Performance Analysis of Suboptimal Receiver Using Shortest Distance Approximation Method for Chaos Shift Keying

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I. Introduction

Research on digital communications systems using chaos becomes a hot topic [1]– [8]. Especially, it is attracted to develop noncoherent detection systems which do not need to recover basis signals (unmodulated carries) at the receiver. In this study, we focus attention on the optimal receiver which is one of typical noncoherent systems [2]. The optimal receiver performs an optimal detection by using the probability density function (PDF) between the received signals and the same chaotic map of the transmitting side. However, the optimal receiver suffers from a computational complexity due to the large chaotic sequence length. Thus, it is important to develop a receiver with performance equivalent to the optimal receiver using different algorithms, i.e., a suboptimal receiver.

In our previous research, we proposed the suboptimal receiver using the shortest distance approximation [9]. Instead of calculating the PDF, the proposed suboptimal receiver approximates the PDFs by calculating the shortest distance between the received signals and the chaotic map. As results of the computer simulations, we confirmed the validity of the proposed suboptimal receiver as an approximation method of the optimal receiver. However, our previous study did not sufficiently investigate the performance and the computational cost of the suboptimal receiver.

In this study, we observe computational costs of the optimal and our suboptimal receiver to evaluate the accuracy of calculation and the processing time.

II. System Overview

We consider the discrete-time binary CSK communication system, as shown in Fig. 1. This system consists of a transmitter, a channel and a receiver.

In the transmitter, a chaotic sequence is generated by a chaotic map. In this study, the transmitter uses a skew tent map which is one of simple chaotic maps, as shown in Fig. 2(a), and it is described by Eq. (1)

$$x_{k+1} = \begin{cases} \dfrac{2x_k + 1 - a}{1 + a} & (-1 \le x_k \le a) \\[2ex] \dfrac{-2x_k + 1 + a}{1 - a} & (a < x_k \le 1) \end{cases} \tag{1}$$

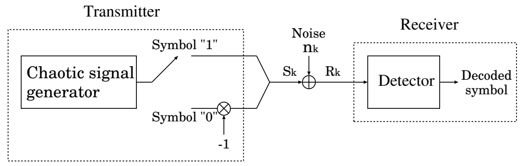
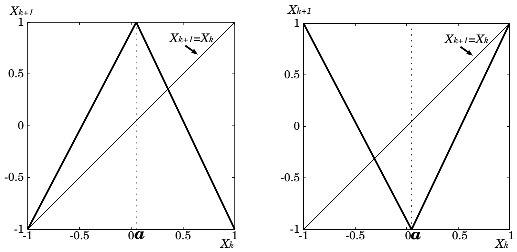


Fig. 1. Block diagram of discrete-time binary CSK communication system.



(a) Symbol “1” (b) Symbol “0”

Fig. 2. Chaos Shift Keying.

where a denotes a position of the top of the skew tent map. The transmitter can generate the different chaotic sequence for every symbol by changing an initial value of chaos. The information symbol is modulated by CSK.

CSK is a digital modulation system using chaos. When the transmitter generates the signals, it is used that chaotic sequences generated by different chaotic maps depending on the value of an information symbol. If the information symbol “1” is sent, Eq. (1) is used (Fig. 2(a)), and if “0” is sent, the reversed function of Eq. (1) is used (Fig. 2(b)). To transmit a 1-bit information, N chaotic signals are generated, where N is chaotic sequence length. Therefore the transmitted signal is denoted by a vector $\mathbf{S} = (S_1\ S_2\ \cdots\ S_N)$.

In the channel, we assume the additive white Gaussian noise (AWGN) with a distribution of mean zero and variance $N_0 = \sigma^2$. AWGN channel is well known as the most popular and basic channel model. Here, the noise signals is denoted by the noise vector $\mathbf{n} = (n_1\ n_2\ \cdots\ n_N)$. Thus, the received signals block is given by $\mathbf{R} = (R_1\ R_2\ \cdots\ R_N) = \mathbf{S} + \mathbf{n}$.

