotic Circuit Using a Ring Oscillator and a van der Pol Oscillator

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Abstract—In this study, we propose a chaotic circuit using a ring oscillator and a van der Pol oscillator. The structure of this circuit is very simple. Therefore, IC implementation of this circuit is not so difficult. The exact solution and its Poincaré map are derived and the largest Lyapunov exponent are calculated. As a result, chaotic phenomena are confirmed. This chaotic circuit is suitable for using investigations of coupled chaotic systems, because this circuit is implemented as a electric circuit and IC implementation is not so difficult.

1. Introduction

Electric circuits are one of the natural physical system and it can be represented by a differential equations. Namely, it is considered that electric circuits are corresponding to many kinds of natural phenomena and it is expected that these phenomena be analyzed by using differential equations. Especially, electric circuits are suitable for investigating large scale chaotic coupled systems. Many investigation results can be obtained by circuit experiments immediately. Generally, large scale coupled systems can be described by high dimensional equations. Therefore, its analysis is not so easy and its computer simulation needs a long time. Additionally, the value of electric circuit elements is comparatively stable. If many same circuits are made, almost circuits will behave the same. These are very important advantages of using electric circuits for investigations of coupled systems.

However, implementing large scale coupled systems is not so easy. The one solution of this problem is IC implementation. Though circuit experiments of the large scale coupled chaotic system become easy by IC implementation, many money and time are needed. Therefore, the chaotic circuit which can be implemented as IC and can be also implemented as normal electric circuit is needed.

In this study, we propose a chaotic circuit using a ring oscillator and a van der Pol oscillator. The ring oscillator included the circuit consists of Op-Amps and resistors. The structures of the ring oscillator and the van der Pol oscillator are very simple. Therefore, IC implementation of this circuit is not so difficult.

2. Circuit Model

2.1. Circuit Schematic

Figure 1 shows the schematic of the proposed circuit. The left side is a ring oscillator which consists of three inverters. Resistor R_{vr1} is for adjusting the amplitude. The right side is a van der Pol oscillator. The Op-Amp part works as a negative resistor. These two oscillators are coupled via diodes which is placed the center.

In order to propose the new chaotic circuit, we used our designing technique. This technique is that two oscillators are coupled via diodes. It is very simple. However, this technique can not determine the parameters. In this study, parameters are determined by circuit experiments.

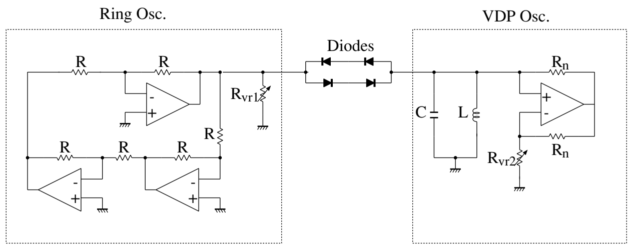

Figure 1: Circuit schematic.

2.2. Circuit Model

In order to make clear the mechanism of generating chaos, the proposed circuit is modeled as follows. Figure 2 shows diodes model of the proposed circuit. Diodes are modeled as a piece-wise linear function shown in Fig 2 (b). Figure 3 shows the circuit model of the proposed circuit shown in Fig. 1. Inverters of the ring oscillator are modeled as a inverter including a time delay. All elements except diodes are modeled as linear elements. It means that diodes play a role as non-linearity which is needed for generating chaos. Additionally, analysis becomes easy because the elements except diodes are linear elements. This is the useful advantage for investigating large scale coupled systems. Using this model, circuit equations are described as

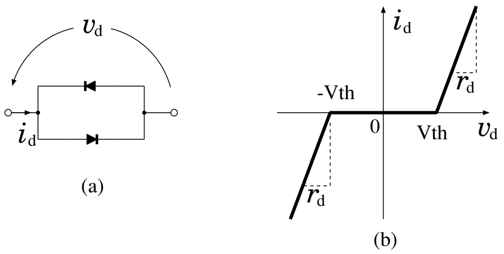

Figure 2: Diodes model.

