Synchronization Patterns Generated in a Ring of Cross-Coupled Chaotic Circuits

Yumiko Uchitani and Yoshifumi Nishio

***Abstract*— Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields. In this study, synchronization patterns generated in a ring of cross-coupled chaotic circuits are investigated. Computer simulations show that this coupled system produces several phase patterns.**

I. Introduction

MANY people have been trying to develop some applications to information processing by exploiting oscillatory phenomena in neural networks. Such neural networks can produce some kinds of phase patterns, and they may be utilized for associative memory or image processing [1]-[4].

On the other hand, since synchronization phenomena in coupled oscillatory systems are good models to describe various higher-dimensional nonlinear phenomena in the field of natural science, studies on synchronization phenomena are extensively carried out in various fields [5]-[13].

In our past studies [14][15], we investigated an interesting state transition phenomenon observed in simple coupled chaotic circuits. In this study, we investigate the phase patterns characterized by synchronization in a ring of cross-coupled chaotic circuits. Computer simulations show that this coupled system produces several phase patterns.

II. Basic Circuit [14][15]

In this section, we review the phenomena observed from simple two cross-coupled chaotic circuits. Figure 1 shows the basic circuit model. In this model, two simple autonomous chaotic circuits [16][17] are cross-coupled via inductors L_2.

By using the following variables and the parameters,

$$\begin{cases} x_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{k1}}{V}, \quad w_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{k2}}{V}, \\ y_k = \dfrac{v_{k1}}{V}, \quad z_k = \dfrac{v_{k2}}{V}, \quad t = \sqrt{L_1 C_2}\,\tau, \\ \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\,G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}\,g, \\ \delta = \dfrac{L_1}{L_2}, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau} \qquad (k = 1, 2) \end{cases} \tag{1}$$

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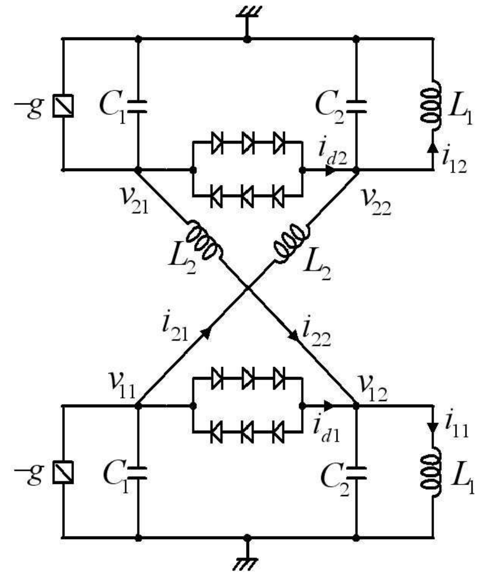

Fig. 1. Basic circuit model.

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k = z_k \\ \dot{y}_k = \alpha\{\gamma y_k - w_k - \beta f\,(y_k - z_k)\} \\ \dot{z}_k = \beta\, f(y_k - z_k) + w_{k+1} - x_k \\ \dot{w}_k = \delta(y_k - z_{k+1}) \\ (k = 1, 2), \end{cases} \tag{2}$$

where z_3 indicates z_1 and f are nonlinear functions corresponding to the $v - i$ characteristics of the nonlinear resistors of the diodes and are described as follows.

$$f(y_k - z_k) = \begin{cases} y_k - z_k - 1 & (y_k - z_k > 1) \\ 0 & (|y_k - z_k| \le 1) \\ y_k - z_k + 1 & (y_k - z_k < -1) \\ (k = 1, 2). \end{cases} \tag{3}$$

A typical example of the observed phenomena is shown in Fig. 2. Figure 2(a) is computer simulated results obtained by integrating Eq. (2) with the Runge-Kutta method and Fig. 2(b) is the corresponding circuit experimental results. In this state, the two circuits exhibited chaos but almost synchronized in in-phase in the sense that the attractor was almost in the quadrant I or III on the $y_1 - y_2$ (or $v_{11} - v_{21}$) plane. The behaviors of the circuits are very interesting

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because the solutions on the $y_i - z_i$ planes seem to be attracted to the fixed points located at around (y_i, z_i)=($\pm$1.2, 0). However, after converging to the fixed points, the solution abruptly moves toward the other fixed point. When one circuit switches to/from the positive region from/to the negative region in this way, the other follows the transition after a few instants.