The receiver recovers the transmitted signals from the received signals and demodulates the information symbol. Since we consider a noncoherent receiver, the receiver memorizes the chaotic map used for the modulation at the transmitter. However, the receiver never knows the initial value of chaos and the information symbol in the transmitter. Before explaining our receiving method, we introduce the optimal receiver and the suboptimal receiver.

A. Optimal Receiver

The optimal receiver was proposed by Hasler and Schimming [2]. This receiver selects the symbol q using the *a posteriori* probability when the received signals block $\mathbf{R}$ is received.

$$\tilde{q} = \arg\max_q \text{Prob}(q \text{ is sent} \mid \mathbf{R}) \tag{2}$$

However, since the *a posteriori* probability is not convenient to calculate, we convert them using Bayes' rule, i.e.

$$\text{Prob}(q \text{ is sent} \mid \mathbf{R}) = \frac{p(\mathbf{R}| \; q \text{ is sent}) \times \text{Prob}(q \text{ is sent})}{p(\mathbf{R})} \quad , \tag{3}$$

where $p(.)$ is the probability density function (PDF). Here, Prob("1" is sent) and Prob("0" is sent) are $1/2$. In addition, since $p(\mathbf{R})$ is independent of q, Eq. (2) can be written as

$$\tilde{q} = \arg\max_q p(\mathbf{R}| \; q \text{ is sent}) \quad . \tag{4}$$

As an example, we explain the case of $N = 2$. In this study, it is assumed that the initial value S_1 of each chaotic signal block $\mathbf{S}$ is chosen randomly in accordance with the natural invariant probability density of f and $-f$, where f is the function of the skew tent map. In addition, since the natural invariant probability density of f and $-f$ is the constant $1/2$ in the interval [0, 1], the PDF $p(\mathbf{R}|$ "+1"is sent) and $p(\mathbf{R}|$ "0"is sent) are also equal to $1/2$ and limited to [0, 1], i.e.

$$p(\mathbf{R}| \text{ "1"is sent}) = \frac{1}{2\pi\sigma^2} \int_{-1}^{+1} e^{-\frac{(R_1 - S_1)^2 + (R_2 - f(S_1))^2}{2\sigma^2}} dS_1 \tag{5}$$

$$p(\mathbf{R}| \text{ "0"is sent}) = \frac{1}{2\pi\sigma^2} \int_{-1}^{+1} e^{-\frac{(R_1 - S_1)^2 + (R_2 + f(S_1))^2}{2\sigma^2}} dS_1 \tag{6}$$

where σ^2 is the variance of noise. Finally the optimal receiver decides the decoded symbol as 1 (or 0) for $p(\mathbf{R}|$ "1"is sent) $>$ $p(\mathbf{R}|$ "0"is sent) (or $p(\mathbf{R}|$ "1"is sent) $<$ $p(\mathbf{R}|$ "0"is sent)).

As mentioned above, two error functions need to be computed for two PDFs in the case of $N = 2$. In other words, to decide the decoded symbol, the optimal receiver has to calculate a total of four PDFs. Because the total number of the PDFs is given as 2^N, the implementation of the optimal receiver becomes quite difficult for large N.

To implement the optimal receiving algorithm with large N, the PDF of the optimal receiver is calculated by using

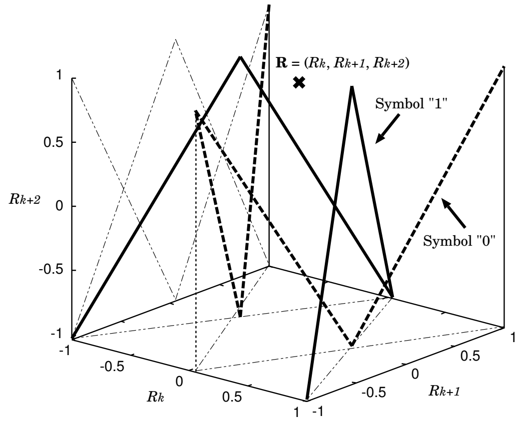


Fig. 3. Proposed detection method.