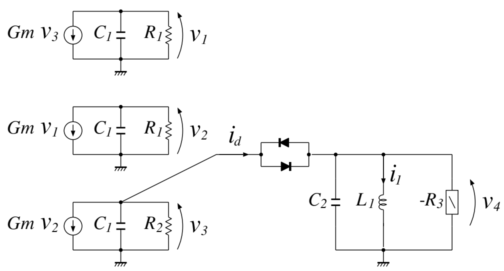

Figure 3: Circuit model.

follows:

$$\begin{cases} C_1\dfrac{dv_1}{dt} &= -\dfrac{1}{R_1}v_1 - G_m v_3, \\ C_1\dfrac{dv_2}{dt} &= -\dfrac{1}{R_1}v_2 - G_m v_1, \\ C_1\dfrac{dv_3}{dt} &= -\dfrac{1}{R_2}v_3 - G_m v_2 - i_d, \\ C_2\dfrac{dv_4}{dt} &= \dfrac{1}{R_3}v_4 - i_1 + i_d, \\ L_1\dfrac{di_1}{dt} &= v_4, \end{cases} \tag{1}$$

where

$$i_d = \frac{1}{r_d}\left\{v_3 - v_4 + \frac{1}{2}(|v_3 - v_4 - V_{th}| - |v_3 - v_4 + V_{th})|\right\}. \tag{2}$$

By substituting the variables and the parameters,

$$x_n = \frac{v_n}{V_{th}}\ (n = 1,2,3,4), \quad x_5 = \frac{R_1}{V_{th}}i_1, \quad x_d = \frac{R_1}{V_{th}}i_d,$$
$$\tau = \frac{1}{C_1R_1}t, \quad \alpha = G_mR_1, \quad \beta = \frac{R_1}{R_2}, \quad \gamma = \frac{R_1}{R_3},$$
$$\delta = \frac{C_1}{C_2}, \quad \varepsilon = \frac{R_1}{R_2} \quad \text{and} \quad \zeta = \frac{C_1R_1^2}{L_1}. \tag{3}$$

Equations (1) and (2) are normalized as

$$\begin{cases} \dot{x}_1 &= -x_1 - \alpha x_3, \\ \dot{x}_2 &= -x_2 - \alpha x_1, \\ \dot{x}_3 &= -\beta x_3 - \alpha x_2 - \varepsilon x_d, \\ \dot{x}_4 &= \delta(\gamma x_4 - x_5 + \varepsilon x_d), \\ \dot{x}_5 &= \zeta x_4, \end{cases} \tag{4}$$

where

$$x_d = x_3 - x_4 + \frac{1}{2}(|x_3 - x_4 - 1| - |x_3 - x_4 + 1|. \tag{5}$$

3. Exact solutions and Poincaré Map

Since the circuit equations (4) are piecewise-linear, solutions in each linear region can be derived. At first, we define three piecewise-linear region as follows.

$$\begin{aligned} \mathbf{R_1}: &\quad x_3 - x_4 > 1. \\ \mathbf{R_2}: &\quad -1 < x_3 - x_4 < 1. \\ \mathbf{R_3}: &\quad x_3 - x_4 < -1. \end{aligned} \tag{6}$$

The eigenvalues in each region are calculated from Eq. (4). The eigenvalues in each region are described as follows.

$$\begin{aligned} \mathbf{R_1} \text{ and } \mathbf{R_3}: &\quad \lambda_{11},\ \lambda_{12},\ \lambda_{13},\ \sigma_{11} \pm j\omega_{11}. \\ \mathbf{R_2}: &\quad \lambda_{21},\ \sigma_{21} \pm j\omega_{21},\ \sigma_{22} \pm j\omega_{22}. \end{aligned} \tag{7}$$

The eigenvalues in R_1 and R_3 are obtained numerically from the 5th order eigenequation of Eq. (4). On the other hand, the eigenvalues in R_2 can be derived from the 3rd and 2nd order eigenequations of Eq. (4). The equilibrium points of each region are calculated by putting the right side of Eq. (4) to be equal to zero. Then, the solutions in each linear region can be described. We omit the description, because it is complicated enough to write here.