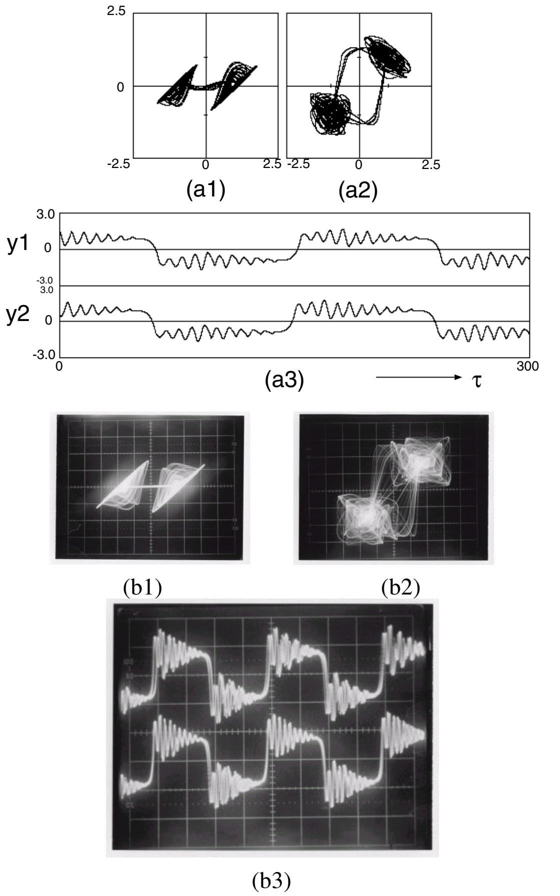

Fig. 2. State transition phenomenon around in-phase synchronization. (a) Computer calculated results. $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.005$. (b) Circuit experimental results. L_1 = 9.93mH, L_2 = 800mH, C_1=32.8nF, and C_2=49.5nF, and g=683mS. (a1) Attractor on $y_1 - z_1$ plane. (a2) Attractor on $y_1 - y_2$ plane. (a3) Time waveform. (b1) Attractor on $v_{11} - v_{12}$ plane. Horizontal and vertical: 1 V/div. (b2) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (b3) Time waveform v_{11} and v_{21}. Horizontal 1.0 ms/div and vertical: 2 V/div.

Similar transition phenomena can be observed around anti-phase synchronization and quadrature-phase synchronization as shown in Fig. 3 and Fig. 4, respectively.

III. Ring of Cross-Coupled Chaotic Circuits

Figure 5 shows the circuit model investigated in this study. In the circuit, n chaotic circuits are cross-coupled with their neighbors via inductors L_2. Note that the connections do not look like crossed but are similar to the basic circuit in Fig. 1.

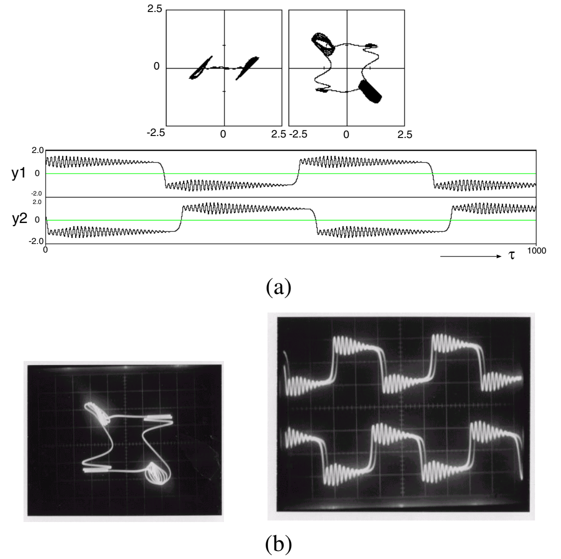

Fig. 3. State transition phenomenon around anti-phase synchronization. (a) Computer calculated results. $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$,and $\delta = 0.0008$. (b) Circuit experimental results. L_1 = 9.93mH, L_2 = 1.2H, C_1=32.8nF, C_2=49.5nF, and g=495mS.