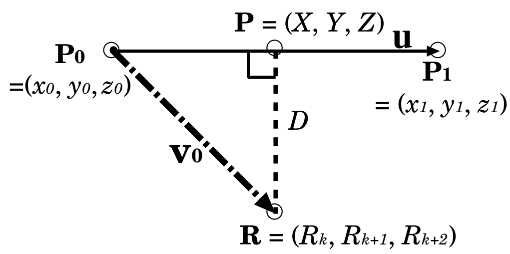


Fig. 4. Calculation of the shortest distance.

the numerical integration method [5]. By applying numerical integration, Eqs. (5) and (6) are rewritten by

$$\begin{aligned} & p(\mathbf{R}| \text{ "1"is sent}) \\ & = \lim_{L\to\infty} \frac{1}{2(2\pi\sigma^2)^{N/2}} \sum_{l=1}^{L} e^{-\frac{\sum_{i=1}^{N} (R_i - f^{(i-1)}(x_l))^2}{2\sigma^2}} \delta x_l \end{aligned} \tag{7}$$

$$\begin{aligned} & p(\mathbf{R}| \text{ "0"is sent}) \\ & = \lim_{L\to\infty} \frac{1}{2(2\pi\sigma^2)^{N/2}} \sum_{l=1}^{L} e^{-\frac{\sum_{i=1}^{N} (R_i - g^{(i-1)}(x_l))^2}{2\sigma^2}} \delta x_l \end{aligned} \tag{8}$$

where $\delta x_l = 2/L$, $x_l = -1 + (k-1)\delta x_k$, $f^{(j)}(\alpha)$ denotes the iteration of the function f and j times with the initial condition α (i.e. $f^{(0)}(\alpha) = \alpha$), $g = -f$. As one can see, the calculation accuracy of the PDF by using the numerical integration method depends on the parameter L. Although the calculation accuracy improves as L increases, the calculation time also increases.

B. Suboptimal Receiver

The suboptimal receiver is a receiving system to implement the optimal receiver using a different algorithm from the optimal receiver. Note that the detection characteristic of the suboptimal receiver can not be superior to the optimal receiver.

III. Shortest Distance Approximation Method

In this section, we explain our suboptimal receiver using a shortest distance approximation [9]. As described in Sec. 2, the core of the optimal receiver's problems is the complexity of the calculation of the PDF due to large N. In other words, to realize a suboptimal receiver, it is necessary to simplify the detection algorithm. In our receiving method, instead of calculating the PDF between received signals and the chaotic map, our suboptimal receiver approximates the PDF by calculating a shortest distance between the received signals and the chaotic map and performs a detection of information. Concretely, the receiver calculates the shortest distance between received signals and the map in the N_d-dimensional space using N_d successive received signals $(N_d : 2, 3, \cdots)$.

As an example, we explain the case of $N_d = 3$. Figure 3 shows the 3-dimensional space of the skew tent map whose coordinates correspond to the three successive received signals $\mathbf{R} = (R_k, R_{k+1}, R_{k+2})$ where $k = 1, 2, \cdots, N-2$. To decide which map is closer to the point $\mathbf{R}$ in the 3-dimensional space in Fig. 3, the shortest distance between the point and the map has to be calculated. Therefore, the receiver can calculate the shortest distance using the scalar product of the vector. Any two points of $\mathbf{P_0} = (x_0, y_0, z_0)$ and $\mathbf{P_1} = (x_1, y_1, z_1)$ are chosen from each straight line in the space of Fig. 3, as shown in Fig. 4.