Now, we are deriving the Poincaré map and the Jacobian matrix.

Let us define the following subspace

$$\mathbf{S} = \mathbf{S_1} \cap \mathbf{S_2}. \tag{8}$$

where,

$$\begin{aligned} \mathbf{S_1}: &\quad x_3 - x_4 = 1. \\ \mathbf{S_2}: &\quad -\alpha x_2 - \beta x_3 - \gamma\delta x_4 + \delta x_5 < 0. \end{aligned} \tag{9}$$

The subspace $\mathbf{S_1}$ corresponds to the boundary condition between $\mathbf{R_1}$ and $\mathbf{R_2}$, while the subspace $\mathbf{S_2}$ corresponding to the condition $\dot{x}_3 - \dot{x}_4 > 0$. Namely, $\mathbf{S}$ corresponds to the transitional condition from $\mathbf{R_1}$ to $\mathbf{R_2}$.

Let us consider the solution starting from an initial point on $\mathbf{S}$. The solution returns back to $\mathbf{S}$ again after wandering $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$. Hence, we can derive the Poincaré map as follows.

$$\mathbf{T}: \quad \mathbf{S} \to \mathbf{S}, \quad \mathbf{x_0} \mapsto \mathbf{T}(\mathbf{x_0}). \tag{10}$$

where, $\mathbf{x_0}$ is an initial point on $\mathbf{S}$, while $\mathbf{T}(\mathbf{x_0})$ is the point at which the solution starting from $\mathbf{x_0}$ hits $\mathbf{S}$ again. $\mathbf{T}(\mathbf{x_0})$ can be derived by using the exact solutions. The Jacobian matrix $\mathbf{DT}$ of the Poincaré Map $\mathbf{T}$ can be also derived.

We can calculate the largest Lyapunov exponent by

$$\mu = \lim_{N\to\infty} \frac{1}{N} \sum_{j=0}^{N} \log |\mathbf{DT}_j \cdot e_j|. \qquad (11)$$

here e_j is a normalized base.

4. Circuit Experiments And Computer Calculations

Figure 4 shows the results of circuit experiments and computer calculated results. Circuit experiment results are shown in Fig. 4 (1). Projections of attractors onto x_3 and x_4 and their Poincaré maps are shown in Figs 4 (2) and (3), respectively. Rows of Fig. 4 are corresponding to each others. In circuit experiments, circuit parameters are fixed as R = 74.0[kΩ], C = 0.015[μF], L = 10[mH], R_n = 2.7[kΩ] and R_{vr2} = 380[kΩ]. The Op-Amp is type TL082. Corresponding parameters of computer calculations are fixed as $\alpha = 4$, $\gamma = 3$, $\delta = 0.2$, $\varepsilon = 100$ and $\zeta = 0.5$. Control parameters are set as R_{vr1} and β. By changing control parameters, periodic orbits are observed in Figs. 4 (a), (b) and (f). Chaotic phenomena are observed in Figs. 4 (c) - (e).

One-parameter bifurcation diagram and the calculated largest Lyapunov exponent are shown in Fig. 5 and Fig. 6, respectively. Bifurcation phenomena of periodic orbits, chaos, window and so on are observed. For $4.20 < \beta < 6.80$, the largest Lyapunov exponent becomes positive though two large windows are observed. By using this result, We can say the generation of chaos is confirmed numerically. By changing other parameters, period doubling bifurcation phenomena, tours and so on are also observed in this circuit.

5. Conclusions

In this study, we proposed the chaotic circuit using a ring oscillator and a van der pol oscillator. By circuit experiments, chaotic attractors have been observed and by using a linearized model, the generation of chaos have been confirmed numerically.

The characteristic of this circuit is a suitable for IC implementation and it is easy to implement as a normal electric circuit. This characteristic is useful for investigations of large scale coupled chaotic circuits. Therefore, it is expected that this circuit contributes to studies of large scale coupled chaotic circuits.