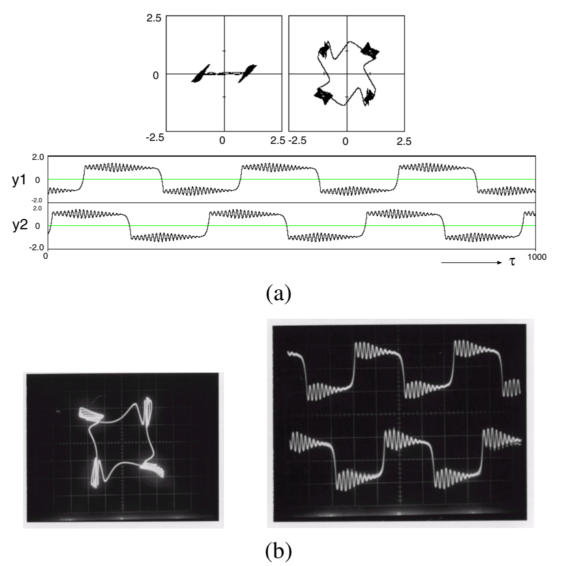

Fig. 4. State transition phenomenon around quadrature-phase synchronization. (a) Computer calculated results. $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$,and $\delta = 0.0014$. (b) Circuit experimental result. L_1 = 9.93mH, L_2 = 1.2H, C_1=32.8nF, C_2=49.5nF, and g=495mS.

Namely a node of each circuit is connected to its neighbor's different node. Hence, we still call this connections as cross-couplings.

As well as the case of the two circuits, we approximate the $v - i$ characteristics of the nonlinear resistors consisting of the diodes by the following 3-segment piecewise-linear