Using Fig. 4, we can calculate the point with the shortest distance $\mathbf{P} = (X, Y, Z)$ and the shortest distance D by the following equations.

$$\mathbf{P} = (X, Y, Z) = (\mathbf{u} \cdot \mathbf{v_0})\,\mathbf{u} + \mathbf{P_0} \tag{9}$$

$$\begin{aligned} D &= ||\mathbf{P} - \mathbf{R}|| \\ &= \sqrt{(X - R_k)^2 + (Y - R_{k+1})^2 + (Z - R_{k+2})^2} \end{aligned} \tag{10}$$

where

$$\text{Unit vector } \mathbf{u} = \frac{\mathbf{P_1} - \mathbf{P_0}}{||\mathbf{P_1} - \mathbf{P_0}||} \tag{11}$$

$$\mathbf{v_0} = \mathbf{R} - \mathbf{P_0}\ . \tag{12}$$

Note that if the point is outside the cube, we calculate the distance between the point and the nearest edges of the maps.

For the 3-dimensional case, there are four straight lines in the space. Therefore, the minimum value in four distances is chosen as the shortest distance D_1 for symbol "1". In the same way, D of symbol "0" is chosen as D_0. the receiver calculates both of D_1 and D_0 for all k and find their summations $\sum D_1$ and $\sum D_0$. Finally, we decide the decoded symbol as 1 (or 0) for $\sum D_1 < \sum D_0$ (or $\sum D_1 > \sum D_0$).

The calculation of the shortest distance can be extended to N_d-dimensional space for $N_d \geq 4$.

IV. Investigation of Computing Cost

To investigate computational costs of the optimal and our suboptimal receiver, we carry out computer simulations with

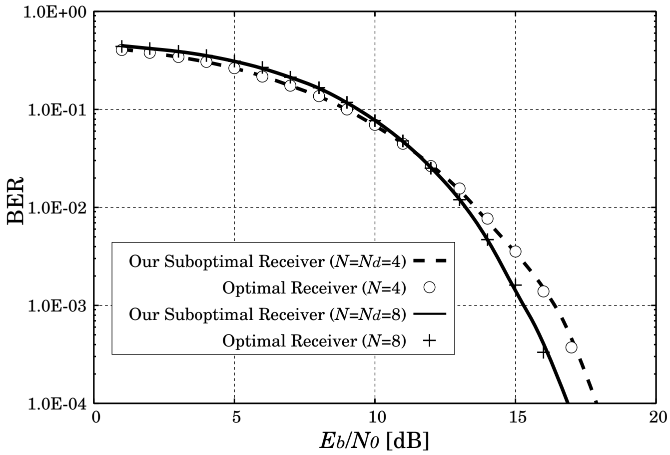


Fig. 5. BER performance (Comparison of optimal receiver and our suboptimal receiver).

following simulation conditions. On the transmitting side, 100,000 symbols are transmitted using chaotic sequences with different initial values. Here, the parameter of the skew tent map is fixed as $a = 0.05$. Also, as the chaotic sequence length, $N = 4$ and 8 are used. The receiver calculates a BER performance under AWGN and a computational time spent to decide all decoded symbols. To compare the performances of different dimensional spaces to calculate the shortest distance, we use 4-dimensional space (4-D) and 8-dimensional space (8-D). Also, to compare the performance, the optimal receiver is changed L from 2^{N-1} and is carried out the simulation. These simulations are carried out using PC implemented CPU: Core2Duo 2.4GHz, RAM2GB.

First of all, to confirm the validity of the our suboptimal receiver as an approximation method of the optimal receiver, we show the BER comparison of the optimal and our suboptimal receiver in Fig 5. Here, to simulate the optimal receiver, we apply the numerical integration method (Eqs. (7) and (8)) with $L = 2000$. It can be observed that the curve of the our suboptimal receiver corresponds that of the optimal receiver when $N = N_d$-dimension. From this result, we consider that our method using the shortest distance in N_d-dimensional space becomes almost identical with the optimal receiver for the case that the dimension N_d is equal to the length of the chaotic sequence N.