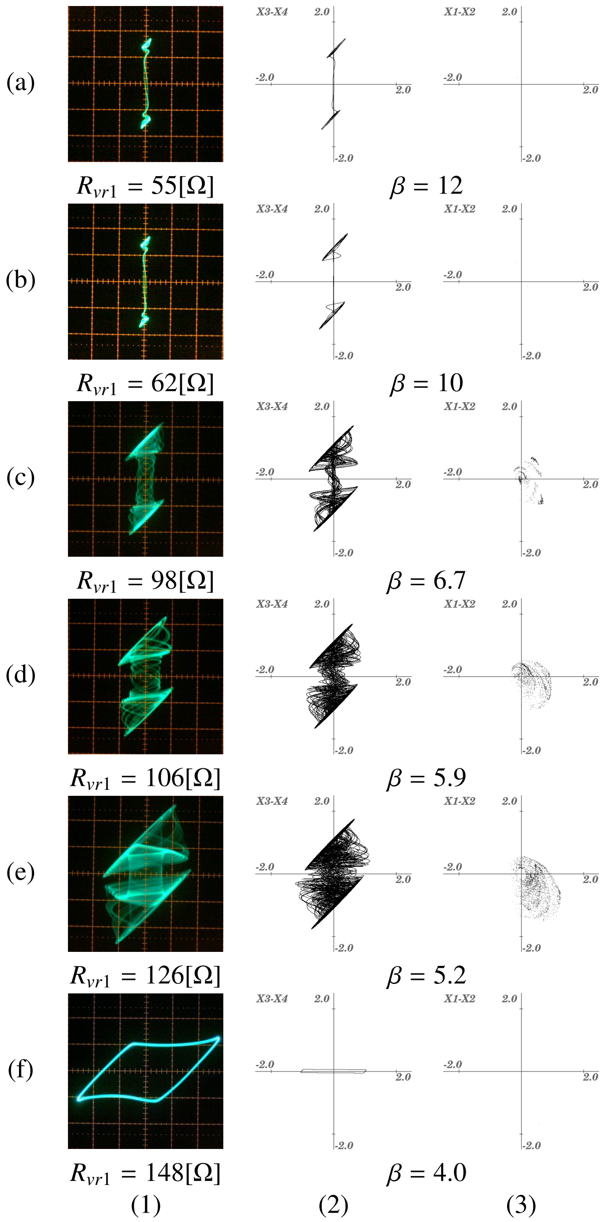

Figure 4: Circuit experiments and Computer calculation results. (1) Circuit experiments. (2) Projections of attractors. (3) Poincaré maps. Circuit experiments: Horizontal axis is v_3 [1.0V/div]. Vertical axis is v_4 [1.0V/div]. R = 74.0[kΩ], C = 0.015[μF], L = 10[mH], R_n = 2.7[kΩ] and R_{vr2} = 380[kΩ]. Computer calculations: $\alpha = 4$, $\gamma = 3$, $\delta = 0.2$, $\varepsilon = 100$ and $\zeta = 0.5$.