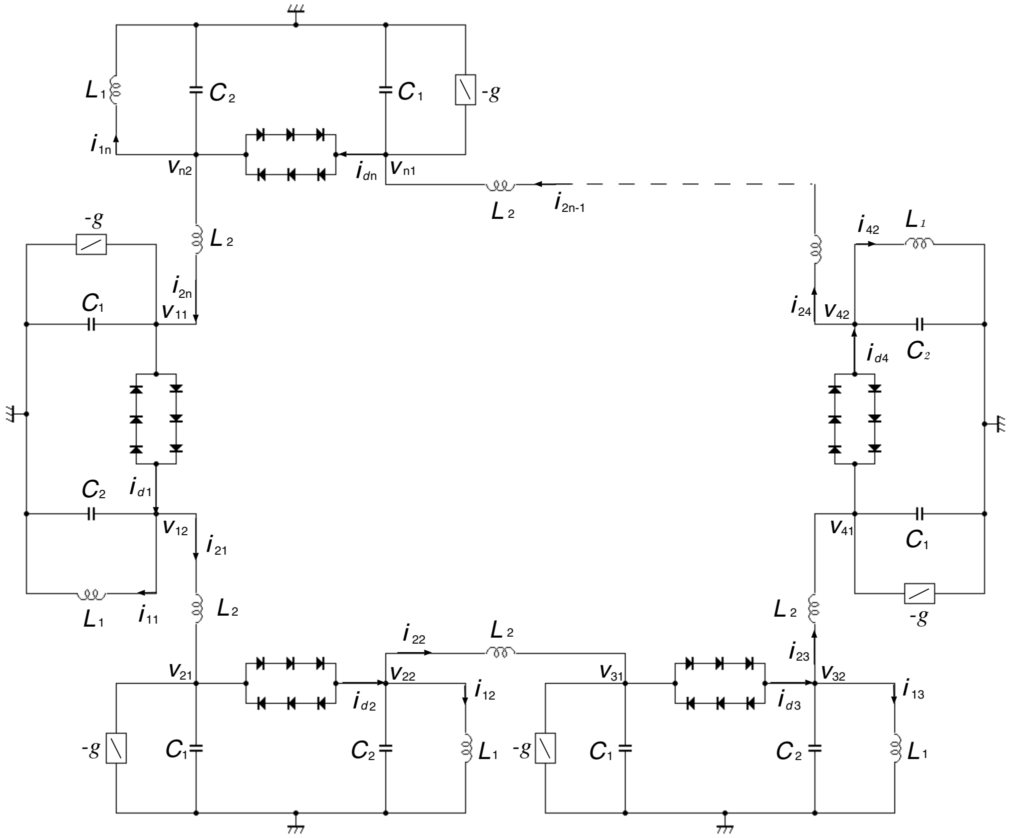

Fig. 5. Ring of cross-coupled chaotic circuits.

functions.

$$i_{dk} = \begin{cases} G(v_{k1} - v_{k2} - V) & (v_{k1} - v_{k2} > V) \\ 0 & (|v_{k1} - v_{k2}| \leq V) \\ G(v_{k1} - v_{k2} + V) & (v_{k1} - v_{k2} < -V) \\ (k = 1, 2, 3, ..., n) & \end{cases} \quad (4)$$

The circuit equations are described as follows.

$$\begin{cases} L_1 \dfrac{di_{1k}}{dt} = v_{k2} \\ C_1 \dfrac{dv_{k1}}{dt} = -i_{dk} + i_{2(k-1)} + g v_{k1} \\ C_2 \dfrac{dv_{k2}}{dt} = i_{dk} - i_{2k} - i_{1k} \\ L_2 \dfrac{di_{2k}}{dt} = v_{k2} - v_{(k+1)1} \\ (k = 1, 2, 3, ..., n) \end{cases} \quad (5)$$

where i_{20} and $v_{(n+1)1}$ indicate i_{2n} and v_{11} due to the ring boundary condition, respectively. By using the same variables and parameters as Eq. (1), the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k = z_k \\ \dot{y}_k = \alpha\{\gamma y_k - w_{k-1} - \beta f\,(y_k - z_k)\} \\ \dot{z}_k = \beta\, f(y_k - z_k) + w_k - x_k \\ \dot{w}_k = \delta(z_k - y_{k+1}) \\ (k = 1, 2, 3, ..., n), \end{cases} \quad (6)$$

where y_{n+1} indicates y_1 and f are the nonlinear functions described by Eq. (3).

IV. Synchronization Patterns

The most common synchronization state observed from the ring in Fig. 5 is the fully in-phase synchronization. Figure 6 shows an example of the fully in-phase synchronization states observed from 5 cross-coupled chaotic circuits. In this state, the movement of all y_i from/to the positive region to/from the negative region almost synchronizes as shown in Fig. 6(g). We consider that a movement of one circuit triggers the neighbors' transition like as the two-circuit case. However, after the transient states we cannot distinguish the starting circuit of the ring.

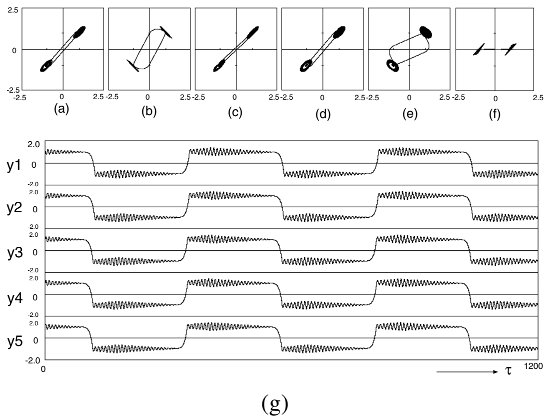

Fig. 6. Fully in-phase synchronization of state transition (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.001$. (a) Attractor on $y_1 - y_2$ plane. (b) Attractor on $y_2 - y_3$ plane. (c) Attractor on $y_3 - y_4$ plane. (d) Attractor on $y_4 - y_5$ plane. (e) Attractor on $y_5 - y_1$ plane. (f) Attractor on $y_1 - z_1$ plane. (g) Time waveform.