Next, we compare a computational cost of our method with that of the optimal receiver. Tables I and II show the BER performance and the actual computational time. Here, E_b/N_0 is fixed as 13dB. From these tables, we can see that the BER performances of the optimal receiver and our method are almost the same when $L = 3 \times 2^{N-1}$. Thus, to obtain the calculation accuracy of the optimal receiver, the minimum required intervals of L needs $3 \times 2^{N-1}$. In addition, the computational time of the optimal receiver with the minimum required interval spends $4 \sim 6$ times of that of our method. From these results, we expect that the computational cost of our method is lower than that of the optimal receiver using the numerical integration method, even if the accuracy of the

TABLE I

COMPUTATIONAL TIME AND BER PERFORMANCE WITH $N = N_d = 4$ (10^6 SYMBOLS, $E_b/N_0 = 13$DB)

	Optimal Receiver			Our Suboptimal Receiver
	L			Number of Straight Line
	$2^{N-1} = 8$	$2 \times 2^{N-1} = 16$	$3 \times 2^{N-1} = 32$	$2^{N_d-1} = 8$
Computational Time [sec]	1.04	1.72	3.23	0.74
BER	1.18×10^{-1}	2.82×10^{-2}	1.61×10^{-2}	1.57×10^{-2}

TABLE II

COMPUTATIONAL TIME AND BER PERFORMANCE WITH $N = N_d = 8$ (10^6 SYMBOLS, $E_b/N_0 = 13$DB)

	Optimal Receiver			Our Suboptimal Receiver
	L			Number of Straight Line
	$2^{N-1} = 128$	$2 \times 2^{N-1} = 256$	$3 \times 2^{N-1} = 512$	$2^{N_d-1} = 128$
Computational Time [sec]	21.06	44.23	90.85	14.28
BER	1.74×10^{-2}	1.26×10^{-2}	1.15×10^{-2}	1.15×10^{-2}

optimal receiver with $N = 16$ and 32 is obtained by increasing the computer performance. Therefore, it can be said that the lower computational cost is advantage of our method.

V. CONCLUSIONS

In this study, to evaluate the accuracy of calculation and the processing time, we have observed computational costs of the optimal and our suboptimal receiver As the simulation results, we have confirmed that the lower computational cost is advantage of our method.

REFERENCES

[1] G. Kolumbán, B. Vizvári, W. Schwarz, and A. Abel, "Differential chaos shift keying: A robust coding for chaos communication," *Proc. NDES'96*, pp. 87-92, Jun. 1996.

[2] M. Hasler and T. Schimming, "Chaos communication over noisy channels," Int . Journal of Bifurcation and Chaos, vol. 10, no. 4, pp. 719-736, 2000.

[3] M. Sushchik, L. S. Tsimring and A. R. Volkovskii, "Performance Analysis of Correlation-Based Communication Schemes Utilizing Chaos," *IEEE Trans. Circuits and Systems Part I*, Vol. 47, pp 1684-1691, Dec. 2000.

[4] F. C. M. Lau and C. K. Tse, "Approximate-optimal detector for chaos communication systems," Int. Journal of Bifurcation and Chaos, vol. 13, no. 5, pp. 1329-1335, 2003.

[5] F. C. M. Lau and C. K. Tse, "On optimal detection of noncoherent chaos-shift-keying signals in a noisy environment," Int. Journal of Bifurcation and Chaos, vol. 13, no. 6, pp. 1587-1597, 2003.

[6] F. C. M. Lau and C. K. Tse, *Chaos-Based Digital Communication Systems*, Springer, 2003.

[7] W. M. Tam, F. C. M. Lau and C. K. Tse, "Generalized correlation-delay-Shift-Keying Scheme for Noncoherent Chaos-Based Communication Systems," *IEEE Trans. Circuits and Systems Part I*, vol. 53, no. 3, pp. 712-721, Mar. 2006.

[8] L. E. Larson, J-M. Liu, L. S. Tsimring, *Digital Communications Using Chaos and Nonlinear Dynamics*, Springer, 2006.

[9] S. Arai and Y. Nishio, "Detection of Information Symbols and Sequence Lengths Using Suboptimal Receiver for Chaos Shift Keying," *Proc NOLTA'06*, pp. 799-802, Sep. 2006.