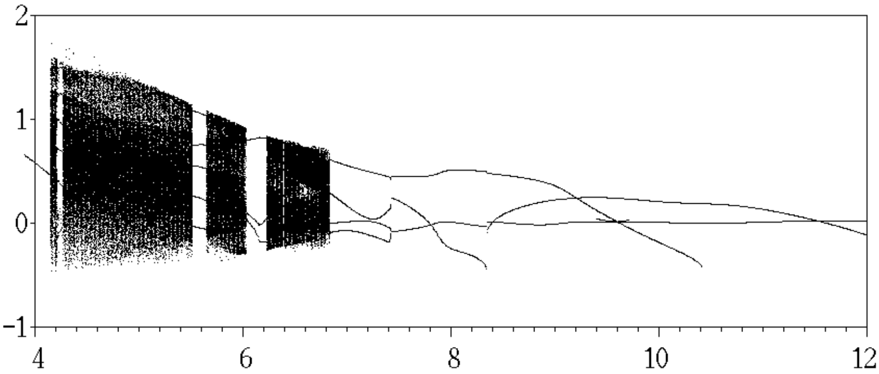

Figure 5: One-parameter bifurcation diagram. Horizontal: β. Vertical: x_1

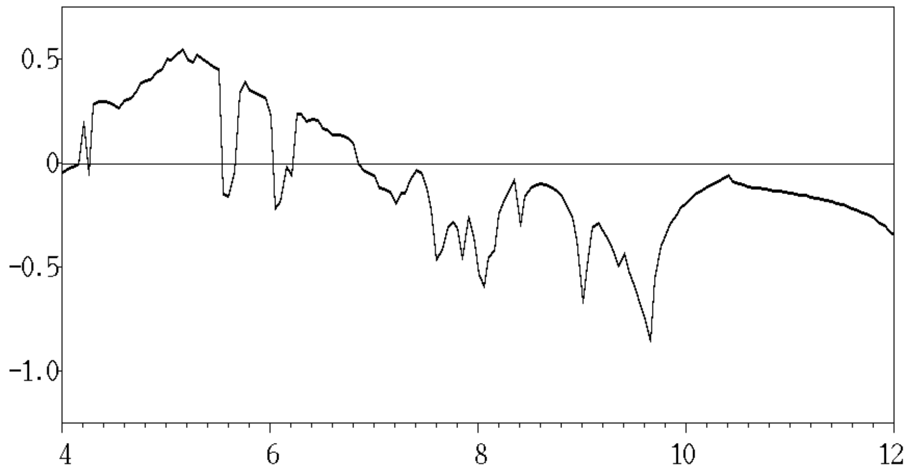

Figure 6: Largest Lyapunov exponents. Horizontal: β. vertical: μ.

References

[1] M. Itoh, "Chaotic Attractors in Nonlinear Circuits," *Tech. Report of IEICE*, Vol.85, No.261, pp.19–24, 1986.

[2] N. Inaba, S. Mori and T. Saito, "Chaotic Phenomena in 3rd Order Autonomous Circuits with a Linear Negative Resistance and an Ideal Diode II," *Tech. Report of IEICE*, Vol.87, No.271, pp.9–15, 1987.

[3] T. Saito, "An Approach Toward Higher Dimensional Hysteresis Chaos Generators," *IEEE Trans. Circuits Syst.*, Vol.37, No.3, pp.399–409, 1990.

[4] K. Mitsubori and T. Saito, "A Four-Dimensional Plus Hysteresis Chaos Generator," *IEEE Trans. Circuits Syst.*, Vol.41, No.12, pp.782–789, 1994.

[5] T. Suzuki and T. Saito, "On Fundamental Bifurcations from a Hysteresis Hyperchaos Generator," *IEEE Trans. Circuits Syst.*, Vol.41, No.12, pp.876–884, 1994.

[6] A. S. Elwakil and M. P. Kennedy, "Systematic Realization of a Class of Hysteresis Chaotic Oscillators," *Proc. NOLTA'99*, Vol. 1, pp.111–114, 1999.

[7] Y. Hosokawa, Y. Nishio and A. Ushida, "Analysis of Chaotic Phenomena in Two Second Order Oscillators Coupled by Diodes," *Proc. NDES2000*, pp.263–267, 2000.

[8] Y. Hosokawa and Y. Nishio, "A Design Method for Chaotic Circuits Using Two Oscillators," *Chaos in Circuits and Systems*, Guanrong Chen and Tetsushi Ueta (Ed.), Chapter 3, pp. 51–69, World Scientific, 2002.

[9] Y. Hosokawa and Y. Nishio, "Simple Chaotic Circuit Using CMOS Ring Oscillators," *Int. J. of Bifurcation and Chaos*, Vol.14, No.7, pp.2513–2524, 2004.

[10] Yasuteru Hosokawa, Yoshifumi Nishio and Akio Ushida, "A Design Method of Chaotic Circuits Using Two Oscillators," *Proc. NOLTA'00*, Vol.1, pp.17–21, 2000.

[11] Yasuteru Hosokawa, Yoshifumi Nishio and Akio Ushida, "A Design Method of Chaotic Circuits Using an Oscillator and a Resonator," *Proc. ISCAS01*, Vol.3, pp.373-376, 2001.