By changing initial conditions, we can observe other types of synchronization states. Figure 7 shows three different kinds of synchronization states observe from 5 cross-coupled chaotic circuits. Because the adjacent two circuits can be synchronized in anti-phase as shown in Fig. 3, the phase states of some circuits can be π with respect to the reference circuit (circuit #1). Figure 7(a) shows the state that only one of 5 circuits has π phase difference to the others. This state can be written as $[0, \pi, \pi, \pi, \pi]$ Figure 7(b) shows the state that the adjacent two circuits has π phase difference to the other 3 circuits. This state can be written as $[0, 0, \pi, \pi, \pi]$ Figure 7(c) shows the state that the second and the fourth circuits has π phase difference to the other 3 circuits. This state can be written as $[0, \pi, 0, \pi, 0]$ We can expect much larger number of combinations of phase states for larger n.

Figure 8 shows how the characteristics of the synchronization state $[0, \pi, \pi, \pi, \pi]$ change as the coupling parameter δ increases. The horizontal axis is δ and the vertical axis is the average length of the period of the transitions between positive and negative. The curve of crosses shows the average period of the state transitions of y_1. The curve of circles shows the average delay time of the state transitions of y_5 when the state transitions of $y1$ are considered to be the reference. We can see that the period decreases monotonically according to the increase of δ. Further, for δ larger than about 0.0021, the state $[0, \pi, \pi, \pi, \pi]$ becomes unstable and only the fully in-phase synchronization state can be observed. The investigation of exact bifurcation scenario is our future research subject.

Because the two cross-coupled circuits can generate the quadrature-phase synchronization as shown in Fig. 4, we can expect some synchronization states including $\pi/2$ phase differences. Figure 9 shows two examples of such synchronization states. Figure 9(1) shows the state that 4 circuits have $\pi/2$ phase difference to the reference circuit and hence

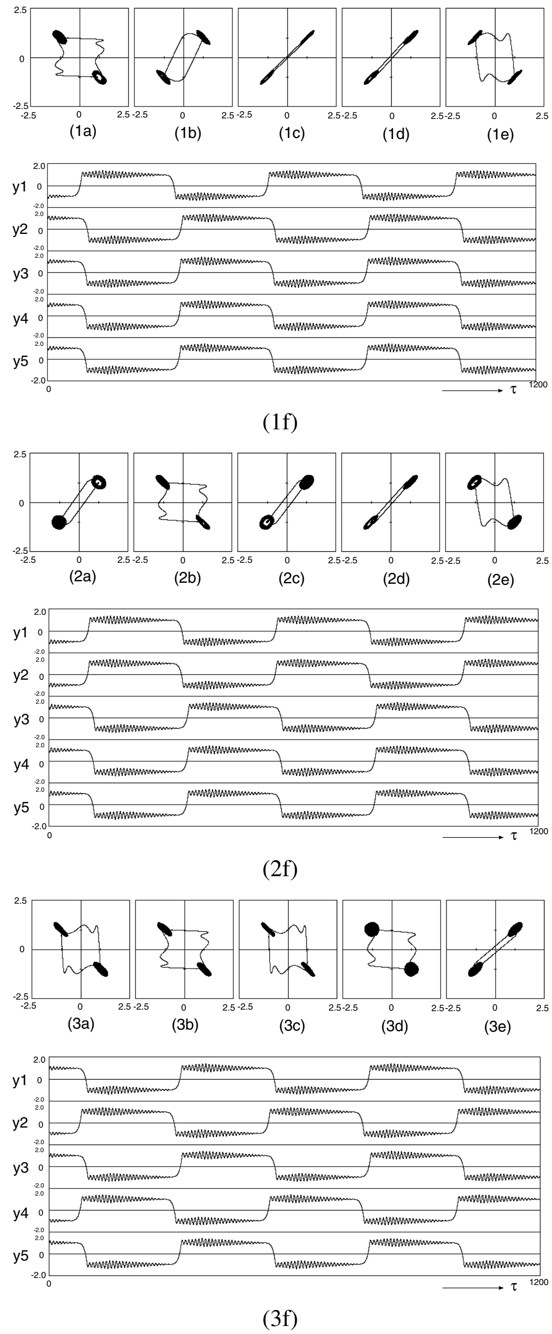

Fig. 7. Several synchronization states including anti-phase synchronizations. (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.001$. (1) [0,π,π,π,π] (2) [0,0,π,π,π] (3) [0,π,0,π,0] (a) Attractor on $y_1 - y_2$ plane. (b) Attractor on $y_2 - y_3$ plane. (c) Attractor on $y_3 - y_4$ plane. (d) Attractor on $y_4 - y_5$ plane. (e) Attractor on $y_5 - y_1$ plane. (f) Time waveform.

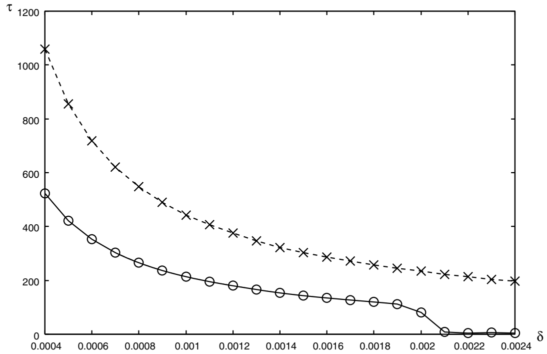

Fig. 8. Average period of the transitions of synchronization states $[0, \pi, \pi, \pi, \pi]$. $\alpha = 2.0$, $\beta = 4.0$, and $\gamma = 0.1$.

this state can be written as $[0, \pi/2, \pi/2, \pi/2, \pi/2]$. In this state, the solutions on $y_1 - y_2$ (or $y_5 - y_1$) plane move to the quadrant I, II, III, and IV (or IV, III, II, and I) in this order as shown in Fig. 9(1a) (or Fig. 9(1e)). While, Fig. 9(2) shows the state that only the first and the fifth circuits have no phase difference and hence this state can be written as $[0, \pi/2, 0, \pi/2, 0]$. In this state, the solutions on all $y_i - y_{i+1}$ planes except $y_5 - y_1$ plane move to the quadrant I, II, III, and IV (or IV, III, II, and I) as shown in Figs. 9(2a)-(2e).

sectionLarge Number of Coupled Circuits

Finally, in this section, some computer simulated results for the case of 20 circuits are introduced. Figure 10 shows two examples of the synchronization states obtained by giving different initial conditions. As we can expect, a huge number of combinations of phase states can be generated. Classifications and stability analysis of these phase states are our future research.

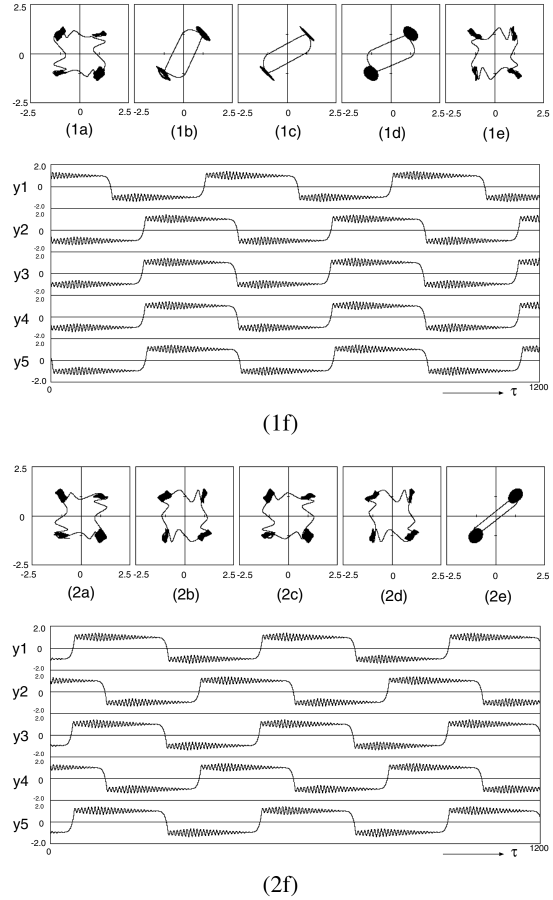

Fig. 9. Two examples of synchronization states including quadrature-phase synchronizations (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.001$. (1) [0,π/2,π/2,π/2,π/2]. (2) [0,π/2,0,π/2,0]. (a) Attractor on $y_1 - y_2$ plane. (b) Attractor on $y_2 - y_3$ plane. (c) Attractor on $y_3 - y_4$ plane. (d) Attractor on $y_4 - y_5$ plane. (e) Attractor on $y_5 - y_1$ plane. (f) Time waveform.

V. Conclusions

In this study, we have investigate the phase patterns characterized by the synchronization states of the interesting state transitions in a ring of cross-coupled chaotic circuits. We confirmed that several patterns could be observed by giving different initial conditions to the circuits. Our future research includes the investigation of the larger size circuits, the clarification of the generation mechanism of the transition, and the application to signal processing using the proposed circuit.

Acknowledgments

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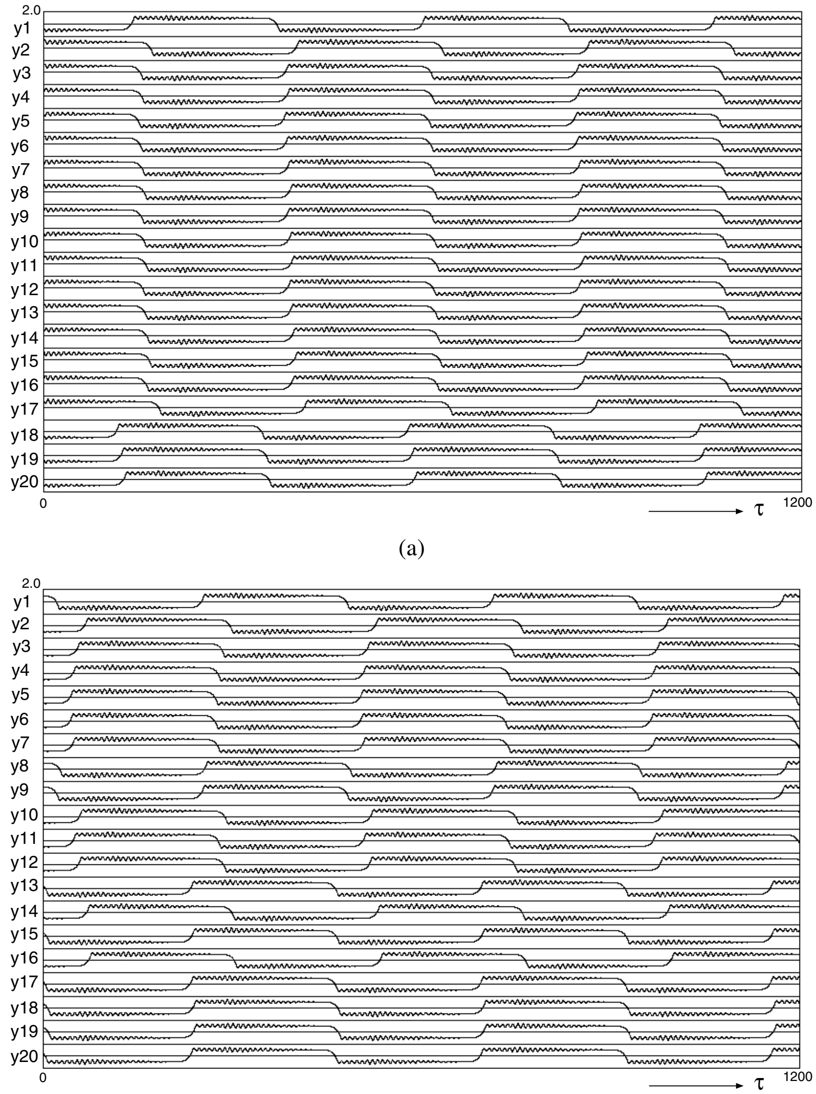

Fig. 10. Two examples of synchronization states observed from 20 cross-coupled circuits (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.001$.

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